# From Mie's Electromagnetic Theory of Matter to Hilbert's Unified Foundations of Physics

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## 1. Introduction

On November 20, 1915, David Hilbert delivered a talk in Göttingen, presenting his new axiomatic derivation of the "basic equations of physics". This talk is often remembered because, allegedly, Hilbert presented in them, five days prior to Einstein, the correct, generally-covariant equations of gravitation that lie at the heart of the general theory if relativity.[1]

The published version of Hilbert's talk opens with the following words:

> The tremendous problems formulated by Einstein, as well as the penetrating methods he devised for solving them, and the far reaching and original conceptions by means of which Mie produced his electrodynamics, have opened new ways to the research of the foundations of physics. (Hilbert 1916)

Historians of science have devoted some efforts to examine the place of Hilbert's article in the history of general relativity and its possible influences on Einstein's work (Earman & Glymour 1978; Mehra 1974; Pais 1982, Chpt. 14). Much less attention has been paid to the place of this work in the context of Hilbert's own career. In particular, the specific role of Mie's theory —the second component explicitly alluded to in the passage quoted above— in consolidating Hilbert's ideas, as well as the background to Hilbert's interest in this theory have hardly been discussed.

In a series of recent articles I have tried to clarify the centrality of Hilbert's interest in physics for his overall scientific view, and in particular his increasing interest, from 1912 on, in questions related to the structure of matter. In the present article I discuss the contents of Mie's electromagnetic theory of matter and how it came to occupy such a central place in Hilbert's work.

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1. This received view, however, has recently been put into question. See Corry, Renn & Stachel 1997.

## 2. Gustav Mie's Electromagnetic Theory of Matter

Gustav Mie (1868-1957) studied in his native city of Rostock and then in Heidelberg, where he doctorated in mathematics in 1892 and served as assistant at the mineralogical institute. He taught physics at the polytechnical institute in Karlsruhe (1892-1902), and in 1897 he received his *Habilitation* from that institution. From 1905 he was professor of physics in Greifswald, from 1917 in Halle, and from 1924 and until his retirement in 1935, in Freiburg (Höhnl 1953, Mie 1948). Mie was a deeply religious man; both his parents came from pastors' families and he himself was strongly connected with the evangelist church throughout his life. His early interest in religion led him to consider the possibility of studying theology, before finally deciding himself for science. He had a good knowledge of German philosophy as well and especially of Kant.

Mie's basic knowledge of theoretical physics was acquired autodidactically while still in Heidelberg. Later on, in Karlsruhe, he had the opportunity to work with the fine collection of experimental devices with which Heinrich Hertz had conducted, several years before, his famous experiments on the propagation of electromagnetic waves. It was the mathematical elegance of the Maxwell equations, however, rather than the experiments connected with his theory that eventually attracted Mie's attention above all. In 1908 he published a path-breaking article in which he computed in strict electrodynamical terms processes of light scattering in spherical dielectrics, as well as in absorbing particles (Mie 1908). His results helped explaining color phenomena in colloidal solutions, and also led to the discovery of the so-called "Mie effect", which found important applications in astronomical as well as in military contexts. In 1910 Mie published a textbook on electromagnetism that soon became a classic and saw two additional editions in 1941 and 1948 (Mie 1910). Mie believed that this was the first textbook in which Maxwell's conclusions were arrived at in a completely inductive way starting from the experimental, factual material. When the first edition was published, Mie took special pride on having been able to "present the Maxwell equations in a complete and exact fashion, expressing himself in plain words, and without having to introduce any mathematical symbols." Later, however, he considered this perspective to have been mistaken, and preferred to lay all his stress on the mathematical aspects of the theory (Mie 1948, p. 739).

Mie sent the first installment of his electromagnetic theory of matter to the *Annalen der Physik* in January 1912 (Mie 1912, 1912a, 1913). At the center of Mie's theory was an articulate attempt to support the main tenets of the so-called "electromagnetic world view," and more specifically, to develop the idea that the electron cannot be ascribed physical existence independently of the ether. Of course, Mie was not the first to advance such an attempt,[2] but his theory was certainly much more mathematically elaborate than most of the earlier ones. Mie had hoped that in the framework of his theory, the existence of the electron with finite self-energy could be derived from the field in purely mathematical terms. What is usually perceived as material particles, he thought, should appear as no more than singularities in the ether. Likewise, compact matter should be conceived as the accumulation of "clusters of world-lines." Mechanics and electrodynamics would thus become the theory of the interaction of the field-lines inside and outside the cluster.

