Financial Risk and Unemployment

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Abstract
There is a strong correlation between corporate interest rates, their spreads relative to Treasuries, and the unemployment rate. We model how corporate interest rates affect equilibrium unemployment and vacancies, in a Diamond-Mortensen-Pissarides search and matching model with capital. Our simple model permits the exploration of US business cycle statistics through the lens of financial shocks. We calibrate the model using US data without targeting business cycle statistics. Volatility in the corporate interest rate can explain a quantitatively meaningful portion of the volatilities of unemployment, vacancies, and market tightness. The strength of model mechanisms is roughly linear in the proportion of capital assumed to be subject to financial shocks. Panel data on corporate firms support the hypothesis that firms facing more volatile financial conditions have more volatile employment.

JEL Classification: E22, E24, E32, E44, J41, J63, J64

Keywords: Equilibrium Unemployment, Search and Matching Models, Business Cycles, Corporate Interest Rates, Interest Rate Spread

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1 Introduction

We document a strong correlation between corporate financial conditions, as measured by the Baa interest rate ($r$) and their spread relative to treasuries, and unemployment ($u$) or vacancies ($v$).\(^1\) Baa interest rates rise during recessions even as Treasury rates and the Federal Funds rate decline, reflecting a countercyclical interest rate spread. Table 1 shows that the corporate interest rate and spread are very volatile, significantly more so than productivity, and have high cross-correlations with unemployment.\(^2\) Our main question is: how important are corporate financial conditions for understanding the volatility of unemployment and vacancies?

Table 1: Summary Statistics of the US Quarterly Time Series Data, 1982–2012

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$r$</th>
<th>Spread</th>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.11</td>
<td>0.12</td>
<td>0.14</td>
<td>0.35</td>
<td>0.01</td>
</tr>
<tr>
<td>Correlation with $u$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>two-quarter lags</td>
<td>0.26</td>
<td>0.71</td>
<td>-0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>contemporaneous</td>
<td>0.32</td>
<td>0.62</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table reports statistics from quarterly US time series data, in HP-log deviations with a smoothing parameter of 1600. The values for the standard deviation of the HP deviations, rather than HP-log deviations, of the interest rate and spread are 0.8% and 1.0%, respectively. The cross correlations (lagged) with unemployment rates is 0.21 (0.17) for the interest rate and 0.60 (0.73) for the spread. Nominal interest rates are converted to real interest rates use realized changes in the core PPI (producer price index). See Appendix A for data definitions.

We address our question using the classic Search and Matching model, as in the Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP) model with capital. Each firm is either vacant or matched with one worker. Firms borrow from banks in order to finance their capital, which is used either by workers or in a vacancy.\(^3\) The firms pay a corporate interest rate and

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\(^1\)Baa is a credit rating of corporate default risk. For the US treasury we use the 5-Year Constant Maturity Rate. We frequently refer to the corporate interest rate as the interest rate, and to the corporate interest rate spread relative to the 5 year treasury interest rate as the spread. We always use real interest rates, under a variety of definitions of inflation. The interest rate spread is unaffected by inflation.

\(^2\)We choose the time period 1982–2012 as Gali and van Rens (2014) claim that during this time, labor productivity became less procyclical, opening the door for other mechanisms to be explored. Jermann and Quadrini (2012) use a similar time period in their analysis of financial shocks and the macroeconomy. We include both lagged and contemporaneous values here for completeness. During the rest of the paper, unless otherwise specified, we use only contemporaneous values.

\(^3\)The assumption that firms use debt to finance investment and rental of capital is common in many studies discussed
cover the depreciation costs of capital. Banks borrow from workers (depositors) and experience a financial intermediation cost while lending to firms. We study exogenous shocks to those costs, which represent monitoring costs, changing default risk and recovery rates, intermediation costs, uncertainty shocks, or other shocks that would affect the interest rate perceived by firms. Free entry of banks pins down the corporate interest rate to be the rate paid to workers plus financial intermediation costs. Workers are risk neutral, are either employed or unemployed, and make a consumption/savings choice. Matches between unemployed workers and vacant firms occur in a frictional matching market. Wages are determined by Nash bargaining based on the dynamic value functions of firms and workers. We solve for a closed-form solution for the equilibrium market tightness ($\theta = \frac{v}{u}$) of the model given the stochastic process of interest rates, which allows us to solve for the general equilibrium of the model, including unemployment, vacancies, and wages.

Interest rates affect firm profits directly by influencing capital costs and thus the incentive to hire. We call this the flow profit channel. The interest rate shock also affects vacancy posting costs as we assume that vacancies require the capital a worker would use to be available. We call this the vacancy cost channel. We follow much of the literature in assuming exogenous and constant separations between workers and firms.

We calibrate the model to US data to match vacancy costs, average job-finding rates, and average labor market tightness. We use the observed Baa interest rate to discipline financial shocks. In our calibration, we do not target business cycle statistics related to unemployment, vacancies, or market tightness in setting parameter values. The main result of our paper is that a simple model with empirically disciplined financial shocks can go a long way towards explaining observed unemployment, vacancies, market tightness and other business cycle volatility.

Quantitatively, our benchmark calibration generates about 60-70% of observed labor market volatility in the sample period of 1982–2012. About 60% of model volatility is explained by the flow profit channel, while the other 40% comes from the vacancy cost channel. Delving deeper into the model (Section 4) shows that our results depend on the assumption that all capital is debt financed, whereas as in the data debt is roughly 40% to 60% of capital. If we take the extreme below, including Jermann and Quadrini (2012), which is closely related to our paper. More recently, Bigio (2015) builds a model in which capital is financed under both limited enforcement contracts and asymmetric information, and applies it to the Great Recession. In a different context, Bocola and Lorenzoni (2017) model the decisions of a central bank with self-fulfilling financial crises. Banks intermediate capital, with the central bank acting as a lender of last resort in multiple currencies.

4See, for instance, Shimer (2005) and Hagedorn and Manovskii (2008). See Shimer (2012) for an empirical study that shows that the job-finding probability accounts for 77% of fluctuations in the unemployment rate since 1948, rising to 90% after 1987.
assumption that the non-debt portion of capital, such as equity, is not subject to cost fluctuations, then the fraction of observed labor market volatility generated by the model is roughly linear in the fraction of capital assumed to be subject to financial shocks. However, if the cost of capital that is not financed by debt moves closely with the Baa interest rate, the results would be roughly equivalent to the results of our benchmark model.\(^5\)

The model generates a correlation between unemployment and financial conditions that is large relative to the data, which we discuss in the context of the DMP literature. Simulating the time series of the model shows that financial shocks are important for the first two recessions of the 2000s but that the model is not consistent with previous recessions. We additionally compare the model implications for investment volatility with empirical counterparts.

Beyond assumptions about debt financing, model results are robust to a wide variety of different parameter values, measures of the real interest rate, and other modeling assumptions such as the assumptions on the production function. Replacing financial shocks with productivity shocks in our benchmark calibration results in negligible unemployment and vacancies volatility. This finding is similar to that found in Shimer (2005), which launched a large literature on whether the DMP model with productivity shocks can create realistic business cycle volatility.\(^6\) We confirm that labor market volatility in the model is due to our use of a large financial shock rather than a small surplus or another mechanism through which a small shock, such as productivity, can create large effects.

It is well known that the volatility of the labor share of income in the data is small as wages do not fluctuate much. The benchmark model, with its simplifying assumptions, overstates the labor share volatility somewhat as wages adjust immediately to large financial shocks. We study two extensions of the benchmark model that significantly reduce labor share volatility. In the first, we introduce long-term contracts between firms and banks such that the interest rate is steady during a match, although we allow firms to refinance should interest rates drop. This reduces the volatility of flow surplus and thus of the labor share of income. This deviation does not have a significant impact on unemployment volatility. In the second, we introduce alternating-offer wage bargaining, as in Hall and Milgrom (2008), to the long-term contract model. This form of bargaining reduces the dependence of the wage on the outside option of the worker and thus further reduces the volatility

\(^{5}\)This argument is consistent with papers that allow for debt to be only a fraction of capital financing, such as Jermann and Quadrini (2012) as we explain below. In Section 4 we extend the discussion on these issues, as well as on whether financial conditions can be taken as exogenous to the firm.

\(^{6}\)See Ljungqvist and Sargent (2017) for a survey explaining the economics behind generating volatility in the DMP model.
of wages and the labor share of income. It also increases business cycle volatility, indicating that
the benchmark model’s overstatement of labor share volatility is not a large concern. We further
discuss the current model’s shortcoming in explaining the cross-correlation of the labor share with
the unemployment rate.

Our model makes a stark prediction about the relationship between interest rate volatility and
employment volatility. To evaluate this prediction, we use Compustat data on firm-level employ-
ment volatility and credit ratings to show that lower credit ratings are associated with both more
volatile employment and higher interest rate volatility. This evidence supports the prediction of a
positive correlation between interest rate volatility and employment volatility. Since the financial
crisis, interest in examining these effects empirically has grown. For example, Chodorow-Reich
(2014), who studies the variation in borrowing and employment for firms depending on precrisis
financial relationships, finds that firms with a precrisis relationship with a bank that became less
healthy after the Lehman crisis had more difficulty obtaining loans, paid higher interest rates on
those loans, and reduced employment to a greater extent than firms linked with banks that per-
formed relatively well.

There is a growing literature that uses financial conditions in macroeconomics. Jermann and
Quadrini (2012) study shocks to firms’ ability to finance through debt, which is cheaper than
equity, and explain hours fluctuation. They infer a time series shock from the Flow of Funds
account. Decreases in a firm’s ability to borrow implicitly raises the cost of capital to that firm, as
it must switch to costlier equity financing. While we abstract from the corporate finance aspects
of debt versus equity financing, implicitly focusing entirely on debt financing, we are similar to
Jermann and Quadrini (2012) in our study of stochastic capital costs, which we discipline with
directly observable Baa interest rate data. We differ, however, in our focus on unemployment and
vacancies rather than on hours worked.

Christiano, Eichenbaum, and Trabandt (2014) integrate a general equilibrium New Keynesian
model with a search and matching framework to study the Great Recession of 2007–2009. They
study four possible shocks including a “financial wedge” shock. The financial wedge is similar
to our shock, as it raises the cost of capital for businesses. They find this shock to be the most
important for explaining the Great Recession. We differ in that we focus exclusively on this shock,
which again, we discipline straight from the data. This allows us to study economic mechanisms
in a simple framework.\footnote{Other related work using general macro models include Aghion, Angeletos, Banerjee, and Manova (2010), who show that, in a panel of 21 OECD countries, tighter credit constraints lead to lower and more volatile growth, providing further support for our mechanism. While we abstract from firm heterogeneity, Khan and Thomas (2013) study a model...}
Recently there are several papers studying financial conditions in a DMP framework. Closest to our paper is that of Hall (2016): studying shocks to the discount rate which affects the willingness of an entrepreneur to invest in creating future jobs, Hall disciplines these shocks by using data on stock market volatility. We differ by including capital in production and vacancies creation and our focus on the Baa interest rate as our measure of financial shocks. Petrosky-Nadeau (2014) amplifies productivity shocks in a DMP model with an endogenous financial constraint that depends on the productivity shock. Wasmer and Weil (2004) add a search dimension for locating capital while Petrosky-Nadeau and Wasmer (2013) quantify these effects. These papers focus on the search aspects of liquidity rather than on the price aspects of interest rates. Schoefer (2015) studies how wage rigidity of existing worker matches affects internal funding; he finds that financial constraints and wage rigidity together make the DMP model very sensitive to productivity shocks. Midrigan, Pastorino, and Kehoe (2014) analyze a similar setup with productivity shocks and discount factor shocks and apply their setting to the Great Recession.

Furthermore, numerous papers have made progress in explaining unemployment volatility using productivity shocks. Hagedorn and Manovskii (2008) do so with a calibrated model exhibiting a small surplus. Other papers, such as Hall (2005), use wage rigidity, which generates significant labor market volatility. Kennan (2010) endogenously creates wage rigidity when allowing for private information in matches. Menzio and Moen (2010) derive wage rigidity when firms optimally insure workers against wage shocks by offering constant wages. This comes in tension with the fact that, sometimes firms wish to offer new hires wages that are low relative to existing matches. If the new hires can replace the old hires, then the firm’s desire to insure workers comes into conflict with the desire to hire new workers at lower wages. Hall and Milgrom (2008) switches the bargaining protocol between workers and firms to an alternating wage-offer protocol from Nash bargaining. Heterogeneity in match quality is another source of volatility; in Menzio and Shi (2011), where on-the-job search is allowed, even if most matches have a large surplus, the marginal matches have a small surplus and are thus sensitive to shocks. These models generate a correlation between productivity and unemployment that is large relative to the data, similar to this.
paper’s counterfactually high correlation between interest rates and unemployment.

There is also a literature that deviates from using productivity shocks as the underlying force for unemployment volatility, as we do here, thus breaking the strong correlation between unemployment and labor productivity. Kaplan and Menzio (2016) develop a model in which a firm hiring a worker generates shopping externalities on other firms. Specifically, newly hired workers have more income to spend and less time to shop for a better deal. This model can generate multiple, self-fulfilling equilibria depending on expectations, which in turn yields unemployment volatility when expectations change, rather than a standard productivity shock. Gervais, Jaimovich, Siu, and Yedid-Levi (2015) consider the speed of technological learning as an alternative shock to productivity. This novel shock affects the time that it takes workers to reach the full potential of technological innovations. Learning-rate shocks have an immediate effect on relatively low productivity workers for whom the value of learning increases, helping to break the correlation between unemployment and productivity.

