The Origin of Contradictory Statements in QED Textbooks

Two well-known textbooks make contradictory statements about relativistic properties of the electromagnetic 4-potential A_μ. One textbook says: "In short,

$$A_\mu = (\Phi, \mathbf{A}) \tag{16.2}$$

is a four-vector. What we call the scalar and vector potentials are really different aspects of the same physical thing. They belong together. And if they are kept together the relativistic invariance of the world is obvious. We call A_μ the *four-potential"* (see [1], p. 25-8). By contrast, another textbook examines the 4-potential of a photon and states that "the fact that A^0 vanishes in all Lorentz frames shows vividly that A^μ cannot be a four-vector" (see [2], p. 251).

Physics is regarded as an exact science that has a consistent mathematical structure. Therefore, the above mentioned contradictory statements deserve an adequate mathematical analysis. QFT theories rely on a Lagrangian density of the following form

$$\mathcal{L}(\psi(x), \psi(x)_{,\mu}). \tag{1}$$

Here $x \equiv (t, \mathbf{x})$ denotes the four local space-time coordinates. This Lagrangian density is a Lorentz scalar. QFT textbooks support this approach: "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [2], p. 300).

The present form of the QED Lagrangian density of a charged lepton and electromagnetic fields is the primary QED expression. Its standard form is (see [3], p. 84, [4], p. 78)

$$\mathcal{L} = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}. \tag{2}$$

This Lagrangian density shows that the electromagnetic 4-potential A_μ is an important QED component. Like (1), the variables of this Lagrangian density depend on local space-time coordinates $(t, \mathbf{x})$.

The Lienard-Wiechert expression for the 4-potential of a charge e is

$$A^{\mu}(x) = e\frac{v^{\mu}}{R_{\alpha}v^{\alpha}}. \tag{3}$$

Here v^{μ} is the retarded 4-velocity of the charge e and $R^{\mu} = x^{\mu} - x^{\mu}_{ret}$ (see [5] , p. 174). It means that the correct expression for the 4-potential A^{μ} depends on local coordinates *and* on retarded coordinates as well. In particular, the electromagnetic fields that are derived from this 4-potential depend olso on differentiation of the retarded coordinates (see [5] , p. 175). Hence, the correct expression of the 4-potential is inconsistent with the locality requirement of the Lagrangian density (1) and of the QED Lagrangian density (2).

This is an inherent contradiction of the present structure of QED. For a more detailed discussion, see [6, 7].

References

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