ABSTRACT

This paper preserves many of the primary features of the standard neoclassical framework while introducing some modifications that transform it into an open economy endogenous growth model with knowledge accumulation. The accumulation of knowledge is determined in part by the extent of knowledge spillovers from abroad, which in turn are affected by commercial policy that regulates the extent of trade between countries. The model predicts that trade liberalization (even if it is unilateral) will increase steady-state output growth in all countries while benefitting the liberalizing country the most in terms of relative income levels.

Correspondence:
Berglas School of Economics
Tel Aviv University
Ramat Aviv, Tel Aviv 69978
Israel
Tel: + 972 3 640-9912
Fax: + 972 3 640-9908
e-mail: bendavid@econ.tau.ac.il
1. Introduction

This paper examines the impact of unilateral trade policy on output levels and growth rates in the steady state. The objective is to formulate an open economy endogenous growth model based upon the standard neoclassical growth model of Solow (1956), Cass (1965), and Koopmans (1965). That model, which is essentially a closed economy model, is characterized by exogenous technological growth. We show how the technology variable, specified here as knowledge and which would grow at a fixed rate were the economy to be closed (as in the Solow model), instead grows at a rate that is endogenous when the economy is open. Thus, in the context of this modified model, a country’s trade policy may influence both its steady-state rate of growth and level of income vis-à-vis other countries.

Why do we return to the neoclassical growth model when there have been so many recent developments in growth theory that endogenize the growth process in other ways?1 The primary reason is that the standard neoclassical growth model appears to be consistent with a considerable number of empirical observations such as income convergence between U.S. states and regions (Ben-David, 1990, Barro and Sala-i-Martin, 1991, and Loewy and Papell, 1996), conditional convergence among countries (Barro, 1991, and Mankiw, Romer and Weil, 1992), as well as unconditional income convergence among developed countries (Baumol, 1986). Furthermore, as Mankiw, Romer and Weil (1992) note, the model correctly predicts the directions of the effects of saving and population growth. Yet, as Mankiw, Romer and Weil also point out, “all is not right for the Solow model.” By virtue of its being dependent on an exogenous growth rate, the standard model is unable to account for the substantial postwar increases in growth among developed countries that have coincided with trade liberalization in those countries. Hence, the focus in this paper is on the impact of trade on the growth process.

Maddison (1982) calculates the average productivity growth rates of the leading countries – that is, countries with the highest output per hour worked – over a span of nearly three centuries and shows that they moved from virtual stagnation in the 1700s to annual growth rates of over 2% in the 1900s.

---

1 Within the trade-growth context, see, for example, Rivera-Batiz and Romer (1991a,b) and Grossman and Helpman (1991).
Ben-David and Papell (1995) search for a structural trend break in the long-run growth paths of real per capita output in 16 OECD countries since 1870. In their calculation of steady-state growth rates along the pre- and postbreak paths, they find that countries grew along their new, postbreak, paths at steady-state rates that were, on average, over twice as high as their prebreak steady-state growth rates.

Table 1 illustrates the long-term behavior of trade and growth by providing some postwar-prewar comparisons for the 16 OECD countries in the Maddison (1991) sample. Each country exhibits an increase in its average postwar growth rate (in comparison with its average prewar growth rate). These ranged from a 38% increase for the United States to an increase exceeding 200% for five other countries. At the same time, the average ratio of exports to output was higher during the postwar for all but one of the countries.2

Can the growth increases depicted in Table 1 actually be tied to greater openness among these countries? Ben-David (1993, 1996) examines subsets of these countries that are formed on the basis of trade ties and finds substantial evidence that movement towards trade liberalization led to heightened trade flows and was accompanied by significant convergence in per capita output among the trading countries.3 Hence, increased openness does appear to affect output. Furthermore, when these trade-related convergence findings are combined with Ben-David and Papell’s (1995) findings of faster growth, the evidence appears to suggest that the convergence did not come at the expense of the wealthier trading countries, but rather that all of the trading countries experienced faster growth — with the poorer traders benefitting the most. This is consistent with the findings in Sachs and Warner (1995) who find that trade liberalization is related to faster growth. The model developed in this paper represents an attempt to...

---

2 The lone country not experiencing an increase in its export-output ratio is Australia. While this ratio remained relatively unchanged following World War II, Australia experienced a massive population inflow that provided many of the same benefits that trade in goods provides in lieu of such migration flows.