We proceed as follows. Section 2 describes and analyzes the model. Section 3 details the calibration strategy and the strengths and weaknesses of the benchmark model. Section 4 delves deeper into underlying assumptions, especially debt financing, breaks down the mechanisms at work in the model, shows model robustness, and compares financial shocks with productivity shocks. Section 5 explores the implications of matching the volatility of the labor share of income. Section 6 shows our firm level empirical analysis linking interest rate volatility with employment volatility. We conclude in Section 7.

2 The Model

Our point of departure is the standard DMP model with capital. Time is discrete, and indexed by \( t \). There is a unit measure of workers who are either employed or unemployed at the beginning of a period. There is a continuum of firms that match with workers in a frictional labor market and a continuum of banks that intermediate capital. We describe the model in full detail in this section.

\[\text{Relatedly, Schaal and Taschereau-Dumouchel (2016) show that, when unemployment affects aggregate demand, productivity shocks can be amplified to create volatility similar to that in the data, and sufficiently large shocks can create jobless recoveries.}\]
2.1 Firms, Banks, and Workers

Firms

At the beginning of period \( t \), firms are either matched with a worker or potentially open vacancies for unemployed workers. At the end of the period a firm that is matched with a worker produces output \( y \) with a Leontief production function using capital per worker such that \( k_t = k_{t-1} = k \). Later we shall relax this assumption using a constant elasticity of substitution production function where \( k_t \) is determined endogenously. Each firm employs one worker for the wage \( w_t \), which is determined by Nash bargaining as we explain below. Each firm rents the capital from a bank at the beginning of period \( t \) for the interest cost \( r_t \) and the depreciation rate \( \delta \).\(^{11}\) Thus, a firm matched with a worker has the per period profits as shown here:

\[
\pi_t = y - w_t - (r_t + \delta)k.
\]  

There is free entry of firms at the beginning of period \( t \) to attract unemployed workers through vacancy posting at the flow cost:

\[
z_t = (r_t + \delta)k + z_l,\]

where \( z_l \) is the non capital cost of searching for an employee. That is, we assume that the vacancy cost comprises two separate costs. First is the cost incurred because the firm must hold the capital, \( k \), that the worker would use should there be a match. Second is the additional cost \( z_l \), for posting a vacancy, which can be thought of as the time a vacant firm dedicates towards search.\(^\text{12}\) The value functions of the matched firms and the vacancy-posting firms will be specified below. The firms’ decisions are vacancy posting and wage bargaining. Assuming free entry, the expected value of a vacancy-posting firm is zero.\(^\text{13}\)

\(^{11}\)Depreciation can be thought of as maintenance costs that the firm faces.

\(^{12}\)The assumption that vacancies include capital is controversial. Hall and Milgrom (2008) argue against it. Hagedorn and Manovskii (2008) use it. The unconvinced reader can see the breakdown between mechanisms in the model in Section 4.3 and choose to ignore this particular mechanism. Notice that we do not include any other financing cost for vacancies. If advertisements or the labor cost of vacancies were to require financing, the economics of the model would be the same.

\(^{13}\)Notice that our model implies that all financing is debt financing rather than equity financing. We discuss this assumption, along with the assumption that all capital is intermediated, further in Section 4.
Banks

Banks are risk-neutral financial intermediaries that borrow capital from workers at rate $r_f^t$ at the beginning of the period and lend to firms at rate $r_t$. We assume that banks incur the financial intermediation cost $x_t$ per unit of capital intermediated every period $t$. Bank period profits per firm, either vacant or matched, are thus given by:

$$\pi^b_t = (r_t - x_t - r_f^t)k.$$  

We take the intermediation cost $x_t$ as an exogenous stochastic process. It is determined by shocks to default risk, recovery rates from default, changes in regulation, or any other shock to financial intermediation that affects the spread between the interest rate that firms pay and the rate that depositors (workers) receive. As can be seen in Appendix B, firm-level uncertainty shocks can also be an underlying cause of fluctuations in firm interest rates.\(^\text{14}\) We do not take a stand on the underlying shocks causing financial volatility, which we take directly from the data as detailed below. Free entry in the banking sector yields zero profits and shows how $x_t$ can be identified from the corporate interest rate. We assume that $x_t$ follows a Markov process.

Workers

Workers maximize their expected lifetime utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t c_t,$$  

where $c_t$ represents the workers’ consumption in a given time period, and $\beta$ is the discount factor.

Workers maximize Equation 4 with respect to $a_t$, their asset holding at the end of the period, subject to the budget constraint of an employed worker:

$$c_t + a_t = w_t + (1 + r_f^t)a_{t-1},$$  

or the budget constraint of an unemployed worker:

$$c_t + a_t = b + (1 + r_f^t)a_{t-1},$$

where $a_{t-1}$ is their asset holding at the beginning of the period; $w_t$ is the wage rate for employed

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\(^{14}\)We thank Gadi Barlevy for the idea that uncertainty shocks could be the underlying shock.
Given the linearity of their utility function, workers are indifferent between consuming and saving when \( r_f^t = \frac{1 - \beta}{\beta} \). Otherwise, they either consume or save all of their resources. Accordingly, in equilibrium, \( r_f^t \) is constant and denoted \( r_f^t = r^f \).

Since all workers are indifferent to the level of asset holdings at the appropriate interest rate, it is without a loss of generality to assume that all workers, employed or not, hold the same amount of assets \( a_t \).\(^{15}\)

### 2.2 Matching and Separations

Vacant jobs, \( v_t \), and unemployed workers, \( u_t \), are randomly matched according to a constant-returns-to-scale matching technology. The matching function, \( M(u_t, v_t) \), represents the number of matches in a period. We follow Ramey, den Haan, and Watson (2000) in picking our matching function:

\[
M(u_t, v_t) = \frac{u_t v_t}{(u_t^l + v_t^l)^{\frac{1}{l}}},
\]

where \( l \) is a parameter that controls the matching technology. This functional form has the desirable properties of the job-finding rate for a worker and the job-filling rate for a firm always being between 0 and 1. The job-finding rate for a worker is \( \lambda^w_t = \frac{M(u_t, v_t)}{u_t} \). Similarly, the job-filling rate for the firm is \( \lambda^f_t = \frac{M(u_t, v_t)}{v_t} \). Note that both rates depend only on market tightness.

Matches created in period \( t \) start producing output only at the end of period \( t + 1 \). A match separates at the end of the period with periodic probability, \( \sigma \), that is time invariant. The evolution of the number of unemployed workers, \( u_t \), is given by:

\[
\begin{align*}
 u_{t+1} = (1 - \lambda^w_t) u_t + \sigma (1 - u_t).
\end{align*}
\]

### 2.3 Wage Setting

Wages are determined by Nash bargaining between workers and firms. In order to solve the bargaining problem, we use the value functions of workers and firms. Let \( E_t \) and \( U_t \) denote the value function of an employed and an unemployed worker, respectively. Workers move between employ-

\(^{15}\)Assuming otherwise would increase the complexity of notation but not change any result, analytic or quantitative, in the model.
ment and unemployment according to the (endogenous) job-finding rate $\lambda^w$ and the (exogenous) separation rate $\sigma$. Workers take probabilities parametrically.

The workers’ wage is $w_t$. Unemployed workers receive a flow utility value of $b$. This represents the value of leisure and home production. Both types of workers receive a dividend income of $r^f a_t$.

The values of employment and unemployment in state $x$, denoted as $E_t$ and $U_t$, are equal to:

$$
E_t = w_t + r^f a_t + \beta \left\{ (1 - \sigma) \mathbb{E}_{x_{t+1}} E_{t+1} + \sigma \mathbb{E}_{x_{t+1}} U_{t+1} \right\}
$$

$$
U_t = b + r^f a_t + \beta \left\{ \lambda^w \mathbb{E}_{x_{t+1}} E_{t+1} + (1 - \lambda^w) \mathbb{E}_{x_{t+1}} U_{t+1} \right\},
$$

where $\mathbb{E}$ is the expectation operator over the subsequent period state. Note that when calculating the surplus for employed workers, $E_t - U_t$, the capital income drops out, as an individual doesn’t lose assets simply by switching states.

Moving to the firm’s demand for workers, the value for posting a vacancy, $V_t$, is:

$$
V_t = -z_t + \beta \left\{ \lambda^f \mathbb{E}_{x_{t+1}} J_{t+1} + \left( 1 - \lambda^f \right) \mathbb{E}_{x_{t+1}} V_{t+1} \right\},
$$

where $J_{t+1}$ is the value of the firm if the vacancy is filled, described next, and the firm discounts the future at the rate $\beta$.

$J_t$ is the value of a firm matched with a worker given the wage $w_t$:

$$
J_t = y - w_t - (r_t + \delta) k + \beta \left\{ (1 - \sigma) \mathbb{E}_{x_{t+1}} J_{t+1} + \sigma \mathbb{E}_{x_{t+1}} V_{t+1} \right\}.
$$

Wages are set period by period as the Nash bargaining solution that solves:

$$
\max_{w_t} (E_t - U_t)^\gamma (J_t - V_t)^{1-\gamma},
$$

where $\gamma \in (0, 1)$ represents the bargaining power of the worker. Recall that, owing to free entry, $V_t = 0$ in equilibrium.

The total surplus of a match between the worker and the firm is denoted as $S_t = J_t + (E_t - U_t)$. As is common in Nash bargaining, the result is splitting the surplus between the worker and the firm according to their relative bargaining powers. The wage $w_t$ is thus picked in order to satisfy $J_t = (1 - \gamma) S_t$ in equilibrium.
2.4 Equilibrium

We now solve for the surplus $S_t$ and market tightness $\theta_t$ for all states. Notice that once we solve for the surplus, we can calculate the wage from Equation 12. The financial shock $x_t$, which is the only exogenous stochastic variable, pins down the stochastic interest rate $r_t$ using Equation 3.

Using the value functions, we arrive at the surplus equation:

$$S_t = y - b - (r_t + \delta)k + \beta \left\{ (1 - \sigma) \mathbb{E}_{x_{t+1}} S_{t+1} - \theta_t \lambda^f_t \mathbb{E}_{x_{t+1}} (E_{t+1} - U_{t+1}) \right\}. \quad (14)$$

From the Nash bargaining solution, we arrive at:

$$\frac{E_t - U_t}{\gamma} = S_t = \frac{J_t}{1 - \gamma}. \quad (15)$$

Using the free-entry condition, $\forall x$, in Equation 11, the number of vacancies, $v_t$, posted in a given state is picked in order that market tightness $\theta_t$ satisfies:

$$\mathbb{E}_{x_{t+1}} S_{t+1} = \frac{z_t}{(1 - \gamma) \beta \lambda^f_t}, \quad (16)$$

where $\lambda^f_t$ is a function of $\theta_t$.

Using the condition from Equation 16 in Equation 14, we arrive at:

$$S_t = y - b - (r_t + \delta)k + \beta \left\{ (1 - \sigma) \mathbb{E}_{x_{t+1}} S_{t+1} - \theta_t \gamma \frac{z_t}{(1 - \gamma) \beta} \right\}. \quad (17)$$

Equations 16 and 17 are the equilibrium conditions for our model. The number of equations of each type is equal to the number of states. Thus, the total number of equilibrium equations is twice the number of states. There is an equal number of variables in those equations, as the two variables are $\{\theta_t, S_t\}$, and the dimension of each is the same as that of the shock $x_t$.

We solve for $\{\theta_t, S_t\}$ using Equations 16 and 17. Using $\theta_t$ we can solve for both $\lambda^w_t = \frac{M(u_t, v_t)}{u_t}$ and $\lambda^f_t = \frac{M(u_t, v_t)}{v_t}$, as they are both a function of only $\theta_t$. From $S_t$, we find $J_t$ for each state using Equation 15. Then, given $J_t$, we solve for $w_t$ using the value for $J_t$ in Equation 12. $E_t$ and $U_t$ can then be solved simultaneously using Equations 9 and 10. The vacancies are determined using current unemployment $u_t$ and $\theta_t$ according to the definition of market tightness. Finally, unemployment in the next period is determined by the standard law of motion for unemployment, as in Equation 8.

Equation 17 is instrumental in understanding the mechanisms through which firms respond to
the financial intermediation shocks, and consequentially how those shocks affect unemployment, vacancies, and market tightness, as follows:

1. When the interest rate rises, there is a smaller flow surplus available to split between the firm and the worker. This is captured by \( r_t k \) in Equation 17. The decline in surplus reduces the incentives for firms to post vacancies, resulting in a rise in unemployment. We call this the *flow profit channel*.

   This channel is closely related to how productivity shocks affect the vacancies decision in the standard model. There, an adverse productivity shock decreases the flow surplus by reducing current and expected revenues. Here, an adverse financial shock affects the cost of acquiring capital, thus increasing the current and expected cost of operation. In both cases, firms reduce the number of vacancies in response to decreasing profits.

2. The capital component of vacancy costs rises proportionally with the interest rate \( r_t \) as is captured by \( z_t \) in Equation 17. The idea underlying this connection is that if a firm needs capital in order to have a position available, then when interest rates rise, this component of vacancy costs rises as well. Firms thus post fewer vacancies, and unemployment rises. We call this the *vacancy cost channel*.

Turning to aggregates, as vacancies and employed workers use the same amount of capital, the aggregate capital stock is given by:

\[
K_t = (1 - u_t + v_t)k, \tag{18}
\]

where \( K_t \) is the aggregate capital stock at a given time, \( 1 - u_t \) is the number of workers employed at that time, \( v_t \) is the number of vacancies, and \( k \) is capital per worker as defined above. Capital accumulates with depreciation \( \delta \) such that \( K_{t+1} = K_t(1 - \delta) + I_t \).