According to Coulomb's law, the field of a charged particle becomes infinite when its radius reduces to zero. Mie's equations generalized those of Maxwell's theory in such a way that the repulsive forces predicted inside the electron are compensated by other forces, of purely electrical nature as well. Moreover, outside the electron the deviation of Mie's equations from Maxwell's becomes undetectable.

Mie opened the presentation of his theory by pointing out that neither the standard laws of electrodynamics nor those of mechanics hold in the interior of the electron. The recent development of quantum theory and the discoveries associated with it suggested the need to formulate some new equations to account for the phenomena that take place inside the atom. Mie's theory was intended as a preliminary contribution to reformulating the necessary, new theory of matter. Its immediate aim was to explain in purely electromagnetic terms the existence of indivisible electrons. At the same time, however, Mie sought to present the phenomenon of gravitation as a necessary consequence of his theory of matter; he intended to show that both the electric and the gravitational actions were a direct manifestation of the forces that account for the very existence of matter.

Mie based his theory on three explicitly formulated basic assumptions. The first one is that the electric and the magnetic field are present inside the electron as well. This means that the electrons are in fact an organic part of the ether, rather than foreign elements added to the ether, as was common belief among certain physicists at the time (e.g., Einstein 1909). The electron is thus conceived as a non-sharply delimited, highly dense, nucleus in the ether that extends continually and infinitely into an atmosphere of electrical charge. An atom is a concentration of electrons, and the high intensity of the electric field around it is what should ultimately explain the phenomenon of gravitation. The second assumption is the universal validity of the principle of relativity (i.e., Lorentz covariance). The third one is that all phenomena affecting the material world can be fully characterized

2. For a description of the early development of the electromagnetic view of nature, with particular reference to the works of Lorentz, Wilhelm Wien, Poincaré, Abraham and Walter Kaufmann, see R. McCormmach 1970.

using the physical magnitudes commonly associated with the ether: the electric field *d*, the magnetic field *h*, the electric charge density ρ, and the charge current *j*.[3] While for Mie the validity of the principle of relativity was beyond any doubt, he considered his third assumption to be in need of further validation. Without stating it explicitly in the introductory section, Mie also assumed as obvious the validity of the energy conservation principle.

An additional constitutive element of Mie's theory is his adoption of the separation of physical magnitudes into "quantity magnitudes" and "intensity magnitudes". This separation, which essentially can be traced back at least to Maxwell (Wise 1979), appears as a central theme in Mie's conception of physics throughout his career, beginning with the first edition of his textbook on electricity and magnetism. "Quantity magnitudes" may be measured by successive addition of certain given units of the same kind: length, time duration, etc. Measuring "intensity magnitudes", on the contrary, is not accomplished by establishing a unit of measurement. Rather, one needs to establish a specific procedure according to which any given measurement of that magnitude can be attained. The foremost example of an intensity magnitude comes from the basic concept of mechanics: force. In the theory of elasticity the tension is an intensity quantity and the deformation is a quantity magnitude; in the kinetic theory the corresponding pair would be pressure and volume (Höhnl 1968).

This separation provides a certain coherence and symmetry to Mie's treatment of the electromagnetic theory of matter, but it does not really alter directly its actual physical content. The magnitudes mentioned in the third basic assumption of the theory, *h*, *d*, ρ, *j*, are four quantity magnitudes. Against them Mie introduced four intensity magnitudes: the magnetic induction, *b*, and the intensity of the electric field, *e*, and two additional ones, φ, and *f*. Mie did not assign any direct physical meaning to the latter two, and he simply stated that the four-vector (*f*, *i*φ) is in the same relation to (*j*, *i*ρ) as the six-vector (*b*, -*ie*) is to (*h*, -*id*). The introduction of these four intensity magnitudes allowed Mie to present an alternative formulation of the third assumption, namely, that all physical phenomena can be described in terms of the ten values involved in the four intensity magnitudes *b*, *e*, φ, and *f*.