3 Ben-David (1993) examines the evolutionary periods of the European Economic Community (EEC), the European Free Trade Association (EFTA), and the US-Canada trade pacts within the framework of the GATT Kennedy Round Agreements while Ben-David (1996) focuses on the general relationship between trade and convergence.
explain these stylized facts within a theoretical framework that tries to maintain the spirit of the standard neoclassical growth model.

The standard model is modified here with the addition of knowledge as a factor of production along with physical capital and labor. While preserving the standard growth model’s assumption that each country produces one good, it is assumed further here that these goods are distinct and that consumers derive utility from the consumption of all goods.

The assumption that each country’s knowledge stock accumulates at a fixed rate in a closed economy preserves the exogenous growth aspect of the standard model. However, given our form of consumer preferences, the countries in this model will be open. Since each country’s good is exposed to competition (both domestically and abroad) from other countries’ goods, there is an impetus to learn and obtain foreign knowledge. This pressure increases as the extent of exposure to foreign goods increases. As in Grossman and Helpman (1991), it is assumed here that trade in goods facilitates the diffusion of knowledge. The premise here, as in Grossman and Helpman, is that knowledge is non-rivalrous and is also non-excludable in many respects. Under these conditions, a country’s commercial policy leads to dynamic terms of trade effects that can have an impact not only on its level of income, but more importantly on its steady-state rates of saving and growth of per capita output and marginal product of capital, all of which now become endogenous. Moreover, the impact of unilateral trade liberalization also improves the steady-state values of these same variables for the country’s trade partners.4

The next section details the model. Section 3 provides a series of propositions which establish the existence of a unique steady-state equilibrium and describe the impact on the equilibrium of a unilateral tariff reduction. Section 4 concludes.

---

4 Devereux (1997) describes a two-country, two-good, one-fixed factor (labor) Ricardian model in which high income growth supports both trade liberalization and a rising trade to output ratio. The emphasis in that paper is on the determination of tariffs within a repeated game framework. We thank an anonymous referee for bringing Devereux’s work to our attention.
2. The Model

Following Dollar, Wolff, and Baumol (1988), Rivera-Batiz and Romer (1991a, b), Grossman and Helpman (1991, 1994), and others, the primary thesis of this paper is that trade serves as a conduit for flows of knowledge. To the extent that increased knowledge acts to raise the productivities of physical capital and labor, it follows that heightened trade has the potential to increase the growth rate of per capita income. Ben-David and Loewy (1998) show that a simplified version of the model presented below is consistent with the observation that higher ratios of total trade to output tend to coincide with faster growth. However, inasmuch as no physical capital is included in that model, it is not possible to consider the effects of trade on the marginal product of capital, saving rate, and process of knowledge accumulation.

The model developed here assumes a world economy comprised of two countries.\(^5\) For each country \(i = 1, 2\), let good \(i\) be the distinct output of country \(i\). As a justification for trade, agents in each country are assumed to derive utility from the consumption of both goods and the marginal utility of consumption of each good satisfies the usual Inada conditions. In order to concentrate on the growth and level effects of commercial policy, in what follows we assume that both countries are identical save for their distinct outputs, their initial conditions, and a possible difference in their tariff rates.

Let \(n\) be the population growth rate in each country. For simplicity, the time \(t\) population size and labor force in country \(i\) are assumed to be equal and are denoted by \(L_i(t)\). Define real per capita consumption in country \(i\) of good \(j\) at time \(t\) as \(c_{ij}(t)\). Then the utility of the agents in country \(i\), \(U_i\), is given by

\(^5\) A more general, multi-country version of the model is available from the authors upon request.
\[
U_i = \int_0^\infty e^{-(\rho - m)t} \left[ \ln c_{i1}(t) + \ln c_{i2}(t) \right] dt
\]  

(1)

where \( \rho \) is the common rate of time preference and the initial population in both countries has been normalized to one.

Each good \( i \) is produced using the physical capital, labor, and knowledge available in country \( i \). Assuming that the production function is linear homogeneous in capital and labor, we write this relationship in per capita terms as

\[
y_i(t) = A k_i(t)^\beta H_i(t)^\epsilon
\]

(2)

where \( y_i(t) \), \( k_i(t) \), and \( H_i(t) \) are per capita output and capital, and the aggregate stock of knowledge in country \( i \) at time \( t \). It is assumed that \( 0 < \beta < 1 \) and \( \epsilon > 0 \).