Investment at a given period \( I_t \) is thus given by:

\[
I_t = K_{t+1} - (1 - \delta)K_t. \tag{19}
\]

Equilibrium in the capital market requires the stock of savings to be equal to the capital stock:

\[
1a_t = K_t, \tag{20}
\]

where 1 is the measure of the workers.
Total output $Y_t$ is given by:

$$Y_t = C_t + I_t + v_t z_t.$$  \hfill (21)

Implicitly, Equation 21 divides investment into two categories. Capital investment $I_t$ includes all capital purchases, both in existing matches and in vacancies (Equation 2). The labor component of vacancy posting $v_t z_t$ is separate in this calculation as it does not add to the aggregate capital stock.

3 \hspace{1em} \textbf{Quantitative Analysis}

We now calibrate the model to US data without targeting any of the business cycle statistics we are looking to explain. To do so, we set some parameters based on \textit{a priori} information and some based on matches between model moments and data moments. We then use the model to generate business cycle statistics related to unemployment, vacancies, and market tightness, and compare what the model delivers to the actual data. To account for aggregation bias, we set the time period as a week.\footnote{Aggregation bias is the bias that comes from looking at unemployment at a quarterly frequency. Should someone lose a job and find a new job within a single quarter, that person’s unemployment spell will not be seen in the data. Setting a model period to a week minimizes this issue and is thus standard in the literature. See, for instance, Hagedorn and Manovskii (2008).} Unless otherwise indicated, data sources can be found in Appendix A.

3.1 \hspace{1em} \textbf{Interest Rate Shocks}

We begin with the financial intermediation shock $x$, which we pick in order to match the volatility of the real Baa interest rate.\footnote{Baa is a rating provided by Moody’s. We take the aggregated interest rate rather than try to break down interest rates by bond duration. We do this as ours is a simple model without a deep theory for why firms might choose one duration of debt versus another. We choose to focus on the aggregate interest rate because it most closely reflects true interest rate costs perceived by firms. Figure 13 in Appendix C shows the relationship between the Baa interest rate and other interest rates, such as the Aaa, Ccc, and US Treasury rates. The Aaa and Baa rates look quite similar in their volatilities.} We choose the real Baa rate, given that 75\% of US firms, representing 50\% of employment, are rated Baa or lower, if they are rated at all, yielding a fairly representative rate for corporate America.\footnote{Authors’ calculation from Compustat data in 2007.} To further demonstrate this point, we note that Gilchrist and Zakrajsek (2012) (henceforth, GZ) calculate a representative nominal interest rate for US businesses that turns out to be almost exactly the same as the nominal Baa interest rate, with a correlation of 0.97.
over the period of our sample, similar means (9.2% for the Baa vs. 8.6% for GZ), and an almost identical standard deviation (2.7% for Baa vs. 2.6% for GZ). Below, in Section 4.1, we discuss further measures of debt to capital and interest rate to revenue ratios, and we find that Baa firms are relatively close to the US aggregates, further confirming their use as our representative firm.

For our measure of inflation, to infer real interest rates, we use the core producer price index (PPI). We choose this deflator because the focus of our analysis is firms. This index measures the average changes in prices received by domestic producers for their output. The core index excludes food and energy.

To implement this approach, we now turn to the mapping of the real Baa interest rate into the model’s shock $x_t$. Equation 3, with free entry for banks, implies that $r_t = r^f + x_t$. As indicated above, we assume a constant $r^f$, which is inferred from the linearity of utility and the constant discount factor $\beta$. Thus, there are no discount factor shocks as in Hall (2016). Since the risk-free rate is constant in the model, $x_t$ can be exactly identified from the real Baa interest rate rather than from the interest rate spread. Accordingly, we estimate the shock process of interest rates in the data and feed those rates into the model. The data is a quarterly series in HP deviations, while the model is weekly. We match the quarterly persistence and unconditional standard deviation of the empirical quarterly process.

To do so, we assume that the HP deviations of $r_t$, denoted $\tilde{r}_t$, follow a first-order Markov process given by $\tilde{r}_t = \rho_{r}\tilde{r}_{t-1} + \epsilon_{r,t}$ with $\epsilon_{r,t} \sim N(0, \sigma_c)$. Given that the model is weekly while the data is quarterly, we need to convert the time frame of the shock process. Accordingly, we pick $\rho_r$ and $\sigma_c$ on a weekly basis such that, after simulating the weekly series and aggregating to quarters, the estimated process of the simulated quarterly interest rate lines up with the process estimated on the quarterly data.

Using deviations implies that we are assuming that firms can respond perfectly to trends in financial conditions but are surprised by shocks at the business-cycle frequency. Using deviations

---

$^{19}$GZ use a sample of 1,112 US nonfinancial firms, rated between AAA and D, covered by the S&P’s Compustat database and the Center for Research in Security Prices, to obtain month-end secondary market prices of their outstanding securities. We thank Simon Gilchrist and Egon Zakrajsek for sharing their data with us. Figure 9 in Appendix C depicts the time series of these two variables, showing our choice to be a reasonable one for use as a representative rate in our macroeconomic study.

$^{20}$We show in Section 4.2 that the model also generates significant volatility under a different inflation specification.

$^{21}$See Appendix A for details.

$^{22}$For completeness, we include Figure 10 in Appendix C, which shows the raw time series data of the real Baa interest rate and unemployment rate. As can be seen, there is a downward trend over time in the interest rate, with no significant trend in unemployment. The jumps in the interest rate are the shocks that we consider, measured using an HP filter.
along with a Leontief production function can be interpreted as firms being able to completely adjust to interest rate trends while having large adjustment costs to shocks at the business-cycle frequency.\footnote{We discuss this more in our robustness analysis in Section 4.2.}

### 3.2 Normalization

We normalize the firms’ revenue net of the average capital cost, \( y - (r + \delta)k \), to be 1 when interest rates are at their mean, or \( r = \bar{r} \), or equivalently, \( \bar{r} = 0 \). This normalization is standard in the DMP literature and gives the interpretation of \( b \) as a replacement rate.\footnote{See, for instance, Hagedorn and Manovskii (2008), who normalize a DMP model with capital in exactly the same way.}

### 3.3 Parameters

We have 12 parameters in this model: \( y, k, \sigma, \beta, z, \gamma, \bar{r}, \delta, l, b, \rho_r \), and \( \sigma_\epsilon \). \( y \) is set to match our normalization described above.\footnote{For details on selecting \( y \), see Appendix A.} We set \( k, \sigma, \beta, z, \gamma, \bar{r} \), and \( \delta \) with \textit{a priori} information; choose \( l \) and \( b \) to match the model moments to empirical moments of the labor market; and pick \( \rho_r \) and \( \sigma_\epsilon \) to match the empirical interest rate process as described above in Section 3.1.\footnote{Specifically, as we describe below, we match the average job-finding rate with the average market tightness in the data.} Table 2 contains the parameter values. Table 9 in Appendix C compares the moments in the data with those in the model to demonstrate the model’s exact fit.

We begin by describing the \textit{a priori} parameters. First, \( k \) is picked to match the capital share of income of \( \frac{1}{3} \).\footnote{For details on selecting \( k \), see Appendix A.} Shimer (2005) calculates the monthly job separation rate, which is 0.0081 at a weekly rate. The discount rate is set to 0.99\( \frac{1}{12} \), representing a quarterly discount rate of 0.99, or a risk-free interest rate of 4%. Hagedorn and Manovskii (2008) calculate the average vacancy cost \( z \) to be 0.584, which comprises an average capital cost of \( (r + \delta)k = 0.474 \) when \( r = \bar{r} \) and a fixed labor cost of \( z_l = 0.11 \). We set \( \gamma \) to be 0.50, following Boeri, Garibaldi, and Moen (2014), who argue that this is a middle ground of the values used in the literature overall. We use an average interest rate, \( \bar{r} \), of 6.6% as calculated in our data. We set \( \delta \) at 6% annually, following Caselli (2005).

We now turn to the internally calibrated parameters, \( b \) and \( l \). We target the average job-finding rate and the average market tightness. Hagedorn and Manovskii (2008) calculate the average job-
Table 2: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>Output per worker</td>
<td>Normalization</td>
</tr>
<tr>
<td>( k )</td>
<td>Capital per worker</td>
<td>Capital share = ( \frac{1}{3} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Job separation</td>
<td>0.0081</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount rate</td>
<td>0.997</td>
</tr>
<tr>
<td>( z )</td>
<td>Vacancy cost</td>
<td>0.584</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Worker bargaining weight</td>
<td>0.50</td>
</tr>
<tr>
<td>( \bar{r} )</td>
<td>Average interest rate</td>
<td>((1 + 0.066)^{\frac{1}{2}} - 1)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
<td>(1 - (1 - 0.06)^{\frac{1}{2}})</td>
</tr>
<tr>
<td>( l )</td>
<td>Matching parameter</td>
<td>0.40</td>
</tr>
<tr>
<td>( b )</td>
<td>Unemployment flow utility</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Shock Processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_e )</td>
<td>Standard deviation interest shocks</td>
<td>0.0318</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>Persistence of interest rate</td>
<td>0.9940</td>
</tr>
</tbody>
</table>

finding rate to be 0.139 and average market tightness to be 0.634. We select \( b \) and \( l \) to minimize the distance between model and data moments, arriving at \( b = 0.60 \) and \( l = 0.40 \).

The flow utility of an unemployed consumer, \( b = 0.60 \), is an intermediate value for this parameter used in the literature. Shimer (2005) uses a low value of 0.4. In contrast, Hagedorn and Manovskii (2008) use a high value of 0.955, creating a surplus small enough to result in strong volatility of the business cycle variables. In accordance with the Frisch elasticity, Hall and Milgrom (2008) reach a value of 0.71. This parameter is essential for controlling the amount of volatility created by the model, as shown below. Our value for \( b \) delivers an average flow surplus that is slightly higher than 0.4, implying that our results are not driven by a small surplus but rather by a large shock. Notice that our normalization allows for comparability of this parameter between our model and others in the literature.
### Table 3: Quarterly Statistics Data versus Model

#### Panel A: US Data

1982:Q1 to 2012:Q4

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.8%</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.94</td>
<td>0.91</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td>Correlation with $u$</td>
<td>1.00</td>
<td>-0.89</td>
<td>-0.97</td>
<td>0.17</td>
</tr>
<tr>
<td>Correlation with $v$</td>
<td>–</td>
<td>1.00</td>
<td>0.97</td>
<td>-0.14</td>
</tr>
<tr>
<td>Correlation with $\theta$</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

#### Panel B: Model

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
<td>0.8%</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.85</td>
<td>0.64</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>Correlation with $u$</td>
<td>1.00</td>
<td>-0.80</td>
<td>-0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Correlation with $v$</td>
<td>–</td>
<td>1.00</td>
<td>0.95</td>
<td>-0.95</td>
</tr>
<tr>
<td>Correlation with $\theta$</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>-0.99</td>
</tr>
</tbody>
</table>

*Notes:* All data are logged and HP filtered. US data: 1982Q1–2012Q4. Model data show the quarterly averages of simulated data (120,000 observations at weekly frequency).

### 3.4 Business Cycle Results

We now describe the business cycle statistics of the calibrated model. We follow the literature in calculating the business cycle statistics of unemployment, vacancies, and market tightness in the calibrated model, and also compare the volatility of the labor share of income in the model to that of the data.\(^{28}\) We finish with a discussion of the volatility of investment in the model as compared to the data.

In Panel A of Table 3, we report the US time series data for the period 1982 to 2012 regarding these series, their standard deviations, autocorrelations, and cross-correlations.

Panel B of Table 3 shows the model counterpart for the US economy. It should be emphasized that these moments are *not* targeted but rather result from the calibration strategy described above in a DMP model with financial intermediation shocks. The main result of the paper is that

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\(^{28}\)For consistency, all variables (unless otherwise indicated) are reported as HP-log deviations with a smoothing factor of 1600.
the simple model presented here can create quantitatively significant labor market business cycle fluctuations. In our benchmark calibration, which we later revisit, the volatility of unemployment, vacancies, and market tightness are all about 80% of that observed in the data. While unemployment, vacancies, and tightness are somewhat less persistent than their empirical counterparts, they are not extraordinarily so when compared to what has been reported in other papers in the DMP literature.  

The cross-correlation of the interest rate and unemployment in our model is substantially higher than that in its empirical counterpart. This is because there is just one shock in the model, and those cross-correlations are almost exactly the same as those in the productivity shock literature, such as reported in Hagedorn and Manovskii (2008). There is a literature that extends the classic DMP model and is able to reconcile this seeming failure of the model; Mitman and Rabinovich (2014), for example, present promising avenues for future research that are outside the scope of the current paper. Another possible interpretation is that the current shock is “too powerful” in the sense that it generates similar volatility as in the data, but empirically the correlation between the shock and the outcome is low. This may be true at the benchmark outcome, but the range of volatility generated by the model reported below indicates that selection of a model could take this point into account.

Next we explore the ability of the model to match the co-movement of labor market variables and to explain past recessions. To do so, we infer shock processes from the data, feed them into the model, and we compare the labor market in the model with the data. In Figure 1, which shows the time series of the data and model simulations, the top panel shows the HP-log deviations of the unemployment rate both in the model and in the data. The middle panel repeats this exercise for vacancies, while the bottom panel does so for market tightness.