Mie's version of the Maxwell equations comprise the following expressions:

$$\mathrm{rot}\,\boldsymbol{h} = \frac{\partial \boldsymbol{d}}{\partial t} + \boldsymbol{j}$$

$$\mathrm{div}\, d = \rho$$

3. Mie's notation is somewhat different from this one, which I use for the sake of simplicity.

$$\operatorname{rot} \boldsymbol{e} = -\frac{\partial \boldsymbol{b}}{\partial t}$$

$$\operatorname{div} b = 0$$

In fact, Mie presented his theory in the language of four- and six-vectors, originally introduced by Minkowski in his work on electrodynamics. This language had later been elaborated into the standard one for relativity theory by Arnold Sommerfeld (1910). In many respects, however, Mie's notation and his usage of these concepts resembled more directly that of Minkowski than that of Sommerfeld. In Mie's formulation, for instance, a possible connection with a tensorial theory of gravitation such as Einstein's was not particularly perspicuous; it only became so, after Born reformulated Mie's theory in 1913 with a more suggestive notation. (See section Born's Formulation of Mie's Theory).

From the issues discussed in Mie's theory of matter, two are especially relevant for discussing Hilbert's later work: the energy principle and gravitation. In Mie's theory, the concept of energy is formulated in terms of a scalar function *W*, the energy density, which, as a consequence of the Maxwell equations must satisfy the field equation

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*S* being the energy current vector. The energy conservation principle demands that *dW* be an exact differential, and Mie showed that this demand is fulfilled whenever *W* can be expressed in terms of the four parameters *d*, *h*, ρ *and j*. Moreover, this function can be determined in terms of a second scalar function *H*, of the same parameters, which must satisfy the equation

$$W = H + h \cdot b + j \cdot f.$$

Mie investigated several aspects of his theory of gravitation, such as the relations between the equations and the energy principle, the invariants that appear in the theory, the principle of action and reaction, and the relation between gravitational and inertial mass. A central point in this d $\frac{\partial W}{\partial t} = -\operatorname{div}\boldsymbol{S}$ the status of the gravitational potential ω. Since the latter appears in the t the basic dynamic variables, it follows that the absolute value of the potential —rather than only potential differences— directly influences physical phenomena. Still, for regions of constant potential, the form of the equations guarantee that its effects can be fully taken into account by suitable rescaling all other dynamic variables. Thus, the effect of a constant gravitational potential could be made to become imperceptible for any given observer. The possibility of doing this is what Mie called "the principle of the relativity of the gravitational potential," which he explicitly formulated as follows:

> If two empty spaces differ from each other only in the fact that in the first one the average value $\omega_0$ of the gravitational potential is very large while in the second one it is zero, then this difference has no influence whatsoever on the size and form of the electrons and of the other material particles, on their charge, on their laws of oscillation, and on other motion laws, on the speed of light, and in general on any physical relations and processes. (Mie 1913, p. 63)[4]

The validity of this principle summarized for Mie the differences between his and other, contemporary theories of gravitation, especially those of Max Abraham and of Einstein.

In concluding this section, Mie expressed the belief that his brief discussion was enough to prove that the basic assumptions of the theory led to no contradiction with experience, even in the case of gravitational phenomena. Preparing the way for a possible empirical confirmation of the law of gravitation, said Mie, was one of the main aims of his article, but he admitted that, at this stage, the results of his research did not really help at that. Two results derived from his theory, which in principle might be thought of as offering that possibility could not as a practical matter do so. The first was the relation obtained in the theory between inertial and gravitational mass. The two are identical, according to Mie, only if there are no motions inside the particle, and in general they are in a relation that depends on the temperature and on the atomic weight. The observable differences between the gravitational acceleration of two bodies of different masses would be, according to this account, of the order between $10^{-11}$ and $10^{-12}$ and therefore they would be of no help in constructing an actual experiment. The second result concerned the existence of longitudinal waves in the ether, also too small to be detectable by experiment (Mie 1913, p. 64). It is note-worthy that these remarks are strikingly similar in both contents and style to those formulated by Minkowski in 1907 to conclude his main paper on electrodynamics and relativity (Minkowski 1911, p. 404).