Per capita expenditure in country \( i \) is simply the sum of per capita consumption of each good plus domestic investment. This expenditure is financed out of per capita income which we define as the sum of per capita net output plus per capita government tariff revenue, \( g_i(t) \), an amount which is transferred back to private agents lump sum. Let \( p_i(t) \) be the price of good \( i \) with good 1 being the numeraire and let \( \tau_{ij} \) be country \( i \)'s tariff on imports from country \( j \) (\( \tau_{ii} = 0 \) by definition). Tariffs are assumed to be determined exogenously and are constant over time. Given these definitions and assuming that the rate of depreciation of capital is set to zero, country \( i \)'s budget constraint is given by

\[
c_u(t) + \frac{p_j(t)\cdot(1 + \tau_{ij})}{p_i(t)} c_{ij}(t) + \dot{k}_i(t) + n k_i(t) \leq A k_i(t)^\beta H_i(t)^\epsilon + g_i(t)
\]

(3)

where

\[
g_i(t) = \frac{p_j(t)\tau_{ij} c_{ij}(t)}{p_i(t)}.
\]

(4)

Following Lucas (1988), per capita growth is obtained by supposing that the technology of knowledge accumulation in country \( i \) is constant returns to scale in the level of knowledge of country \( i \).
In order to provide a means for knowledge dissemination to affect growth, it is assumed further here that this technology is also constant returns to scale in the level of knowledge present in other countries. Furthermore, the effect that country \( j \)'s knowledge has on the rate of knowledge accumulation in country \( i \) depends upon (i) the extent to which country \( i \) can access country \( j \)'s knowledge and (ii) on country \( i \)'s ability to absorb and utilize the accessible part of country \( j \)'s stock of knowledge.

Turning first to the notion of accessibility, we follow Grossman and Helpman (1991) and assume that the share of country \( j \)'s knowledge that country \( i \) can access, a variable that we denote as \( v_{ij} \), is an increasing function of the volume of trade between the two countries. Specifically, we define \( v_{ij} \) to be the ratio of country \( i \)'s total trade with country \( j \) (i.e., bilateral imports plus bilateral exports) to country \( i \)'s aggregate output. Thus,

\[
v_{ij}(t) = \frac{L_i(t)\frac{P_i(t)}{P_j(t)}c_j(t) + L_j(t)c_i(t)}{L_i(t)v_i(t)}.
\]

Second, let \( a_{ij} \) be a constant \( (0 \leq a_{ij} \leq 1) \) that represents the share of country \( j \)'s accessible knowledge that country \( i \) can in fact utilize as part of its own knowledge stock. In some respects, \( a_{ij} \) captures Abramovitz’s (1986) notion of “social capability,” or the ability of a country to utilize existing technologies.

With these definitions in hand, we write the accumulation of knowledge of country \( i \) as

\[
\dot{H}_i(t) = \phi\left[a_{ij}v_{ij}(t)H_j(t) + H_i(t)\right],
\]

where \( \phi > 0 \) represents a cross-country productivity parameter. Henceforth, we set \( a_{ij} = 1 \) to conserve on notation.

---

6 Lucas (1993) suggests a related technology of knowledge accumulation. In his model the level of knowledge in other countries affects knowledge accumulation in country \( i \) through the average level of knowledge worldwide.

7 While a case can be made for assuming that \( a_{ij} \) is a function of \( H_i/H_j \), such an approach complicates considerably the model’s dynamics without adding new insights.
(6) implies that if no country were to trade (an outcome that is ruled out by the form of the utility function), then each country’s growth rate of knowledge would simply be $\phi$. This rate corresponds to the exogenous growth rate of technology found in a closed-economy neoclassical growth model. In this model, however, $\phi$ represents the lower bound on the growth rate of knowledge. To the extent that countries do trade and are able to absorb each others’ knowledge, then each country’s stock of knowledge grows at a rate that exceeds $\phi$. As will be shown below, in steady state this growth rate is common to both countries and varies with the two tariff rates despite the fact that tariffs do not directly enter into the definition of $v_{ij}$. Instead, tariffs affect the rate of knowledge accumulation by changing the relative price of goods, $p_j/p_i$, and the quantities of imports and exports which appear in the expression for $v_{ij}$.