First, it is apparent that the model simulation for the co-movements of unemployment, vacancies, and market tightness sit well with the results posted in Table 3. The cross-correlations of these variables are close to 1 (in absolute value) in both the model and the data. That is, there is a high correlation between the model series of each of the three panels: unemployment in the model is high when vacancies and tightness in the model are low. The same is true of the data. Vacancies are slightly more volatile than unemployment, with market tightness being somewhat more volatile.

---

29 It should be noted that we have not added any mechanisms to the model to extend persistence, such as habit formation, time to build for capital, job/occupation-specific human capital, etc. We leave these notions for promising future research.

30 Mitman and Rabinovich (2014) include government policy changes. During recessions, unemployment benefits become more generous, yielding jobless recoveries, which in turn lower the correlation between productivity and unemployment.
than vacancies in both the model and the data. Thus, these figures clearly show the success of the model in matching the basic business cycle statistics of the US data.

But while the model effectively matches the co-movements and volatilities of unemployment, vacancies, and tightness, does it match the timing of these movements? It seems that the model does better in capturing the more recent recessions (since about 2000) than it does the previous ones, especially those in the 1980s. In particular, the model’s prediction for unemployment lines up quite well with the data for the 2008 recession and reasonably well with the data for the 2000 recession. The model does not generate jobless recoveries in either of these recessions, with unemployment falling faster in the model than in the data.\footnote{This can be seen in Table 3 with the low persistence of unemployment in the model as compared with the data.} However, the model does not appear to do a good job in the 1980s, with the unemployment series in the model and the data are negatively correlated for much of the decade. In Appendix D we redo this exercise where we feed in an exogenous process to match the corporate interest rate spread rather than the corporate interest rate, and we discuss the properties of that exercise. However, given the mechanisms at work in the model, namely capital costs, we feel that the exercise reported here is the most relevant.\footnote{For the empirical relationship between the HP-filtered financial data and labor market data, see Appendix C, Figures 11 and 12.}

Next, we turn to the model fit of the labor share of income. By construction, the steady state of that variable in the model is $\frac{2}{3}$. However, we do not target the volatility of the labor share of income in our quantitative analysis. Our empirical measure is the standard deviation of the HP-log deviations of the labor share series in the data. We find a value of 0.011. The model counterpart, which is 0.031, is the standard deviation of the HP-log deviations of $\frac{w}{y}$, wages divided by output per worker.\footnote{The labor share of income is $\frac{wL}{yL}$, where $L$ is total employment. The numerator is total labor income. The denominator is aggregate income, $Y = Ly$, where $y$ is output per worker. Thus, the labor share of income is $\frac{w}{y}$.} While in absolute terms this is not dramatically higher than our value, the difference is worthy of greater exploration, which we do in detail in Section 5. For now, we simply note that there is nothing in the model to generate wage rigidity of any sort. It is also worth noting that the cyclicality of the labor share in the model is not well matched to that in the data. There, the cross-correlation between detrended unemployment and the labor share is $0.186$, while in the model this figure is $-0.94$. The wrong sign is due to the fact that the model, where the labor share is $\frac{w}{y}$, has a procyclical wage rate and, by construction, an acyclical labor productivity. In the data, wages grow less quickly than productivity over the business cycle, yielding the opposite sign.\footnote{Section 5 presents modified versions of the model that are able to match the labor share volatility in the data. Since these versions maintain the assumption of constant labor productivity, they are unable to match the cross-correlation of unemployment and the labor share.}
Figure 1: US Time Series Data and Model, Baa Interest Rate

Notes: The top panel is the HP log deviation of unemployment rates both in the data and model. The middle panel is the same for vacancies, while the bottom panel is the same for market tightness.
absolute value of the correlation is due to the model having a single shock, as discussed above.

Finally, we turn to investment statistics. The mean (standard deviation) of investment as a percent of GDP is empirically 17.5 (1.8). The model equivalent is 16.6 (4.3). So while the level of investment is quite close, the volatility is much higher than in the data. One possible reason for this is the way investment is calculated in the model. We infer investment rates from changes in capital stocks. Capital stocks are in turn inferred from the employment rate and vacancy postings, as in Equation 18. This implies highly volatile investment, as unemployment and vacancies are both volatile and strongly negatively correlated. Mechanically speaking, an increase in unemployment has a strong negative impact on investment as firms sell off their capital in the model.

In the data, this phenomenon is captured by both a decrease in investment rates and a decrease in capital utilization rates. It is well known that capital utilization rates are highly volatile over the business cycle (Greenwood, Hercowitz, and Huffman (1988)), reflecting various adjustment costs to capital stocks that are unmodeled. Another way of comparing the model to the data is to look at changes in the utilized capital stock after taking depreciation into account, and inferring an investment rate from that. Recalculating the empirical investment as a percentage of aggregate capital stock in this manner yields a mean (standard deviation) of 8.8 (3.2). The model equivalent is 6.0 (0.8), which is less volatile than the data. However, there is the potential issue that allowing for capital utilization would reduce the need to hire new capital when interest rates are high, since old capital can be employed more intensively. Taking this into account explicitly in the model would tend to weaken the interest rate mechanisms studied here.

4 Delving Deeper into the Model

We begin this section by discussing further assumptions in the model, such as that all capital is debt financed. We then study the sensitivity of our results to different assumptions and parameterizations of the model. We continue by breaking down the mechanisms that are responsible for generating business cycle volatility in the model. We then compare the same model with productivity shocks to the benchmark model with financial shocks. Finally, we perform an analytic exercise to study our mechanisms theoretically, along the lines of Ljungqvist and Sargent (2017), complete with a comparison of financial shocks to productivity shocks.

35 The data source is FRED St. Louis, investment as a percentage of GDP time series data, from 1982Q1 to 2012Q4.
36 That is, $I_t = \hat{K}_{t+1} - (1 - \delta)\hat{K}_t$, where $\hat{K}_t$ is the capital stock multiplied by the utilization rate in time $t$. 
4.1 Debt Financing

One strong assumption in the model is that all capital is debt financed and thus subject to interest rate expenses and volatility. Additionally, we assume that interest rate movements, regardless of their underlying cause, reflect capital costs for the firm, and that the term structure of debt is fixed. We discuss these assumptions here.

We begin by addressing the assumption that all capital is debt financed. Empirically, this is not even close to true. Using Compustat data from 1986 to 2011 (see Section 6) we calculate the average debt-to-capital ratio for firms. This ratio, which is assumed to be 1 in the model, is empirically (weighted by employment) 0.57 (0.38). Interest expenses in the model are approximately 15% of revenue; empirically, they are 6% (3.5%) of revenue.

The question at hand is: what is the appropriate price of capital to be used in the model? If firms borrow capital for the marginal employee, the corporate interest rate is then the relevant price for the model. Under this assumption, fluctuations in the corporate interest rate determine whether the firm wants to undertake an extra hire, which then determines unemployment.

If one does not make such an assumption, the next natural question to ask is: does the capital stock that is not financed by debt also experience price volatility as financial conditions change? Many models would deliver a positive answer. For example, Jermann and Quadrini (2012) argue that equity financing is even more expensive than debt financing owing to tax incentives. We note that, owing to cyclical changes in stock prices, equity financing is more expensive during recessions, yielding higher capital costs. This is presumably true for other forms of capital financing, such as leasing, which may depend on general financial conditions. It is not clear that the corporate interest rate is the relevant price to be used for other forms of financing. However, one possible interpretation of the benchmark exercise is that the corporate interest rate pins down the opportunity cost of all firm capital, even that which is not directly debt financed. To understand this point, consider a firm that has an outstanding bond. If it has cash on hand, from whatever source, it can either invest in hiring a worker or repurchase its bond. The interest rate on the bond dictates the opportunity cost of hiring the worker and thus is the relevant price for the firm.

Debt is defined as the sum of “Debt in Current Liabilities – Total” and “Long-Term Debt – Total.” Capital is the sum of “Property, Plant and Equipment – Total (Gross),” “Investment and Advances – Equity,” “Investment and Advances – Other,” “Intangible Assets – Total,” and “Inventories – Total.”

Notably, the comparable numbers for Bbb firms are 4.8% and 3.2%, respectively, where Bbb is the rating available in Compustat, as opposed to Moody’s Baa. These two ratings are roughly similar. The similarity between Bbb and the general set of US firms is further evidence that the Baa is a good source to use for this study.

Notice that, in this case, the relevant interest rate is the contemporaneous one, rather than the rate at the date the bond was issued. If the interest rate rises, the price of the bond decreases, raising the opportunity cost of hiring for the firm.
If we reject the assumption that debt is the marginal form of capital financing and assume that the portion of capital not financed by debt is immune to financial volatility, then the appropriate way of examining the effects of interest rate volatility on capital costs may be to use the average debt-to-capital ratio. Under this assumption, only 40%–60% of capital is financed by debt, as found above, and subject to interest rate volatility. We report in the next section a robustness exercise using this assumption below, and we indeed find that financial shocks create substantially less unemployment volatility under this specification, with volatility seemingly a linear function of the debt-to-capital ratio.

The next question we turn to is: is it safe to assume that interest rate movements matter for firm capital costs? This need not be the case if default drives interest rates for firms. Consider a world where firm owners take loans and may simply run off with the money. The probability of this sort of default increases interest rates but does not increase the ex-ante cost of capital from a firm’s point of view; when the owner defaults, (s)he keeps the money. Under this scenario, interest rates overstate the capital costs to firm owners, making our methodology an upper bound as to how interest rate shocks affect capital costs. An alternative scenario with default would not be subject to this critique. If firms invest the capital into a project that fails with a certain probability, then increases in this probability increase both interest rates and capital costs, since the firm owner receives nothing in the event of failure. In this case, the firm owner would not only experience higher capital costs, but also implicitly face a higher separation rate, as default would result in the matched firm being taken away by creditors. Both of these mechanisms would move in the direction of firms posting fewer vacancies, though only the former is modeled. We proceed with the caveat that if the first worldview is a more correct statement of reality, our analysis is an upper bound.

The final question is: what duration bonds should be studied? We take our data from Moodys, which has an average duration of about 5 years. Actual employment averages 2.5 years. An interesting question is: how do firms match the expected duration of employment with the term structure of their debt? For instance, a tenure-track professor might be expected to last 7 years on the job, justifying a longer term loan. A consultant might last only two years before switching jobs. Do universities and consulting firms issue different duration debt? Do firms that hire all sorts of workers adjust their relative employment of “long-term” workers to “short-term” workers when long-term interest rates rise? We leave these questions to future research and instead focus on the representative interest rate reported by Moodys.
4.2 Robustness

In order to understand the sensitivity of the model to various parameter and modeling choices, we perform robustness checks on the parameter values of $b, \gamma,$ and $\delta$; the interest rate assuming a different measure of inflation; an alternative measure of the corporate interest rate, a constant elasticity of substitution (CES) production function; and alternative assumptions regarding firm financing. When performing robustness exercises, we do not recalibrate in order to show the effects of just changing these parameter values. Table 4 describes the results.

The flow utility value of unemployment, $b$, is a key parameter in the literature. Its importance comes from its potentially strong effect on surplus. When lowering this value to 0.40, as in Shimer (2005), we receive somewhat lower, yet still significant, volatility.\footnote{It is well known that for the canonical model with productivity shocks, increasing $b$ from 0.40 to 0.58 would not make a large difference because the magnitude of volatility is small in any case. In comparison, the magnitude of volatility here is substantial, and therefore this parameter matters.} Using a value of 0.71, which is in line with recent literature (e.g., Hall and Milgrom (2008)), leads to higher volatility than that in the data. This verifies the intuition that a lower surplus yields higher volatility of unemployment.

Workers’ bargaining weight, $\gamma$, is an additional parameter that affects volatility. Although this parameter does not affect the surplus itself, it affects the share of the surplus that goes to firms. We check this using both a higher value for $\gamma$, 0.72, as in Shimer (2005), and a lower value, 0.28, which is the same absolute deviation from our benchmark, simply in the other direction. In both cases, the results are about as strong as in the benchmark, indicating robustness to this parameter choice.\footnote{Recall that we are not recalibrating the model. Increasing worker’s bargaining power has an indirect effect of reducing average market tightness. Had we recalibrated, $b$ would have been larger, yielding more volatility in the model. The analysis in Section 4.5 confirms that this parameter value is not important for our quantitative results.}

We continue with the depreciation rate, $\delta$. The reason that this parameter value matters is that it dictates the fraction of capital costs that is subject to shocks.\footnote{Specifically, total capital costs to a firm are $r + \delta$. Thus, $\frac{1}{r+\delta}$ of the cost of the capital fluctuates with respect to a financial shock. Higher values of $\delta$ accordingly correspond to less volatility.} The result can be seen in Table 4, where we set $\delta = 8\%$. Volatility is somewhat lower but still substantial.