4. Unless otherwise stated, all translations from German are mine.

The theory contained several difficulties that Mie was never able to work out successfully, yet he never really abandoned his belief in its validity. The most serious shortcoming of the theory is connected with the fact that it depends on an absolute gravitational potential, and therefore the equations do not remain invariant when we replace the potential *w* by a second potential *w+const.* Under these conditions, a material particle will not be able to exist in a constant external potential field. Moreover, in retrospective it is also clear that Mie's theory did not account for either red shift or light bending, but these issues did not really become crucial until much later.[5]

Mie published his electromagnetic theory of matter at a critical time from the point of view of the development of a relativistic theory of gravitation by Einstein and by others. In (1913) Einstein published together with Marcel Grossmann the so-called *Entwurf* paper, containing the first serious, articulate effort to formulate a "generalized theory of relativity and a theory of gravitation". Einstein and Grossmann proved in their first publication that the equations of gravitation were invariant only with respect to linear transformation. Later, in early (1914), they also proved the invariance of the equations with respect to non-linear transformations of a restricted kind. However, already by August 1913, Einstein harbored some doubts concerning his *Entwurf* theory and he was ambivalent about its validity. In a letter to Lorentz he confessed that his theory had only one 'dark spot', i.e., that it was not generally covariant.[6] On September 23, 1913, Einstein lectured at the 85th Congress of the German Natural Scientists and Physicians in Vienna, and discussed the current state of his research on gravitation (Einstein 1913). Several other physicists that attended the meeting were also working at the same time on gravitation. Among them was Mie, and also Max Abraham and Gunnar Nordström were present. Einstein's paper gave rise to a heated discussion.[7] Einstein had been recently collaborating with Nordström in developing some central ideas of the latter's work. Abraham, on the other hand, had been involved in 1912 in a caustic debate with Einstein; Einstein had had a hard time finding the right arguments to defend himself against Abraham's attack on his early attempts to formulate a relativistic theory of gravitation. Still, Abraham was one of the few physicists whose opinion concerning his own theory, Einstein really valued.[8]

In Vienna, Einstein declared that, among the theories represented in the meeting, he favored that of Nordström most, because it complied with several physical principles which his own *Entwurf* theory also satisfied, and among which was the principle of equivalence of the inertial and the gravitational masses. In fact, Einstein did not even mention Mie's theory. Following a question of the latter, Einstein explained that unlike the others,

5. For an historical account of light bending as a test for relativity, see Earman & Glymour 1980. For a parallel account of red shift, see Earman & Glymour 1980a.
6. On August 16, 1916. See Norton 1984, p. 150 (ftn. 1).
7. See Pais 1982, pp. 228-238. The discussion was published in the *Physikalische Zeitschrift* as an appendix to Einstein 1913. See M. Klein et al (eds.) 1995, Doc. 18.
8. See Cattani & De María 1989.

Mie's theory did not satisfy the principle of equivalence and therefore he had not really studied it in detail.[9] More privately, in a letter written to Erwin Finlay Freundlich that same year, Einstein confided that Mie's theory was "fantastic and has, in my opinion, a vanishingly small inside chance" of being right.[10]

In December of 1913 Mie wrote a detailed criticism of Einstein's theory that was published in the *Physikalische Zeitschrift*, where also Einstein's talk at the Vienna meeting had appeared. Among other things, Mie claimed that the relevant perspective from which to consider the invariance of Einstein's theory was that offered by the principle of relativity of the gravitational potential, rather than that of a generalized principle of the relativity of motion. Moreover, Mie stressed the difficulties implied by a tensorial theory over a scalar one, difficulties he considered not to be justified by any evident advantages of the former approach (Mie 1914, pp. 169-172).[11] Einstein replied to Mie's criticism in the same issue of the *Zeitschrift* (Einstein 1914), but his reply was essentially a further clarification of his own theory, rather than a direct criticism of Mie's theory or a rebuttal of his arguments.