3. Equilibrium

This section begins with the definition of an equilibrium for the economy described above. It is then followed by a series of propositions which provide the paper’s main results. Proposition 1 establishes the existence of a unique steady-state equilibrium in which the growth rate of knowledge is common to both countries. The effect of a unilateral change in tariffs on steady-state marginal products of capital, saving rates, and rate of growth of knowledge are stated in Proposition 2. In particular, it is shown that each of these is a decreasing function of tariffs. Given the assumption of identical economies (save for tariff rates and initial conditions), it follows that output and consumption of the two goods all grow at a common rate as well. Finally, Proposition 3 establishes that in addition to its growth effects, a unilateral change in tariffs also produces the expected level effects for the liberalizing country.

As a first step towards defining an equilibrium, briefly consider the problem being solved in each country. Suppose, as does Lucas (1988), that the population of each country is sufficiently large that private agents are atomistic. Thus, agents treat the time path of knowledge as beyond their influence. This in turn implies that the problem for the agents of Country 1, $C1$, is to choose $c_{11}(t), c_{12}(t)$, and $k_1(t)$ to maximize (1) subject to (3) given $g_1(t), H_1(t), p_2(t), t \geq 0$, and $k_1(0)$. Defining the problem for the
agents of Country 2, C2, in a symmetric fashion, an *equilibrium* for this economy is defined by time paths for all of the endogenous variables such that C1 and C2 are solved and commodity markets clear. Placing the last condition within the context of good 1, this entails that

$$c_{1i}(t) + \frac{L_2(t)}{L_1(t)} c_{2i}(t) + \dot{k}_i(t) + n k_i(t) = y_i(t) \quad .$$

(7)

Note that market clearing, together with the private and government budget constraints, implies that trade is balanced. In other words, (7), (3), and (4) imply that

$$L_1(t) p_2(t) c_{12}(t) = L_2(t) c_{21}(t) \quad .$$

(8)

The definition of an equilibrium now follows immediately:

**Definition 1:** Given $H_i(0)$ and $k_i(0)$ for $i = 1, 2$, an equilibrium consists of time paths for $H_i(t)$, $k_i(t)$, $c_{ii}(t)$, $c_{ij}(t)$ for $i,j = 1,2$, and $p_2(t)$ such that C1 and C2 are solved and Equation (7) holds for all $t \geq 0$.

3.1. Existence

Consider first the implications of Problem C1. Let $z_i = y_i/k_i$ and $\chi_i = c_{ii}/k_i$ where the time argument is dropped both here and below to simplify the notation. Standard manipulations of the Euler equation for $k_1$ and the first-order condition for $c_{11}$ imply that

$$\gamma_{z_{11}} = \beta z_1 - \rho$$

(9)

where $\gamma_x$ denotes the growth rate of $x$. Next, after substituting (4) into (3), dividing both sides by $k_1$, and making use of the first-order conditions for consumption, one obtains
\[ \gamma_{k_1} - z_1 - n - \left[ 2 - \frac{\tau_{12}}{1 + \tau_{12}} \right] \chi_1. \tag{10} \]

Consider next the law of motion for knowledge, (6). Using the trade balance condition to substitute imports for exports and then the first-order conditions for consumption to write imports in terms of domestic consumption, it follows that \( v_{12} = 2\chi_1/[z_1(1 + \tau_{12})] \). Substituting this expression into (6) implies that

\[ \gamma_{H_1} = \phi \left[ \frac{2\chi_1}{z_1(1 + \tau_{12})} \frac{H_2}{H_1} + 1 \right]. \tag{11} \]

In what follows, we concentrate solely on steady-state equilibria. Letting * denote steady-state values, then (9), (10), and the definition of a steady state imply that \( z_1^* \) and \( \chi_1^* \) each equal a constant. Hence, \( \gamma_{y_1}^* = \gamma_{k_1}^* = \gamma_{c_{i1}}^* \) and by (2) these in turn equal \( \epsilon \gamma_{H_i}^*/(1 - \beta) \). Furthermore, since the left-hand side of (11) must equal a constant in steady-state, it follows that the same is true of \( H_2^*/H_1^* \). Thus, the steady-state growth rates of the two knowledge stocks must be equal to each other. (9) and the fact that Country 1 and Country 2 are taken to be identical (save for their initial conditions and possibly their tariff rates) implies that the steady-state marginal products in the two countries, \( \beta z_i^* \), are also equal.