Our model requires using real interest rates. As described above, our measure of inflation is the core PPI. As an alternative index we use the core Consumer Price Index (CPI), in which case the implied volatility of the interest rate falls from 0.8% to 0.6%, and model volatility becomes lower for all variables. The difference reflects differences in volatilities of the underlying price indices: the standard deviation of HP-log deviations of the CPI is 60% of that of the PPI. We use the PPI measure as our benchmark as producers are the relevant decision makers in our model.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
Flow utility value of unemployment ($b$) & 0.40, 0.71 \\
Workers’ bargaining weight ($\gamma$) & 0.72, 0.28 \\
Depreciation rate ($\delta$) & 8\% \\
Core PPI implied volatility of interest rate & 0.8\%, 0.6\% \\
\hline
\end{tabular}
\caption{Robustness parameter values.}
\end{table}
Table 4: Robustness

<table>
<thead>
<tr>
<th>Robustness</th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.8%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
<td>0.8%</td>
</tr>
<tr>
<td>$b = 0.40$</td>
<td>0.05</td>
<td>0.06</td>
<td>0.11</td>
<td>0.8%</td>
</tr>
<tr>
<td>$b = 0.71$</td>
<td>0.09</td>
<td>0.10</td>
<td>0.18</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\gamma = 0.28$</td>
<td>0.06</td>
<td>0.08</td>
<td>0.14</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\gamma = 0.72$</td>
<td>0.07</td>
<td>0.09</td>
<td>0.14</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\delta = 0.08$</td>
<td>0.06</td>
<td>0.07</td>
<td>0.12</td>
<td>0.8%</td>
</tr>
<tr>
<td>CPI</td>
<td>0.05</td>
<td>0.06</td>
<td>0.11</td>
<td>0.6%</td>
</tr>
<tr>
<td>GZ</td>
<td>0.07</td>
<td>0.09</td>
<td>0.15</td>
<td>0.8%</td>
</tr>
<tr>
<td>CES</td>
<td>0.05</td>
<td>0.06</td>
<td>0.11</td>
<td>0.8%</td>
</tr>
<tr>
<td>Financing</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Notes: Standard deviations for various robustness tests. All data are logged and HP filtered. Model data show the quarterly averages of simulated data (120,000 observations at a weekly frequency).
Next, we use the interest rate calculated in Gilchrist and Zakrajsk (2012) as our measure of corporate financial conditions (see “GZ” in Table 4). The estimated shock process is slightly more volatile and less persistent, with the net result being slightly higher volatility for unemployment.

For our next robustness exercise, we relax our assumption of a Leontief production function and replace it with CES production function:

\[ y(k) = A((1 - \alpha) + \alpha k^\rho)^{\frac{1}{\rho}}, \]

where \( \alpha \) controls capital per worker, \( A \) controls total factor productivity, and \( \rho \) controls the elasticity of substitution between capital and labor. See Appendix A for details of the calibration of \( A \) and \( \alpha \), which are picked in order to match a capital share of income of \( \frac{1}{3} \) and the same normalization as before. We pick \( \rho \) to match the volatility of labor productivity. We take the HP-log deviations of labor productivity from 1982Q1 to 2012Q4 and calculate the standard deviation to be 0.0095. We match this value exactly with \( \rho = -2.2. \)

Our value for \( \rho \) suggests high complementarity between capital and workers, as in the benchmark Leontief case. This is compatible with the literature on putty-clay models, which attempts to explain the low elasticity of substitution between these inputs at business cycle frequencies. See Figure 2 for evidence that hiring and investment are strongly correlated. We find quantitatively similar results for the CES and Leontief cases.

The final robustness exercise (see “Financing” in Table 4), looks at the assumption that all capital is debt financed, as discussed above. We redo our exercise under the assumption that 40% of capital is debt financed, as this reflects the average debt-to-capital ratio of US firms when weighting by employment. It also is the more conservative of the numbers reported above. We redo our benchmark exercise as follows. We assume that 40% of capital is debt financed and thus subject to the interest rate shocks as before. The remaining 60% we assume is financed at \( \bar{r}. \) We find the volatility to be about 40% of the benchmark, implying that the strength of the mechanism is roughly linear in the debt/capital ratio assumed, when non debt-financed capital is assumed to have constant financing costs. Thus, firms that borrow little, especially for marginal workers, are

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43 An alternative approach to matching the volatility of labor productivity would be to introduce adjustment costs for capital. Using CES maintains the simplicity and transparency of the model. We also note that our approach works against the model creating volatility. To understand this point, note that capital in the model decreases too much when interest rates rise. Considering the cost of capital is \((r + \delta)k\), this over-reduction in capital works to mitigate the rise in capital costs. With adjustment costs, firms would end up adjusting their capital by relatively little and would thus experience a greater rise in capital costs. We therefore consider our framework to be the conservative approach between the two.

44 See, for instance, Gilchrist and Williams (2000).

45 We assume constant financing at \( \bar{r} \), rather than \( r_f \), in order to maintain the same capital share of income as in the benchmark exercise.
little affected by interest rate volatility. Under this view of the world, financial risk has little effect on unemployment volatility.

4.3 Breakdown of Mechanisms

In Table 5, we break down the strengths of our two mechanisms in order to learn their relative strengths. We focus only on the standard deviation as the other statistics are similar to those in Table 3. The “Data” row shows the US data reported in Table 3. The “All” row shows the model standard deviations reported in Panel B of Table 3. The “Flow profit” row shows the standard deviation of each series when only the capital cost for production fluctuates and vacancy costs remain constant. The “Vacancy cost” row describes the opposite case, where only the vacancy cost fluctuates and the capital cost for production remains constant.

These results show that most of the volatility in the model comes from the flow profit channel. There does not seem to be an interaction between the two channels, as summing the effects of each individually results in the total volatility the model generates.
Table 5: Breakdown of Mechanisms

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>u</th>
<th>v</th>
<th>v/u</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>All</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>Flow profit</td>
<td>0.04</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Vacancy cost</td>
<td>0.03</td>
<td>0.03</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Notes: Standard deviation of key variables. All data are logged and HP filtered. US data is 1982Q1–2012Q4. Model data show the quarterly averages of simulated data (120,000 observations at a weekly frequency).

Table 6: Productivity Shocks

<table>
<thead>
<tr>
<th>Robustness</th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>r</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.8%</td>
<td>0.01</td>
</tr>
<tr>
<td>Interest Rate Shocks</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
<td>0.8%</td>
<td>0.00</td>
</tr>
<tr>
<td>Productivity Shocks</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: Standard deviations for various robustness tests. All data are logged and HP filtered. US data is 1982Q1–2012Q4. Model data show the quarterly averages of simulated data (120,000 observations at a weekly frequency).

4.4 What About Productivity Shocks?

We now demonstrate that our benchmark calibration of the model with productivity shocks, measured empirically using the HP log-deviations of labor productivity using seasonally-adjusted real average output per person in the nonfarm business sector, does not cause quantitatively significant volatility. This result is similar to that in Shimer (2005), who explores a similarly calibrated canonical DMP model. Using our benchmark calibration described above, we feed in shocks to labor productivity estimated in the data and find low volatility of the labor market. This confirms that indeed it is the magnitude of our shock that is driving the results; the surplus is large.

Table 6 shows the results for these analyses. The “Data” row is the same as in Table 5 and simply adds in the volatility of interest rate shocks. The row labeled “Interest Rate Shocks” is
our benchmark model, as described above. The row below that, “Productivity Shocks,” shows the model with only productivity shocks. Notice that our findings are quite similar to those in Shimer (2005).\footnote{There are two main differences between our approach and Shimer’s. First, we use a different time period to ensure consistency with the rest of our exercises. Second, we use an HP filter with a smoothing parameter of 1600 rather than 10000. Accordingly, we get somewhat smaller shock values for productivity. However, our results are about the same as Shimer’s in that they generate only about 10% of the empirical volatility of unemployment.} That is, there is a small effect of productivity shocks.

We conclude by pointing out that our calibration, which features a large surplus as does Shimer (2005), leaves little room for productivity shocks to have a meaningful quantitative effect. Our results come from the highly volatile nature of interest rates.

### 4.5 Analytic Discussion

As we have demonstrated, the model generates strong results that are quantitatively robust with financial shocks. How do we understand this finding theoretically? To answer this question, we now analytically study the two theoretical mechanisms – namely, flow profit and vacancy cost – following the methodology of Ljungqvist and Sargent (2017). We derive the elasticity of market tightness with respect to the shock using the equilibrium condition of the steady-state model. When multiplied by the standard deviation of the shock, this elasticity can be a good predictor of the quantitative results with respect to volatility.

To illustrate how our shocks create large volatility of unemployment, vacancies, and market tightness, we compare our results with the productivity-shock literature. For comparability with productivity shocks, we use HP log-deviations of the real Baa interest rate, as reported in Table 1, as our measure of volatility. Accordingly, we first analyze both of our mechanisms separately, and then perform the same analysis as if there were productivity shocks instead. The main takeaway is that for productivity shocks to matter, the elasticity of market tightness to productivity must be large in the canonical DMP model.\footnote{While we are showing that our model generates volatility through a large shock rather than a small surplus, there are other methods of creating volatility in the DMP model. Ljungqvist and Sargent (2017) survey this literature and discuss small surpluses, sticky wages, financial accelerators, bargaining, unemployment insurance, and layoff costs in creating volatility.} In contrast, for financial shocks to matter, the elasticity of market tightness to interest rates can be small, as interest rate shocks are larger.

We begin the analysis with a steady-state version of our model. By combining the right-hand side of Equations 16 and 17, we get the following equation, which can be solved for the market tightness at the steady state, dropping the time index:
\[
\frac{1 - \gamma}{z(r)} (y - b - (r + \delta)k) = \frac{r^f + \sigma}{\lambda^f(\theta)} + \gamma \theta,
\]

where we make explicit that the vacancy cost \(z\) is a function of \(r\) and that the job-filling rate \(\lambda^f\) is a function of \(\theta\). For simplicity, we neglect the labor cost associated with the vacancy cost \(z_l\).\(^{48}\)

The elasticity of market tightness \(\theta\) with respect to \(r\) is given by the following expression:

\[
\frac{\partial \log \theta}{\partial \log r} = - \left( \frac{rk}{y - rk - \delta k - b} + \frac{r}{r + \delta} \right) \ast \frac{r^f + \sigma + \gamma \theta \lambda^f}{\eta(r^f + \sigma) + \gamma \theta \lambda^f},
\]

where \(\eta\) is the elasticity of matching with respect to unemployment.

The right term, \(\frac{r^f + \sigma + \gamma \theta \lambda^f}{\eta(r^f + \sigma) + \gamma \theta \lambda^f}\), which is denoted by \(\Upsilon\), is discussed in Ljungqvist and Sargent (2017), who study various versions and calibrations of the DMP model. They argue that although \(\Upsilon\) is, by construction, greater than a unity, it cannot be much larger than unity under any reasonable calibration. This is the case in our calibration as well.

The remaining two terms correspond to the two mechanisms we have in the model. The term \(\frac{rk}{y - rk - \delta k - b}\) provides the elasticity of market tightness relative to the flow profit mechanism. Under that mechanism, changes in the interest rate affect capital expenditure, \(rk\). The magnitude of that effect is measured relative to the flow surplus, \(y - rk - \delta k - b\). The higher the flow surplus, the better the ability of the economy to absorb the shock in capital cost, implying a low labor market volatility.

The second term, \(\frac{r}{r + \delta}\), provides the elasticity of market tightness to the interest rate that is owing to the vacancy cost channel. Since total capital costs are \((r + \delta)k\), only a fraction \(\frac{r}{r + \delta}\) of the vacancy cost is affected by the volatility of the interest first. When the interest rate changes, this fraction of the vacancy cost changes, affecting the firm’s decision on opening vacancies. Notice that \(\Upsilon\) is simply the elasticity of \(\theta\) with respect to the vacancy cost \(z\).

To see the potential for volatility of market tightness in our model, notice first that the elasticity is unit free. To allow comparison to a model without capital, we normalize \(y - (r + \delta)k\) to 1. Then, using an average (annual) interest rate and depreciation of 6% each, and assuming a capital share, \(\frac{(r+\delta)k}{y}\) of \(\frac{1}{3}\) gives \(y = 1.5\), \(rk = 0.25\), and an elasticity of 1.1. Multiplying this by the standard deviation of the Baa interest rate gives an approximation of the standard deviation of

\(^{48}\)This abstraction does not have any effect on the quantitative argument made below, once rounding is taken into account (results available from the authors). The abstraction does, however, have the benefit of simplifying the calculation.
market tightness of 0.15, which is quite close to 0.14 found in the benchmark model and to 0.22 in the data.

We now turn towards labor productivity, \( y \). The elasticity of market tightness to productivity is given by \( \frac{y}{y+b} \). Because in our approximation above, we normalized \( y - (r + \delta)k \) to 1, the two elasticities can be compared.\(^\text{49}\) Ljungqvist and Sargent (2017) emphasize the importance of the \( y - b \) term and call it the fundamental surplus. For the given value of the parameter \( b \) used above, the elasticity of market tightness with respect to productivity is 2.5 – considerably higher than the elasticity of market tightness with respect to the interest rate of 1.1.

What this tells us, in the terminology of Ljungqvist and Sargent (2017), is that the fundamental surplus in our model is large – at least significantly larger than the one in the model with productivity shocks. While such a low transmission of a shock to volatility of market tightness would lead to very little volatility in the standard DMP model, our model is able to produce considerable volatility. The reason is that the shock that we use, the interest rate, is about 14 times stronger than that of productivity (a standard deviation of 0.14 compared with 0.01 for productivity shocks).

\section{The Labor Share, Debt, and Bargaining}

As shown in Section 3.4, the benchmark model overstates the volatility of the labor share of income. This is because the model does not have any source of wage rigidity, and wages are empirically very stable. For our purposes, we are concerned with whether matching the labor share volatility in the model would affect the volatility of unemployment, vacancies, and market tightness. We explore this issue with extensions to the model in order to understand the sensitivity of our results to matching this empirical moment.

