A Problem

1. It is very well known that all terms of a physical expression must have the same dimensions (otherwise, a change in the unit system destroys numerical balance). Now, let $F(q)$ be an analytic function used in a description of a physical relation. If the power series of $F(q)$ contains more than one term (e.g. $aq^m + bq^n$, where a and b are nonzero pure numbers and $m \neq n$), then it is required that q be dimensionless. Thus, for example, the exponential factor used in the Maxwell-Boltzmann distribution takes the form $e^{-E/KT}$ and the product KT has the dimensions of energy. If $F(q)$ belongs to a relativistic expression then covariance arguments prove that q must also be a Lorentz scalar. The wave function's phase $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ satisfies the two requirements. As is very well known, a student who violates these laws fails in exams.

2. Let $\Phi(x^\mu)$ be a gauge function and its 4-derivative $\Phi(x^\mu)_{,\nu}$ is subtracted from a 4-potential in a gauge transformation. In quantum mechanics, the charged particle's sector contains the gauge dependent factor $e^{ie\Phi(x^\mu)}$ (see [1], p. 78). Note that the symbol e in the exponent denotes the particle's electric charge. Now, in the system of units used here the electric charge is a pure number $e^2 \simeq 1/137$. Thus, the analytic properties of the exponential function and the laws described in the previous paragraph prove that in quantum mechanics, the gauge function $\Phi(x^\mu)$ must be a dimensionless Lorentz scalar. As of today, this restriction is not implemented and the standard gauge transformation used in the literature regards $\Phi(x^\mu)$ as a free function of space-time coordinates (see [2], p. 52; [1], p. 78). This example provides a reason for the need of an investigation of the role of gauge transformations in electrodynamics.

References:

[1] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Mass., 1995).

[2] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Elsevier, Amsterdam, 2005).