Next, equate (11) to its Country 2 counterpart to solve for \( H_2^*/H_1^* \). Substituting the resulting expression back into (11), using (10) and its Country 2 counterpart to substitute out the two \( \chi_i^*/[z_1^*(1 + \tau_{ij})] \) terms implies that (11) can be written as

\[ \gamma_{H_i}^* = \phi \left[ 1 - \frac{\epsilon \gamma_{H_i}^*(1 - \beta)^{-1} + n}{z_i^*} 2T + 1 \right], \tag{12} \]

where \( T = [(2 + \tau_{12})(2 + \tau_{21})]^{-0.5} \). 8

---

8 Note that the term inside the parentheses on the right-hand side of Equation (12) is simply one minus the common steady-state saving rate, \( s^* = s_{i1}^* = (k_i^* + nk_i^*)/y_i^* = \epsilon \gamma_{H_i}^*(1 - \beta)^{-1} + n/z_i^* \), for \( i = 1, 2 \).
(12) provides one equation in $\gamma_H^*$ and $z^*$. To obtain a second equation, substitute $\varepsilon \gamma_H^*(1 - \beta)$ for $\gamma_{e,-}$ and $\gamma_H^*$ for $\gamma_H^*$ in (9), an expression which for completeness is repeated as (13):

$$\frac{\varepsilon \gamma_H^*}{1 - \beta} = \beta z^* - \rho \quad (13)$$

Using these two equations, we can prove the following:

**Proposition 1:** There exists a unique steady-state equilibrium. Furthermore, $\gamma_H^* \in (\phi, \phi(2T + 1))$.\(^9\) (The proof of this and paper’s other two propositions appear in Appendix 1.)

To gain some further insight into the determination of $\gamma_H^*$, note that $\gamma_H^*$ corresponds to the maximum eigenvalue of the 2×2 system defined by (6) when all of the endogenous variables are evaluated at their steady-state levels, namely,

$$\gamma_H^* = \phi \left[ 1 + \left( \frac{v_{12}}{v_{21}} \right)^{0.5} \right] \quad (14)$$

which shows that the steady-state growth rate is an increasing function of the $v_{ij}^*$'s. Since each of the $v_{ij}^*$'s is itself a function of $\gamma_H^*$ (because $\chi_i^*$ and $z_i^*$ are), (12) and (13) effectively solve a particular fixed point problem. By the same token, the steady-state relative level of the stocks of knowledge corresponds to the ratio of the elements of the eigenvector associated with $\gamma_H^*$, namely $H_2^*/H_1^* = (v_{21}^*/v_{12}^*)^{0.5}$. Moreover, each of these endogenous variables is a function of, among other things, the two tariff rates. How changes in these tariffs affect the economy's steady-state is the subject of the next subsection.

---

\(^9\) Given our choice of utility function, the restrictions on parameters such that Proposition 1 holds are quite weak: $0 < \beta < 1$, $\varepsilon > 0$, $\rho > n$ which must hold to satisfy the transversality condition, and $\tau_y \geq 0$ which is necessary given the absence of any source of government revenue other than tariffs.
3.2. The Effects of Trade Liberalization

The emphasis now shifts to a description of the effects of trade liberalization on the steady-state magnitudes of the economy’s endogenous variables. To the extent that a given change in tariffs may affect the \( v_{ij}^* \)'s in different directions, then (14) would appear to suggest that there is some ambiguity in liberalization’s impact on steady-state growth. As is shown in Appendix 2, a unilateral reduction in one tariff \( \tau_{ij} \) — does, in fact, lead to an increase in \( v_{ij}^* \) and to a decrease in \( v_{ji}^* \). The combined impact on steady-state growth, however, proves to be unambiguous; it increases.

The following proposition addresses the question of the effects of changes in tariffs on productivity, output growth, saving, and welfare. Specifically, we show that a unilateral tariff reduction raises the steady-state average (and hence marginal) product of capital and the rate of growth of knowledge. The latter in turn implies that the steady-state growth rate of per capita output rises. Not surprisingly, a higher return to capital induces an increase in the steady-state saving rate. Since the steady-state growth rates of consumption and capital rise as well, it follows that welfare increases. Hence, the growth effects of even a unilateral tariff reduction are widespread.