The labor share of income is mostly determined by wages, which in turn reflect the flow surplus, \( y - (r + \delta)k - b \), and the outside option of the worker, given labor market conditions. If either of these components is volatile, then the wage, and thus the labor share, will also be volatile. Accordingly, we first introduce long-term contracts between banks and firms in order to reduce the volatility of the interest rate during a match and thus reduce the volatility of flow surplus. We then introduce an alternating-offer wage bargaining solution, as in Hall and Milgrom (2008), to the long-term contract model in order to reduce the dependence of the wage on the outside option of the worker. We show the results of each of these model extensions separately.

\(^{49}\)Equivalently, consider the standard DMP model with capital, in which case the elasticity of market tightness to productivity would be \( \frac{y-(r+\delta)k}{y-(r+\delta)k-b} \).
5.1 Long-Term Contracts

We introduce long-term contracts into the model. The interest rate is picked at the date of the match and remains fixed. We allow firms to refinance at a lower interest rate should interest rates drop, which conservatively mitigates the adverse effect that high interest rates have on hiring. Thus, the interest rate during a match is the lowest observed interest rate since the beginning of the match. This keeps the flow surplus of the model relatively constant. At a slight abuse of notation, we denote a variable to be a function of another variable with a subscript, for example $w_r$ means that the wage is a function of the interest rate. We develop the model formally in Appendix E. Intuitively, we replace Equation 12 with:

$$J_{r_t, \hat{r}} = y - w_r - (\hat{r} + \delta)k + \beta (1 - \sigma) E_{x_{t+1}, J_{r_t+1, \hat{r}}},$$

where $\hat{r}$ is the lowest interest rate seen before the current period, and $\tilde{r} = \min\{r_t, \hat{r}\}$, where $r_t$ is the current period interest rate.

Equation 11 remains as before; the relevant cost for posting new vacancies depends on current financial conditions. We are assuming that the contract with the bank begins when the vacancy is filled, not when it is created, and ends when the match breaks up.

5.2 Alternating-Offer Wage Bargaining

We next introduce an alternating-offer wage bargaining solution, instead of Nash bargaining, into the long-term contract model. This formulation reduces the influence of the worker’s outside option on the wage bargain, reducing the volatility of wages. See Hall and Milgrom (2008) for a more detailed discussion. Here we give the general framework. We assume that firms and workers take turns offering each other wage contracts. If an offer is turned down, the match must wait a period to renegotiate. During this period, the worker receives the flow utility of being unemployed $b$ while the firm pays cost $\omega$ associated with keeping the match vacant. There is a probability $\nu$ that the match breaks apart. We denote the wage offered by the firm by $w$ and the one offered by the worker by $w^*$. With a slight expansion of notation, the worker’s value function, as in Equation 10, is $E_{r, \hat{r}}^w$ if the accepted wage is that offered by the firm and $E_{r, \hat{r}}^{w^*}$ if it is that offered by the worker.$^{50}$

The indifference condition for a worker in state $\{r_t, \hat{r}\}$ when contemplating an offer $w_{r_t, \hat{r}}$, is set to

$^{50}$Recall that $r_t$ is the current period interest rate while $\hat{r}$ is the minimum interest rate during the match between worker and firm.
make the worker indifferent as to accepting the wage or making a counteroffer, is given by:

\[ E_{r_t, \hat{r}}^w = \nu U_{r_t} + (1 - \nu) \left[ b + r^f a_{r_t} + \beta \left\{ (1 - \sigma) \mathbb{E}_{r_{t+1}} E_{r_{t+1}, \hat{r}}^w + \sigma \mathbb{E}_{r_{t+1}} U_{r_{t+1}} \right\} \right]. \]  

(26)

Similarly, and with a similar expansion of notation, should the worker reject the firm’s offer and make a counteroffer, the indifference condition for the firm is given by:

\[ J_{r_t, \hat{r}}^{w^*} = (1 - \nu) \left[ -\omega + \beta (1 - \sigma) \mathbb{E}_{r_{t+1}} J_{r_{t+1}, \hat{r}}^w \right]. \]  

(27)

In equilibrium, the firm makes a wage offer that makes the worker indifferent, and the worker accepts that offer.

5.3 Calibration and Results

We now discuss our calibration of the extended models and results.

We view the long-term contract model as a means of testing the sensitivity of our benchmark model to a slightly different specification. Accordingly, we maintain the same parameter values throughout this exercise as we used in our benchmark above. The one exception is \( b \), which we adjust from 0.60 to 0.63, because, under the benchmark model, when \( r = \bar{r} \), \( y - (r + \delta)k \) remains constant in expectation throughout the match. The long-term debt features a downward drift in interest rates, implying that when \( r = \bar{r} \), \( y - (r + \delta)k \) increases in expectation. The interpretation of \( b \) being the replacement rate requires increasing \( b \) so that \( \sum_{t=0}^{\infty} ((1 - \sigma)\beta t \frac{b}{y-(r_t+\delta)k} \) remains constant between the two models.

When we add the alternating-offer wage bargaining procedure, two added parameters, \( \omega \) and \( \nu \), need to be set. We take these parameter values from Hall and Milgrom (2008) (HM), who set \( \omega = 0.23 \) and \( \nu = 0.0055 \) at a daily rate, or \( 0.0379 \) at a weekly rate.

Turning to the results, Table 7 compares the volatility of labor market variables and the labor share of income in the two extensions with the benchmark model and the data. We refer to the model with long-term contracts and alternating-offer wage bargaining as “Long-Term Contract + HM.”

Beginning with the long-term contract model, the volatility of labor market parameters is almost identical to the benchmark model.\(^{51}\) the benchmark model generates 0.031 while the long-term contract model generates a lower value of 0.026. The lower volatility of the labor share is as

\(^{51}\)Table 10 reproduces Table 3 for this model in Appendix E.
Table 7: Comparison of Models

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>Labor Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.011</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
<td>0.031</td>
</tr>
<tr>
<td>Long-Term Contract</td>
<td>0.08</td>
<td>0.09</td>
<td>0.16</td>
<td>0.026</td>
</tr>
<tr>
<td>Long-Term Contract + HM</td>
<td>0.13</td>
<td>0.19</td>
<td>0.31</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Notes: Standard deviation of key variables. All volatility variables are in HP-filtered log deviations. The data period is 1982Q1–2012Q4. Model entries show the quarterly averages of simulated data (120,000 observations at a weekly frequency).

expected, given the lower volatility of surplus within a match.

It is worth discussing briefly why this version of the model still generates strong volatility of unemployment, vacancies, and market tightness as the benchmark model, even if interest rates fluctuate less during a match. Consider an extreme case in which the interest rate remains completely fixed during a match: then, when the interest rate is high, firms are strongly opposed to hiring, while when interest rates are low, firms have a strong incentive to hire. Thus, while it is true that the surplus within a match may be lower with these contracts, the surplus of new matches is still quite volatile.\(^5\)

Introducing alternating-offer wage bargaining further reduces the volatility of the labor share, almost to the level seen in the data, while greatly increasing the volatility of other labor market variables. This result is exactly in line with the findings of Hall and Milgrom (2008) and the general intuition that reducing wage volatility increases the volatility of unemployment in the DMP model. Wage rigidity forces firms to bear shocks rather than share them with workers, which increases the volatility of profits and the incentive to post vacancies. This point has been made clearly in the

\(^5\)Which model is a better model? We argue that each model represents different ways that firms borrow. The implicit assumption behind the benchmark model is that firms borrow by issuing bonds. This assumption leads to the current interest rate being the relevant interest rate when considering the opportunity cost of capital for firms. To understand this point, assume that a firm issued a bond when interest rates were low but that interest rates subsequently rose. The price of this bond should fall, making it relatively cheap for the firm to repurchase this debt. That is, the opportunity cost of capital increases with the interest rate. Thus, the current interest rate rather than the interest rate at the time of the match, is relevant for firms that raise money by selling bonds. The long-term contract model is more akin to firms borrowing from banks. When interest rates rise, banks do not necessarily give incentives to firms to repay outstanding debt. Thus, the opportunity cost of capital is defined by the interest rate on the date when the firm borrowed from the bank, rather than by the current period’s interest rate. Incorporating refinancing is also realistic in a world where firms borrow from banks.
Matching the volatility of the labor share of income increases the volatility of unemployment, vacancies, and market tightness. Thus, we conclude that the fact that the benchmark model overstates the volatility of the labor share makes the quantitative results a lower bound on how important financial shocks are for understanding unemployment.

6 Corporate Interest Rates and Employment Volatility

Our model makes a stark prediction of a positive correlation between interest rate volatility and employment volatility. Empirically, this relationship has not been widely studied.\textsuperscript{53} In this section, we document a positive correlation between interest rate and employment volatilities using disaggregated data. To do so, we study US firms to show that lower credit ratings are associated with more volatile interest rates and that firms with lower credit ratings have more volatile employment.

It is well known that lower credit ratings are associated with higher interest rates. However, we

\textsuperscript{53}A notable exception is Chodorow-Reich (2014), as discussed in the Introduction.
are concerned with the volatility of rates, as this volatility measures financial risk.\textsuperscript{54} We document that lower credit ratings are associated with higher interest rate volatility.\textsuperscript{55} Accordingly, we next turn to Federal Reserve Economic Data to show that lower credit ratings are associated with more volatile interest rates. Taking interest rates by rating from 1997Q1 to 2015Q3, we report the standard deviations of the HP deviations of the Bank of America Merrill Lynch US corporate ratings effective yield series. This statistic, which summarizes the extent of business cycle-level volatility in interest rates, is reported in Figure 3 and confirms that lower credit ratings are associated with stronger levels of volatility.

We next show that firms with lower credit ratings experience more volatility in their employment. Using Compustat North America annual data on 4,418 US firms from 1986 to 2012, we divide firms by their credit ratings in each year.\textsuperscript{56} Our observation is at the firm-year level, yielding 38,144 observations once those firms rated lower than B– are dropped.\textsuperscript{57} Table 8 shows summary statistics on the average and standard deviation of employment, as well as on the number of observations by rating. We group firms into six ratings, for which we simply do not consider pluses and minuses as separate ratings. For example, ratings AA+, AA, and AA– are all considered AA for the purpose of our analysis. We use Compustat rating data to add firm ratings to each observation, taking the firms’ ratings at the beginning of each year as our measure. Grouping the firms into these aggregated ratings allows us to make a more direct comparison with the interest rate volatility described above as well as to reduce measurement error.

For each firm-year observation, we compute the absolute value of the percentage change of employment for the subsequent year and denote this variable $y$. We do not include firms that drop from our sample, and so we are looking at the intensive margin of employment change rather than at firm closures. We run the following regression:

$$y_{it} = \beta_0 + \sum_{r=1}^{5} \beta^r \text{RATING}_{it} + \sum_{t=1986}^{2012} \beta^t \text{YEAR}_t + \beta^c \text{CONTROLS}_{it},$$  \hspace{1cm} (28)

where RATING$_{it}$ is a vector of five rating dummy variables on which firm $i$ could be rated at time

\textsuperscript{54}We also speculate that firms with higher interest rates have capital expenditures that form a higher share of their revenue and are thus more sensitive in general to financial conditions. We leave this hypothesis for future research.

\textsuperscript{55}Notice our deviation from the rest of the paper, where we use Moody’s ratings. Here, we use Bank of America Merrill Lynch ratings to achieve consistency with Compustat ratings, while we use Moody’s for the rest of our analysis because more data are available.

\textsuperscript{56}To give an idea of the scale of coverage of the US economy, these firms employed over 46 million workers in 2007.

\textsuperscript{57}Firms rated in the C range, for instance, contain 1.5% of observations, representing 0.2% of employment in 2005. As a result, standard errors of estimates related to these variables are large when included in our sample.
Table 8: Summary Statistics

<table>
<thead>
<tr>
<th>Rating</th>
<th>Mean (Employment) (Thousands)</th>
<th>Std (Employment) (Thousands)</th>
<th>Observations All years</th>
<th>Observations 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>85</td>
<td>99</td>
<td>678</td>
<td>11</td>
</tr>
<tr>
<td>AA</td>
<td>53</td>
<td>137</td>
<td>3,118</td>
<td>77</td>
</tr>
<tr>
<td>A</td>
<td>34</td>
<td>58</td>
<td>9,026</td>
<td>303</td>
</tr>
<tr>
<td>BBB</td>
<td>23</td>
<td>45</td>
<td>10,900</td>
<td>538</td>
</tr>
<tr>
<td>BB</td>
<td>14</td>
<td>27</td>
<td>7,461</td>
<td>367</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>19</td>
<td>6,402</td>
<td>300</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>37,585</td>
<td>1,596</td>
</tr>
</tbody>
</table>

$t$ as rated by Standard and Poor’s Ratings Services, $r \in \{AA, A, ..., B\}$, in each year. Notice that AAA is excluded from our rating dummies, so that our estimates are all relative to AAA. YEAR$_t$ is a series of dummy variables for each year $t \in \{1986, .., 2012\}$.$^{58}$ CONTROLS$_{it}$ include a cubic polynomial in employment level to control for firm size.$^{59}$

The intuition behind this exercise is that lower-rated firms will experience more volatility in general. Volatility of employment rates is measured by the magnitude of changes (both decreases and increases) in employment. To measure the magnitude of these changes, we use the absolute value of percentage employment change. It is possible to think that lower-rated firms may be smaller and thus may experience more volatility. This is especially important given that lower credit ratings are associated with lower average firm size, as seen in Table 8. To control for this, we include the employment controls.

