This has all happened in the first sixteen pages, and it is time to move on. The next section begins with a mention of Pythagoras and Fermat's last theorem, which are interesting because they are ‘always true’ (or surely, in the latter case, ‘always false’). Is this an example of mathematics ‘with an evolutionary advantage’? Apparently not, since it is akin to someone who, having once had an unpleasant encounter with a tiger, is automatically uncomfortable the next time this happens, and decides that a quick exit is a better strategy than ruminating on whether the first tiger's antisocial behaviour was an exception to a general rule. Similarly, the mathematical theory of infinity is not an example of evolutionary progress since a primitive man in a region teeming with tigers is uninterested in whether there are $\aleph_0$ of them. Nor is rigour a necessity to survival in a hostile world; a man who sees a tiger's head poking out from a bush is unlikely to speculate on whether it is attached to the rest of the animal. So not all mathematical activity is a result of evolutionary change. It is good to know that, since it seems to my mind to make it more rather than less worthwhile, but I am not sure I needed convincing of it, and particularly not with these examples.

I am afraid that I gave up reading carefully at this point. I did, indeed, take a random look at further developments, cherry-picking references to Plimpton 322 and the Rhind papyrus, the proof of the irrationality of $\sqrt{2}$, pentagonal numbers and regular polyhedra, Platonism versus Formalism and Kepler's laws, all of which turn up in the first ninety pages. The remainder of the book follows the same model; you will find here almost every anecdote which is told about mathematics and the whistle-stop tour includes celestial mechanics, electricity and magnetism, quantum theory, probability and statistics, game theory and economics, computation, logic, pedagogy and the nature of mathematical research.

Inevitably, none of these topics is given more space than a few pages and nowhere did I discover that anything particularly original was being said. If you want to read something which tells you a little bit about almost everything in mathematics, and you are prepared to ignore the embarrassing stuff about evolution at the start, this might be just up your street. I, however, was not impressed.

10.1017/mag.2017.95

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A brief history of numbers, by Leo Corry, pp. 309, £24.99 (hard), ISBN 978-0-19-870259-7, Oxford University Press (2015).

The title of this book was inspired by Stephen Hawking's *A Brief History of Time*, but Leo Corry had no similar intention of it being a popularisation. For his readership, he had in mind mathematics undergraduates, historians, teachers and mathematicians in general. However, the book's innocuous title provides little clue as to the richness and clarity of its historical discourse.

Focusing mainly upon European mathematics, the story begins with a brief outline of Babylonian and Egyptian number systems, and it ends in the early 20th century with an account of the transfinite arithmetic of Georg Cantor. And, although the writing reflects great historical scholarship, readers aren't burdened with a plethora of footnotes. Instead, there is an extensive bibliography containing recommended reading for each chapter and for further study in general.

Two central themes pervade the text. Firstly, there is the crucial issue of mathematical notation in relation to associated concepts. Then there is the fact that number systems have developed in the context of particular areas of mathematics (geometry, algebra, analysis). Moreover, the evolution from ancient ideas on number

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to our logically sound modern number systems is shown to have been a long circuitous process involving very many famous (and lesser-known) mathematicians.

Another matter is the degree of controversy surrounding changing viewpoints on the nature of number. The Pythagoreans were outraged by the introduction of incommensurables, and the infinitesimals of Newton and Leibniz were scornfully dismissed by Bishop Berkeley. As for negative numbers, 200 years after Bombelli had recognized that 'più di meno via più di meno fa meno' (i.e. $\sqrt{-1} \times \sqrt{-1} = -1$), many mathematicians, such as the great John Wallis (1616-1703), still regarded negative numbers as 'fictions'. Much later (1901), the foundations of Frege's *Grundgesetze der Arithmetik* collapsed due to the intrusion of the Russell paradox.

Up to the 17th century, the concept of number was closer to that of the ancient Greeks than to 19th century formalised number systems. Prior to that, much preoccupation with equation solving had gradually revealed the need for broader classes of number–and it simultaneously led to the development of algebra as an identifiable branch of mathematics (the principle participants in this process being Viéte, Descartes, Oughtred, Harriot and Wallis). Later on, the emergence of calculus and real analysis raised questions about the nature of the real number system that were only satisfactorily resolved in the late 19th century by Dedekind, Cantor, Frege and Peano.

Use of complex numbers became established when Euler employed them as powerful tools in calculus and number theory. Gauss subsequently improved upon Argand's method of their geometric representation; without complex numbers he could never have proved the fundamental theorem of algebra. He also bequeathed us new numerical entities, $\mathbb{Z}[i]$ (Gaussian integers).

Leo Corry also explains how Hamilton, hoping to avoid dependence upon the debatable nature of $\sqrt{-1}$, chose to define complex numbers in terms of ordered pairs of real numbers (with no use of 'i'). But isn't it strange that, in a subsequent moment of insight into the basis of his quaternions, he should carve the equation '$i^2 = j^2 = k^2 = -1$' on a bridge over the River Liffey?

But what of errors and omissions? Apart from a few obvious typos (e.g. $6 \times 216 = 393{,}216$ on p. 158) the book has been produced to an excellent standard. But it would not have been remiss to mention Jost Burgi as a co-inventor of logarithms alongside Napier and Briggs. Finally, I feel that, for teachers, construction of the real number system via equivalence classes of rational Cauchy sequences would have more pedagogical relevance than Dedekind cuts.

Quibbles aside, Leo Corry has to be congratulated for producing this unique history of number from Greek arithmetic to Gödel's deflation of Hilbert's ideal. I shall return to it time and again.

10.1017/mag.2017.96
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Measure and integration by Hari Bercovici, Arlen Brown and Carl Pearcy, pp. 300, £46.99 (hard), ISBN 978-3-31929-044-7, also available as e-book, Springer Verlag (2016).

There is quite a variety of approaches to measure theory and in particular to the Lebesgue integral available in many texts. The preface of this one makes the claim that 'any beginning graduate student who has reached a certain level of mathematical maturity, which may be taken to mean the ability to follow and construct $\varepsilon - \delta$ arguments, should have no difficulty in mastering the material in this book'. 'No difficulty' is not a phrase I would have used. Indeed the text is austerely abstract from the outset, with results and exercises densely packed, leaving little room for motivation.

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