**Proposition 2:** \( \frac{\partial z^*}{\partial \tau_{ij}} < 0; \frac{\partial y^*_H}{\partial \tau_{ij}} < 0; \frac{\partial s^*}{\partial \tau_{ij}} < 0; \frac{\partial U_i^*}{\partial \tau_{ij}} < 0. \)

The next proposition shows that the level effects of a tariff reduction coincide with the growth effects just established. Hence, the liberalizing country experiences an increase in its steady-state level of output relative to that of its trade partner. This occurs not just because the level of knowledge in the liberalizing country rises relative to that of its trade partner, but because the same also holds for the relative level of capital. Therefore, during the transition to the steady state it is possible for an initial income gap to be increased, eliminated or reversed depending upon the pre-liberalization and post-liberalization relative magnitude of the tariffs.

**Proposition 3:** \( \frac{\partial (k_i/k_j)}{\partial \tau_{ij}} < 0; \frac{\partial (y_i/y_j)}{\partial \tau_{ij}} < 0. \)
Proposition 3 and (13) provide some insight into the question posed by Lucas (1990) on why capital doesn’t flow from rich countries to poor countries. To the extent that countries differ from one another in their commercial policies, Proposition 3 implies that the country with the higher tariff will be the poorer of the two. However, while the two countries may be at different stages of development, (13) implies that their steady-state returns to capital, $\beta z^*$, will be the same. Finally, since the real wage for a given level of knowledge is $(1 - \beta)y^*$, it follows that labor will seek to migrate from the poor to the rich country.

As Easterly and Rebelo (1993) show, developing countries tend to tax trade more than do developed countries. This evidence is consistent with Proposition 3 inasmuch as different initial tariff policies on the part of the two countries result in a negative relationship between tariff rates and their levels of development. A trade agreement between the countries which serves to reduce either or both tariffs will in turn lead to an outcome of faster growth for both countries, a result which is consistent with the long-run increases in growth rates reported by Maddison (1982) and Ben-David and Papell (1995). Should the agreement yield tariffs which are equalized across the two countries, then the faster growth will be accompanied by income convergence.\(^1\)

4. Conclusion

This paper modifies the standard neoclassical closed-economy exogenous growth model by extending it into an open economy model that includes knowledge as a factor of production in addition to the traditional capital and labor inputs. The model developed here facilitates the analysis of the growth and level effects of commercial policy. These effects cause changes in trade flows and as such impact the extent of knowledge spillovers between countries.

\(^1\) For examples of countries where liberalization brings about income convergence and increased growth, see Ben-David (1993) and Sachs and Warner (1995).
The model preserves the conditional convergence feature of the standard model while extending the conditions for such convergence to occur. Similar countries that adopt identical trade policies will converge to the same growth path, regardless of their initial factor endowments. Countries that implement differing trade policies will converge to different, though parallel, growth paths.

While the conditional convergence aspects of the model are inherently similar to the standard model, the growth implications are not. Though closed economies in this model — were they allowed to exist — would grow at exogenous rates as in the standard model, the degree of trade openness determines how much faster (than the exogenous autarkic growth rate) the economies will grow in the steady state.

Specifically, technological change is modeled here as the accumulation of knowledge. This accumulation process is assumed to be driven by the degree to which each country is able to apply the knowledge spillovers coming from its trading partners to its own knowledge stock. As Grossman and Helpman (1991) suggest, knowledge spillovers are taken to be directly related to the degree of openness among countries. The model provides an analysis of the impact of trade liberalization on the steady-state rate of growth of per capita output, marginal product of capital, saving rate, and the relative levels of income.

Assuming that economies possess similar technologies, it is shown that rates of return to capital are the same in all countries, regardless of their level of development. Returns to labor on the other hand, vary among countries — with the highest rates of return appearing in the most developed, and hence, wealthiest countries. Countries that tax trade heavily will be poorer in the steady state than countries adopting more liberal trade policies. Commercial policy emphasizing trade liberalization should have a positive affect on knowledge accumulation and as a result, on economic growth as well.