Our variables of interest are $\beta^r$, which show the average firm size changes by rating relative to our baseline group of rating AAA. The higher the number, the more the firm is changing its employment, our measure of volatility. Figure 4 reports our results and shows clearly that lower-rated firms are associated with more volatile employment. The figure reports confidence intervals for each variable with respect to AAA but not to one another. Next, we show that this curve is indeed increasing in a statistically meaningful way. Accordingly, we calculate that \{\beta^{AA}, \beta^{A}\} are significantly different at the 1% level, as are \{\beta^{A}, \beta^{BBB}\}, \{\beta^{BBB}, \beta^{BB}\}, and \{\beta^{BB}, \beta^{B}\}. We cannot rule out reverse causality or selection as firms that are more volatile may be put in a lower

$^{58}$Ideally, we would exploit an exogenous financial shock for identification. To the best of our knowledge, this has never been done effectively on a macro scale. One current example is Bergman, Iyer, and Thakor (2015), who exploit weather shocks to identify cash flow shocks to firms (farms).

$^{59}$Standard errors are clustered at the firm level.
credit rating. However, this evidence suggests a link.\footnote{In Appendix D we provide further suggestive evidence of the time-series relationship between interest rates, or their spread relative to treasuries, and labor market variables.}

The idea that investment is related to this employment volatility is also supported by the data. To show this point, we rerun the regression in 28 where the dependent variable is the absolute value of the change in investment rather than employment. We show the results in Figure 5, which confirms higher investment volatility for lower-rated firms, supporting a link between investment and hiring, by rating.

One potential criticism of this exercise is that lower-rated firms may simply be experiencing different average growth rates. For instance, if all B-rated firms are growing at 10% per year while AAA-rated firms are growing at 1% a year, the relevant $\beta^B$ will be equal to 0.09. Yet this is not volatility. To demonstrate that, we run the same exercise where $y_{it}$ is now the percentage change in employment levels, not in absolute values. The results are reported in Figure 6. There is no relationship between growth rates of firms and credit ratings in our analysis.\footnote{A second way of controlling for this issue is to redo Figure 4 after detrending firms’ growth rates by rating. The results are virtually identical. Details are available from the authors upon request.}

Our final method of linking credit ratings and hiring is to look at 2008 as a case study, centered around the beginning of the financial crisis. We again group firms into ratings based on their credit ratings in 2008, and we calculate their percentage change in employment over the subsequent year.

Notes: The data period is 1986–2012. The dependent variable is the absolute value of a firm’s percentage change in employment over a year. Standard errors are clustered at the firm level. This figure shows the point estimate and a 95% confidence interval of each rating as compared with AAA.
Figure 5: Absolute Percentage Change in Investment by Rating

Notes: The data period is 1986–2012. The dependent variable is the absolute value of a firm’s percentage change in investment over a year. Standard errors are clustered at the firm level. This figure shows the point estimate and a 95% confidence interval of each rating as compared with AAA.

Denoting this variable $y_i$, we run the following regression:

$$y_i = \beta_0 + \sum_{r=1}^{5} \beta_r \text{RATING}_i + \beta^c \text{CONTROLS}_i,$$  \hspace{1cm} (29)

where $\text{RATING}_i$ is firm $i$’s rating. Again, our variables of interest are $\beta_r$. Our results are reported in Figure 7. It is evident that firms with lower credit ratings experienced greater declines in employment. For example, firms with a B rating decreased their employment by about 8% more than firms with an AAA rating. We limit the range of credit ratings used in this study because of data limitations. This case study of the Great Recession provides further evidence of the link between a firm’s financial conditions and its employment choices.\footnote{62}{In Appendix C we replicate the results in Figures 4, 6, and 7 for disaggregated ratings.}

It should be noted that this entire section is focused on the intensive margin of hiring. That is, we are looking at how differently rated firms respond to volatility in interest rates. We do not study the extensive margin; that is, we do not look at firm exits by rating. Presumably, this channel would also work in our favor as low-rated firms are rated as such because they have a high probability of defaulting and shutting down. Empirically, this is difficult to measure as it is not always clear why a firm leaves our sample; for example, it may have shut down, been acquired by or merged with another firm, become private, etc.
To summarize, in this section we set out to document a positive correlation between interest rate volatility and employment volatility. We first demonstrate that lower credit ratings are associated with higher interest rate volatility and then show that these ratings are also associated with higher employment volatility. We combine these two results graphically in Figure 8. Interest rate volatility is positively correlated with employment volatility, in accordance with the theory presented in this paper.

7 Conclusion

In this paper, we ask how much of the US business cycle volatility of unemployment, vacancies, and market tightness can be accounted for by fluctuations in corporate financial conditions. To this end, we build a parsimonious equilibrium model of how financial shocks, in the form of interest rates interest rates, affect unemployment. The main result of the paper is that a simple model can produce quantitatively significant business cycle fluctuations. We discuss the assumptions under which this result does and does not hold – in particular, regarding assumptions on corporate financing. The strength of model mechanisms is roughly linear in the fraction of capital assumed to be subject to financial shocks.

In the course of our analysis, we study two mechanisms by which interest rates affect unem-
Figure 7: Percentage Change in Employment by Rating, 2008–2009

Notes: The data period is 2008–2009. The dependent variable is a firm’s percentage change in employment over a year. Standard errors are clustered at the rating level. This figure shows the point estimate and a 95% confidence interval of each rating as compared with AAA.

employment. Adverse interest rate shocks reduce the flow surplus of matches between workers and firms, leading to less vacancy creation. Higher interest rates also increase vacancy costs, which also increase unemployment. We show that the results are robust to many different quantitative approaches and to the assumption of a Leontief production function. We complement the analysis with an analytic exercise to formally show how our model, which is based on a large surplus, can generate large volatility where others fail. The key is the use of large financial shocks as opposed to small productivity shocks.

We discuss the (un)importance of matching the volatility of the labor share of income. We also present empirical evidence that interest rate volatility and employment volatility are positively correlated at the firm level.

This paper contributes to the recent literature that links financial risk with the business cycle properties of the labor market, and opens the door for a larger future research agenda. One avenue would be to include habit formation, time to build, and other mechanisms that increase the persistence of model variables. Another would be to include heterogeneity in firm size, with a particular focus on small firms that are highly dependent on debt. Finally, one could embed our model into a Dynamic Stochastic General Equilibrium model in order to study the effects of monetary policy on unemployment through the mechanisms studied in this paper.
Figure 8: Relationship between Interest Rate and Employment Volatility

Notes: The standard deviation of HP-filtered interest rates is the same as in Figure 3. The absolute values of employment changes by rating are the coefficients $\beta_r$ in Equation 28, as reported in Figure 4.
References


Appendices

A Computational and Production Function Details and Data Definitions

In this appendix, we explain our method for picking $A$ and $\alpha$, the level of total factor productivity and weight on capital in the production function, respectively, for the constant elasticity of substitution (CES) robustness; for picking $y$ and $k$ for the benchmark calibration; and the details of our computational techniques.

A.1 Picking $A$ and $\alpha$ for the CES Robustness

As described in the text, we pick these parameters in order that $y - (r + \delta)k = 1$ and the capital share of income is $1/3$ when $r = \bar{r}$ (the steady state). We do so by first calculating what $k$ and $y$ must be at the steady state and then picking parameter values accordingly.

Since we have assumed that vacancies and employed workers use the same amount of capital, the total capital in use at the steady state is thus $((1 - \bar{u}) + \bar{v})\bar{k}$, where $\bar{\cdot}$ represents the steady-state value of a variable. The cost of capital is $\bar{r} + \delta$, where $\bar{r}$ is the average interest rate. Thus, capital income is given by $((1 - \bar{u}) + \bar{v})\bar{k}(\bar{r} + \delta)$.

We denote $\alpha_l = \frac{1}{3}$ as the capital share of income, as commonly used in the macro literature. Output in this model is equal to $y(1 - \bar{u})$, so capital income should be equal to $\alpha_ly(1 - \bar{u})$. Our normalization that $1 = y - (\bar{r} + \delta)k$ gives capital income to be $\alpha_l(1 - \bar{u})(1 + (\bar{r} + \delta)k)$.

The value for $k$ is thus given by solving:

$$(1 - \bar{u}) + \bar{v})\bar{k}(\bar{r} + \delta) = \alpha_l(1 - \bar{u})(1 + (\bar{r} + \delta)k).$$

We solve by noting that the steady-state unemployment rate is 0.055, the steady-state market tightness is 0.634, and our value of depreciation and our mean interest rate are 6% and 6.6%, respectively. Thus,

$$\bar{k} = \frac{\alpha_l(1 - \bar{u})}{(1 - \bar{u} + \bar{v} - \alpha_l(1 - \bar{u}))((\bar{r} + \delta))} = 195.85,$$

which in turn implies that $\bar{y} = 1.478$.

This translates to a capital/output ratio of roughly 3.75 over the course of a year. We now turn
to calibrating our production function. Output per worker is given by:

\[ y = A((1 - \alpha) + \alpha k^\rho)^\frac{1}{\rho}. \]

Our calibration strategy picks \( \rho \) in order to match the volatility of labor productivity. We therefore take \( \rho \) as given when calculating \( A \) and \( \alpha \). Accordingly, we pick \( \alpha \) and \( A \) so that the steady-state value of output above is:

\[ \bar{y} = A((1 - \alpha) + \alpha \bar{k}^\rho)^\frac{1}{\rho} = 1.478, \]

and the marginal product of capital (\( \frac{dy}{dk} \)) equals the cost of capital (\( \bar{r} + \delta \)) when \( k = \bar{k} = 195.85 \):

\[ \bar{r} + \delta = A((1 - \alpha) + \alpha \bar{k}^\rho)^\frac{1}{\rho} - 1 - \frac{1}{\rho} \alpha \bar{k}^\rho - 1. \]

The solution is that \( A = 0.0115 \) and \( \alpha = 1.3685e - 6 \). While our value for \( \alpha \) may seem extreme, we note that \( \alpha k^\rho \sim \frac{1}{2}(1 - \alpha) \), consistent with a labor share of \( \frac{2}{3} \).

### A.2 Picking \( y \) and \( k \) for the Benchmark

Continuing with the above calculations, it is easy to see that, in the Leontief case, \( k = 195.85 \) and \( y = 1.478 \).

### A.3 Computation: Discretization and Converting Time Frames

We solve the model numerically by discretizing the state space following Tauchen (1986) into 50 points, uniformly distributed with +/- 3 standard deviations from the mean interest rate.

As described in the text, we need to convert our shock process from a quarterly process to a weekly process. Accordingly, we estimate a quarterly AR(1) process on the HP-log deviations of the quarterly real Baa interest rate from 1982Q1 to 2012Q4 using a smoothing factor 1600.

The algorithm to convert the estimated process to a weekly process is as follows:

1. Guess a persistence and standard deviation of an AR(1) process for the log interest rate at the weekly level.
2. Simulate the interest rate for 10,000 quarters (120,000 weeks).
3. Aggregate the weeks into quarters by taking averages of every 12-week block.
4. Take the HP-log deviations of the simulated quarters, after “burning” the first 500 quarters.

5. Estimate the income process on the simulated data.

6. Repeat until the estimates on the simulated quarterly data match the estimates on the actual quarterly data.

The results are shown in Table 2, which include the estimated shock process in the data, the weekly process fed into the model, and the estimates on the simulated quarterly process.

### A.4 Data Definitions

Unless otherwise indicated,

- The Civilian Unemployment Rate is measured by the Bureau of Labor Statistics (BLS).
- The Baa Interest rate is based on Moody’s Seasoned Baa Corporate Bond Yield.
- The spread is the Baa rate relative to the 5-Year Treasury Constant Maturity Rate.
- Vacancies are based on the help-wanted advertising index, constructed by the Conference Board.
- Labor productivity data are the seasonally-adjusted real average output per person in the nonfarm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics.
- The labor share of income is from the Bureau of Economic Analysis, Non Financial Corporate Sector, calculated as compensation divided by gross value added less taxes on production and imports.
- Core PPI is taken from the Federal Reserve Bank of St. Louis (FRED Economic Data). Realized inflation is calculated quarter by quarter as the percentage change in the core PPI from the same quarter in the previous year. This inflation rate is then subtracted from nominal baa yields in order to infer real baa interest rates.

Notice that, while interest rates (nominal and real) and the spread can be negative, and thus problematic for calculating HP-log deviations, unemployment, vacancies, and productivity are always strictly positive.
B Uncertainty and Endogenous Interest Rates

In this appendix we describe a simple model that illustrates how uncertainty in the return to firms’ productivity endogenously affects the interest rate. We can therefore think of uncertainty shocks as being another source of interest rate shocks leading to volatility of $x$ in the model. Thus, the findings of Bloom (2009) can also imply shocks to interest rates, with effects as described in this paper. The channel from uncertainty to interest rates works through default. We end with a discussion of an additional mechanism by which default may affect unemployment in a DMP model and compare this mechanism to that studied in Hall (2016).

There is a measure one continuum of firms that engage in risky projects. Each project requires an input that is normalized to unity. The output $\omega$ is distributed $\omega \sim G = \text{unif}(\mu - \sigma, \mu + \sigma)$. We assume that the output $\omega$ is non-negative, that is, $\mu \geq \sigma$.

Firms have access to competitive financial intermediaries that finance the cost of the input. These intermediaries have access to a risk-free asset at a periodic cost of $r_f$ and set the risky interest rate $r$ for firms. We denote $R = 1 + r$. Firms whose return on investment is below $R$ cannot pay back the entirety of their loan and declare bankruptcy. The measure of firms that go bankrupt is then $G(R)$. We assume for simplicity that bankrupt firms do not pay back the financial intermediaries (e.g., because of agency/monitoring problems).