The model’s implications are consistent with: (1) the increases in economic growth seen during the postwar period among countries that have actively sought to liberalize trade — even after adjusting for the slowdown in growth along the path to steady state; and (2) the income convergence among countries that engaged in extensive trade liberalization with one another. As such, the model can serve as a
theoretical link between either closed- and open-economy growth models or exogenous and endogenous growth models that has the capacity to confront many of the trade-related “stylized facts” regarding convergence or lack thereof across both growth rates and levels of per capita income.

Acknowledgements

The authors thank participants at the 1996 Meetings of the Society for Economic Dynamics and Control and the 1997 CEPR ERWIT conference for their many useful comments.
References


This appendix provides proofs of the paper’s three propositions.

Proposition 1: There exists a unique steady-state equilibrium. Furthermore, $\gamma^*_H \in (\phi, \phi(2T + 1))$. Proof: Solving (12) and (13) for $z$ as a function of $\gamma_H$ and denoting the resulting functions as $G(\gamma_H)$ and $E(\gamma_H)$ (for the growth rate of $H$ and the Euler equation for $k$), one obtains

$$G(\gamma_H) = \frac{2T\phi[\psi\gamma_H + n(1 - \beta)]}{(1 - \beta)[\phi(2T + 1) - \gamma_H]}$$

and

$$E(\gamma_H) = \frac{[\psi\gamma_H - \rho(1 - \beta)]}{(1 - \beta)\beta}.$$  \hspace{1cm} (1.1)

To prove existence and the restriction on $\gamma^*_H$, it suffices to show that $\exists \gamma_H \in (\phi, \phi(2T + 1))$ such that Equations (A1) and (A2) hold simultaneously. To see that this is indeed the case, note that these expressions imply that $G(\phi) = e\phi(1 - \beta) + n < [e\phi(1 - \beta) + \rho]/\beta = E(\phi)$, $\lim_{\tau_H \to \phi(2T + 1)} G(\gamma_H) = \infty$, and $E(\phi(2T + 1)) < \infty$. Since $\gamma^*_H = \phi$ when trade yields no spillovers, the continuity of $G(\cdot)$ and $E(\cdot)$ establishes these results. To see that such an equilibrium must be unique, note that Equations (A1) and (A2) imply that both $G(\cdot)$ and $E(\cdot)$ are strictly increasing over the relevant domain with the former being strictly convex and the latter being linear. \blacksquare

Proposition 2: $\partial z^*/\partial \tau_{ij} < 0$; $\partial \gamma^{*}_H/\partial \tau_{ij} < 0$; $\partial s^*/\partial \tau_{ij} < 0$; $\partial U_i^*/\partial \tau_{ij} < 0$.

Proof: Recall that $T = [(2 + \tau_{12})(2 + \tau_{21})]^{-0.5}$. Hence, without loss of generality, it suffices to show that $\partial z^*/\partial T > 0$, $\partial \gamma^{*}_H/\partial T > 0$, and $\partial s^*/\partial T > 0$. Totally differentiating (1.1) and (1.2), applying Cramer’s Rule, and noting that $G'(\gamma_H) > E'(\gamma_H)$, establishes the first two inequalities. (13) implies that $s^* = \beta - (\rho - n)/z^*$ which proves the third inequality. By (9) and symmetry, $\gamma^*_c = \gamma^* = \beta z^* - \rho$. Solving these \textit{expressions for} $c_H(t)$ and $c_\theta(t)$, \textit{substituting into} (1) \textit{and integrating}
yields \( U_i^* = [\ln c_{i1}(0) + \ln c_{i2}(0)](\rho - n)^{-1} + 2[\beta z^* - \rho](\rho - n)^{-2} \) where the static allocation between consumption and saving implies that both \( c_{i1}(0) \) and \( c_{i2}(0) \) are decreasing functions of \( s^* \). Direct calculation then shows that the growth effect operating through the marginal product of capital dominates the level effect operating through the saving rate which proves the final inequality.

**Proposition 3:** \( \partial (k_i^*/k_j^*) / \partial \tau_{ij} < 0; \partial (y_i^*/y_j^*) / \partial \tau_{ij} < 0. \)

**Proof:** \( z_1^* - z_2^* \) implies that \( k_1^*/k_2^* - y_1^*/y_2^* = (H_1^*/H_2^*)^{(1 - \beta)} \). Since (10) and (11) imply that \( H_1^*/H_2^* = [(2 + \tau_{12})/(2 + \tau_{21})]^{0.5} \), the result follows.