The revenue of financial intermediaries is $(1 - G(R)) R$. Given the uniform distribution, the revenue can be written as $\frac{(\mu+\sigma)R-R^2}{2\sigma}$. This term is an inverse U-shape. When $R = 0$, the revenue is also zero because while no firm declares bankruptcy, they all pay back zero. When $R = \mu + \sigma$, the revenue is zero because all firms are bankrupt.

The financial intermediaries set $R$ competitively so that the average return on the loan is equal to $R_f(= 1 + r_f)$. The zero profit condition is:

$$R_f = (1 - G(R)) R. \tag{34}$$

Assuming uniform distribution of the return, this can be written as:

$$R_f = \left(1 - \frac{R - (\mu - \sigma)}{2\sigma}\right) R \tag{35}$$

$$R^2 - (\mu + \sigma) R - 2R_f\sigma = 0.$$

We make the parametric assumption that $R_f \leq \frac{(\mu+\sigma)^2}{8\sigma}$, where $\frac{(\mu+\sigma)^2}{8\sigma} = \max_R \frac{(\mu+\sigma)R-R^2}{2\sigma}$. This assumption is critical because financial intermediaries will otherwise never lend to firms.
The zero profit condition therefore has two solutions: \( R^- = \frac{\sigma + \mu}{2} - \frac{1}{2} \sqrt{(\sigma + \mu)^2 - 8R_f\sigma} \) and \( R^+ = \frac{\sigma + \mu}{2} + \frac{1}{2} \sqrt{(\sigma + \mu)^2 - 8R_f\sigma} \). We rule out \( R^+ \) since firms will strictly prefer to borrow at \( R^- + \varepsilon \), so that \( R^- \) is the only possible equilibrium in this economy.

We now differentiate \( R^- \) with respect to \( \sigma \) to study the effect of uncertainty on the endogenous interest rate:

\[
\frac{\partial R^-}{\partial \sigma} = \frac{1}{2} \left\{ 1 - \frac{\sigma + \mu - 4R_f}{\sqrt{(\sigma + \mu)^2 - 8R_f\sigma}} \right\}.
\]

We now show that this derivative is positive. If Case I: \( \sigma + \mu - 4R_f \leq 0 \), the derivative is trivially positive. It is left to show that if Case II: \( \sigma + \mu - 4R_f > 0 \) holds, the derivative is positive:

\[
1 - \frac{\sigma + \mu - 4R_f}{\sqrt{(\sigma + \mu)^2 - 8R_f\sigma}} > 0
\]

\[
\sqrt{(\sigma + \mu)^2 - 8R_f\sigma} > \sigma + \mu - 4R_f
\]

\[
(\sigma + \mu)^2 - 8R_f\sigma > (\sigma + \mu)^2 - 8(\sigma + \mu)R_f + 16R_f^2
\]

\[
R_f < \frac{\mu}{2}.
\]

This condition is guaranteed as follows:

\[
R_f < \frac{\sigma + \mu}{4} \leq \frac{\mu + \mu}{4} = \frac{\mu}{2}.
\]

The first inequality is defined in Case II. The second inequality is based on the assumption of non-negative output \( (\mu > \sigma) \).

This example illustrates how taking an interest rate shock (and the implied spread shock) can be a reduced form for a more primitive uncertainty shock.

We now turn to a possible extension of the model that would have default directly influence vacancy postings by firm owners. Recall Equation 12:

\[
J_t = y - w_t - (r_t + \delta)k + \beta \left\{ (1 - \sigma)E_{x_{t+1}}J_{t+1} + \sigma E_{x_{t+1}}V_{t+1} \right\}.
\]

If there is default risk and the owner receives zero in the event of default, then from the owner’s
point of view the value of the match decreases. Mathematically and economically, this is similar to a decrease in $\beta$. An owner that discounts the future more will be less willing to post vacancies, leading to an increase in unemployment.

Hall (2016) studies shocks to the discount rate $\beta$ and disciplines these shocks by using data on stock market volatility. In our paper, we did not focus on any mechanisms related to the discount rate of firm owners, nor did we use stock market data to discipline our model, making our approach decidedly different from that of Hall (2016). In a working paper preliminary to this one, available from the authors by request, we included default shocks, disciplined by the data on default risk, and found their effects to be quantitatively small. The mechanism worked as described above: default risk was equivalent to a decrease in $\beta$. The reason that this mechanism is weak quantitatively is that default is small relative to the number of separations that occur naturally in the economy. Shocks to the default rate do not move the separation rate that owners experience by very much.
Figure 9: Baa Interest Rates vs. General Corporate Interest Rate

Notes: The data period is 1982M1–2012M12. The Baa interest rate is based on Moody’s Seasoned Baa Corporate Bond Yield. The average yield is the representative corporate interest rate calculated by Gilchrist and Zakrajsek (2012).

Table 9: Data and Model Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job-finding rate</td>
<td>0.139</td>
<td>0.139</td>
</tr>
<tr>
<td>Market tightness</td>
<td>0.634</td>
<td>0.634</td>
</tr>
<tr>
<td>Persistence of quarterly interest rate</td>
<td>0.794</td>
<td>0.794</td>
</tr>
<tr>
<td>Standard deviation of quarterly interest rate</td>
<td>0.137</td>
<td>0.137</td>
</tr>
</tbody>
</table>
Notes: The data period shown is 1982Q1–2012Q4 and lagged by two quarters for the Baa interest rate. The figure depicts the following quarterly US data: the civilian unemployment rate as measured by the US Bureau of Labor Statistics (BLS) and Moody’s Seasoned Baa Corporate Bond Yield in real terms.
Notes: The data period shown is 1982Q1–2012Q4 and lagged by two quarters for the Baa interest rate. The figure depicts the following quarterly US HP-log deviations with a smoothing parameter of 1600: the civilian unemployment rate as measured by the US Bureau of Labor Statistics (BLS); Moody’s Seasoned Baa Corporate Bond Yield in real terms; and the labor productivity data, which are the seasonally adjusted real average output per person in the nonfarm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics.
Figure 12: US Time Series Data, Filtered

Notes: The data period shown is 1982Q1–2012Q4 and lagged by two quarters for the corporate interest rate spread. The figure depicts the following quarterly US HP-log deviations with a smoothing parameter of 1600: the civilian unemployment rate as measured by the US Bureau of Labor Statistics (BLS); the credit spread is equal to the difference between Moody’s Seasoned Baa Corporate Bond Yield and the 5-Year Treasury Constant Maturity Rate; and the labor productivity data, which are the seasonally adjusted real average output per person in the nonfarm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics.
Figure 13: US Time Series Data

Notes: The data period shown is 1982Q1–2014Q4. The figure depicts the following quarterly US HP-log deviations with a smoothing parameter of 1600: the civilian unemployment rate as measured by the US Bureau of Labor Statistics (BLS); the 5-Year Treasury Constant Maturity Rate; Moody’s seasoned Aaa corporate bond yield; Moody’s seasoned Ccc corporate bond yield.
Notes: The data period is 1986–2012. The dependent variable is the absolute value of the percentage change in employment of a firm over a year. Standard errors are clustered at the firm level. This graph shows the point estimate along with a 95% confidence interval of each rating as compared with AAA.

Notes: The data period is 1986–2012. The dependent variable is the percentage change in employment of a firm over a year. Standard errors are clustered at the firm level. This graph shows the point estimate along with a 95% confidence interval of each rating as compared with AAA.
Figure 16: Percentage Change in Employment by Rating, 2008–2009

Notes: The data period is 2008–2009. The dependent variable is the percentage change in employment of a firm over a year. Standard errors are clustered at the rating level. This graph shows the point estimate along with a 95% confidence interval of each rating as compared with AAA.
D Longitudinal Relationship between Corporate Spread and Economic Variables

In this appendix, we redo the time series simulation exercise shown in Figure 1 in Section 3.4 using the spread instead of the interest rate series. Theoretically, the identification for our shock $x$ comes from the difference between the Baa interest rate and the risk-free rate. As noted in Section 3.1, the relevant equation is $r_t = r_f^f + x_t$. That is, $x_t$ is the interest rate spread. In our benchmark exercise, we were careful to match the time series properties of the Baa interest rate, implicitly assuming that $r_f^f$ is constant over the business cycle. What if, instead, we were to match the interest rate spread assuming that $r_f^f$ is stochastic? We show the results in Figure 17.

This figure shows a much better match between the model and the data. From the point of view of this paper, there are two issues with this simulation. First, our studied economic mechanisms are regarding the interest rate and not the interest rate spread. We do not have a solid theoretical framework by which the spread in and of itself should cause unemployment fluctuation, rather than through its affect on the interest rates. The second is a causality issue. If the Federal Reserve lowers interest rates when unemployment rises, then the correlation could well represent reverse causality. We report this figure here for completeness.
Figure 17: US Time-series Data and Model, Baa Interest Rate Spread

Notes: The top panel is the HP log deviation of unemployment rates in both the data and the model. The middle panel is the same for vacancies, while the bottom panel is the same for market tightness.
E Long-Term Contract Model, Details

In this appendix, we describe in more detail the long-term contract model. We use subscripts to make clear which variables are functions of other variables. For instance, $\theta_r$ denotes that $\theta$ is a function of $r$. We do this at a slight abuse of notation; implicitly, everything is just a function of the history of the intermediation shock $x$. We now describe the deviations from the benchmark model.

We introduce long-term contracts into the model. The interest rate is set at the date of the match and remains fixed until a separation occurs. We allow firms to refinance to a lower interest rate should interest rates drop, which conservatively mitigates the adverse effect that high interest rates have on hiring. The interest rate during a match is thus the lowest observed interest rate since the beginning of the match.

Since the worker’s problems have not changed at all in this version of the model, we begin by describing how this formulation affects firms. We replace Equation 12 with:

$$J_{r,\hat{r}} = y - \tilde{w} - (\tilde{r} + \delta)k + \beta (1 - \sigma) \mathbb{E}_{x_{t+1}} J_{r_{t+1},\hat{r}} + \beta (1 - \sigma) \mathbb{E}_{x_{t+1}} J_{r_{t+1},\tilde{r}},$$

where $\hat{r}$ is the lowest interest rate seen before the current period and $\tilde{r} = \min\{r_t, \hat{r}\}$.

Equation 11 remains as before; the relevant cost for posting new vacancies depends on current financial conditions. We are assuming that the contract with the bank begins when the vacancy is filled, not when it is created, and ends when the match breaks up.

We now return to the banking sector, the purpose of which is simply to close the capital market: banks intermediate capital between consumers and firms, given a period cost $x_t$ that endogenously generates $r_t$. Practically speaking, the interest rate $r_t$ is the exogenous shock that firms face in this model when making their capital choice and hiring decisions, which are the only nontrivial choices in the model. The processes for calculating $r_t$ from $x_t$, which was simply $r_t = r^f + x_t$ in Section 2, is now a bit more complicated, since banks are offering firms an interest rate with the understanding that they are committed to offering that rate to the firms again in the future, even if $x_t$ rises.

The contract offered by a bank is one sided: the bank must honor a past interest rate, while the firm can always refinance with another bank, should the current period interest rate ever be lower. Competition drives the bank to simply allow the firm to refinance at this period’s interest rate. The
value of a loan of a unit of capital, from the point of view of the bank, is given by:

\[ V_{x_t,\hat{r}}^b = \tilde{r} - x_t - r^f + \beta(1 - \sigma)\mathbb{E}_{x_{t+1}} V_{x_{t+1},\tilde{r}}^b, \]  

(39)

where \( \tilde{r} = \min(r_{x_t}, \hat{r}) \) as before, \( x_t \) is the current period financial intermediation cost, \( r^f \) is the rate at which the bank borrows from consumers, and \( r_{x_t} \) is the interest rate offered on new loans when the current state is \( x_t \). The bank discounts the future payments at rate \( \beta \), which happens should the match continue, as it does with probability \( 1 - \sigma \). Free entry sets the interest rate \( r_{x_t} \) to be such that \( V_{x_{t+1}}^{b,\hat{r}} = 0 \). In other words, the value to a bank of making a new loan, when the value of intermediation costs is \( x_t \), is zero.

Using the free-entry condition, \( \forall x \), the number of vacancies \( v \) posted in a given state is picked so that market tightness \( \theta \), which implies a job filling rate \( \lambda_f \), satisfies:

\[ \mathbb{E}_{x_{t+1}} S_{x_{t+1},r_{x_{t+1}}} = \frac{z_{r_{x_t}}}{(1 - \gamma)\beta\lambda_f}. \]  

(40)

We can now rewrite the surplus equation, 17, for the long-term contract model:

\[ S_{x_t,\hat{r}} = y - b - (r_{\tilde{r}} + \delta)k + \beta \left\{ (1 - \sigma)\mathbb{E}_{x_{t+1}} S_{x_{t+1},\tilde{r}} - \frac{\theta r_{x_t} \gamma z_{x_t}}{(1 - \gamma) \beta} \right\}. \]  

(41)
Table 10: Quarterly Statistics Data versus Model – Long-Term Debt

<table>
<thead>
<tr>
<th></th>
<th>Panel A: US Data</th>
<th>Panel B: Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1982:Q1 to 2012:Q4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td>$v$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.94</td>
<td>0.91</td>
</tr>
<tr>
<td>Correlation with $u$</td>
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<td>-0.89</td>
</tr>
<tr>
<td>Correlation with $v$</td>
<td>--</td>
<td>1.00</td>
</tr>
<tr>
<td>Correlation with $\theta$</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Notes: All data are logged and HP filtered. US data comes from 1982Q1 to 2012Q4. Model data show the quarterly averages of simulated data (120,000 observations at weekly frequency).