**APPENDIX 2**

This appendix analyzes the impact of tariff changes on the trade-output ratios, \( v_{ij}^* \)'s. Unilateral trade liberalization in a two-country world has a differential impact on the steady state values of the openness variables with one rising and the other falling. This can be seen by looking first at the steady-state ratios of domestic consumption to capital, \( \chi_i^* \). Since \( \gamma_{k_1}^* = \gamma_{k_2}^* = \gamma_k^* \) and \( z_1^* = z_2^* = z \), (9) and (10) imply that \( \chi_i^* = [z^*(1 - \beta) + \rho - n](1 + \tau_{ij})/(2 + \tau_{ij}) \). The term in the square brackets is simply the ratio of consumption's (domestic plus imports) share of income to capital and constitutes the “income effect” stemming from trade liberalization that both countries receive. By Proposition 2, this term unambiguously increases as either tariff decreases. The ratio multiplying the term in square brackets constitutes the “substitution effect” that the liberalizing country experiences from reducing its tariff. Since this ratio is increasing in the tariff, the substitution effect from trade liberalization serves to decrease \( \chi_i^* \). Consequently, the income and substitution effects tend to offset each other for the liberalizing country.

Turning now to the openness terms, recall that \( v_{ij}^* = 2\chi_i^*/[z^*(1 - \tau_{ij})] \). Making use of the above expression for \( \chi_i^* \), this can be written as \( v_{ij}^* = 2[1 - \beta + (\rho - n)/z^*](2 + \tau_{ij})^{-1} \). As in the preceding paragraph, there is an “income effect” (the term inside the square brackets) and a “substitution
effect.” By Proposition 2, a tariff reduction serves to decrease the term in square brackets, and hence both $v_{ij}^*$'s, while the liberalizing country also experiences an offsetting substitution effect.

Which of the two effects dominates in the case of the liberalizing country? On the one hand, lowering $\tau_j$ lowers $v_{ij}^*$. On the other hand, Proposition 2 states that such action raises the steady-state rate of growth of knowledge. Hence, it follows from (14) that $v_{ij}^*$ must in fact increase and the substitution effect necessarily dominates the income effect.
Table 1

Changes in Rates of Growth and Changes in Export-GDP Ratio for 16 OECD Countries
Postwar (1950-1989) versus Prewar (1870-1939)

<table>
<thead>
<tr>
<th>Country</th>
<th>Ratio of Postwar Average to Prewar Average</th>
<th>Growth Rates</th>
<th>EX/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td></td>
<td>3.75</td>
<td>0.96</td>
</tr>
<tr>
<td>Austria</td>
<td></td>
<td>3.38</td>
<td>2.37</td>
</tr>
<tr>
<td>Belgium</td>
<td></td>
<td>3.12</td>
<td>2.63</td>
</tr>
<tr>
<td>Canada</td>
<td></td>
<td>1.74</td>
<td>1.24</td>
</tr>
<tr>
<td>Denmark</td>
<td></td>
<td>1.62</td>
<td>2.02</td>
</tr>
<tr>
<td>Finland</td>
<td></td>
<td>2.26</td>
<td>1.31</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td>2.44</td>
<td>2.15</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td>2.09</td>
<td>1.16</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td>3.51</td>
<td>2.34</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td>3.14</td>
<td>3.15</td>
</tr>
<tr>
<td>Netherlands</td>
<td></td>
<td>2.38</td>
<td>2.21</td>
</tr>
<tr>
<td>Norway</td>
<td></td>
<td>2.00</td>
<td>1.97</td>
</tr>
<tr>
<td>Sweden</td>
<td></td>
<td>1.64</td>
<td>1.94</td>
</tr>
<tr>
<td>Switzerland</td>
<td></td>
<td>1.66</td>
<td>1.48</td>
</tr>
<tr>
<td>U.K.</td>
<td></td>
<td>2.55</td>
<td>1.03</td>
</tr>
<tr>
<td>U.S.</td>
<td></td>
<td>1.38</td>
<td>1.31</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>2.42</td>
<td>1.83</td>
</tr>
</tbody>
</table>