It is worth noticing that Mie wrote his criticism of Einstein's theory at a time when he was working on a broader report on the current state of research on gravitation. This report had been commissioned for the *Jahrbuch der Radioaktivität und Elektronik* by the editor Johannes Stark. On December 10, 1913 Mie wrote to Stark accepting with pleasure the invitation to write the report. However, on March of the next year, Mie explained in a second letter to Stark that the article he had written for the *Physikalische Zeitschrift* had consumed too much of his energies and he preferred to be released from his earlier commitment. Mie also explained how he saw now the relationship between his and Einstein's theory of gravitation. In the letter he wrote:

> You may have perhaps wondered why I haven't contacted you again concerning the report on the theory of gravitation. You have meanwhile received a reprint of my article on Einstein's theory that cost me a great deal of effort. As you probably know, Einstein has answered this article, while claiming that I haven't understood his main ideas. He is right to the extent that I have not touched upon them in my article. I have done this deliberately, and I must admit that I don't fully understand Einstein's point of view. I have the feeling that the way he is embarking upon is very alluring and interesting but nevertheless an incorrect one. Meanwhile I think that some time will still have to pass by before that becomes clear.

9. Klein et al (eds.) 1995, p. 506.
10. Quoted in Pyenson 1985, p. 186. See also M. Klein et al (eds.) 1993, Doc. 468.
11. A similar criticism appears in Mie 1915 p. 252.

> In any case, I could write a report on gravitation only from my point of view, which, to begin with, is completely different from Einstein's. Obviously, I should also present Einstein's views, but under the present circumstances I could not give them a correct appreciation. If that helps you, I could write the report over the summer semester. Yet, I must tell you this one thing: that the theory of gravitation is by no means the main question that I address in my theory of matter, but rather there are other problems that seem to be much more important than that.[12]

Mie added in his letter that he was planning to start in the winter writing a new book on the basic ideas of his theory and its consequences. Instead of the requested report for the *Jahrbuch*, Mie proposed to send a review of his forthcoming book. In his view this would be more important than a general report on gravitation. Stark, from his side, finally decided to ask Max Abraham to write the desired report, which the latter sent for publication in December of 1914 (Abraham 1915).[13]

Whatever projected book Mie had in mind when writing his letter to Stark, he did not get to publish it over the following years. His comments on a lesser interest in gravitation than on other issues may have been caused by resentment following the Vienna conference and its aftermath. As already mentioned, Mie wrote very explicitly in his early articles that an explanation of gravitation would be an important by-product of his theory, though he certainly did not present it as the main task. But Mie continued to lecture and publish on gravitation over the years (Mie 1917, 1921) and, in fact, to relate to certain aspects of Einstein's work in a somewhat critical attitude (Hentschel 1990). In a letter written to Hilbert on February 13, 1916, shortly after the publication of the latter's first article on relativity, Mie still referred back to the discussions held in the Vienna meeting. He manifested his general skepticism towards the idea of a "general relativity", but at the same time he confessed that Hilbert's recent article helped him realize that Einstein had been very close to the truth from the beginning. Still, Mie did not believe that Einstein

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12. These two letters of Mie to Stark, and an additional one dated August 17, 1917, are preserved in Stark's *Nachlass*, Staatsbibliothek Berlin.

13. A letter from Abraham to Stark, dated October 10, 1914, confirming his agreement to write the report is preserved in Stark's *Nachlass*, Staatsbibliothek Berlin, Preußische Kulturbesitz.

would attain what he had announced as the aim of his research.[14] In 1921 Mie published a short monograph on Einstein's theory, where he admitted that the current development of the theory was satisfactory from his point of view. Concerning the validity of the postulate of invariance under arbitrary coordinate transformations, he wrote:

> I think that many of my [non-mathematical] readers will be astonished that it might be possible at all to satisfy that postulate. In fact, I believe that many professionals will have to concede that at the time when Einstein was still looking for the correct way to apply it, they doubted that he would possibly succeed. The author of this essay must confess that he himself belonged to these skeptics. It took Einstein many years until the problem had attained the clarity that led to its solution. Finally, however, he found the way to rely on the geometrical research of several mathematicians, and especially of the genial Riemann, that had worked out the most general geometries of many-dimensional continua. Einstein filled up the formerly pure mathematical thoughts of these researchers with physical contents and thus finally obtained his theory. [15]

Mie's theory of matter, then, and his attempt to explain gravitation in electromagnetic terms, had a rather convoluted and unfortunate development. Still, it succeeded in attracting the attention of Hilbert from very early on. In fact, the sequence of events that led to Hilbert's foundational, unified physical theory started with his interest in Mie's theory as a viable theory of matter. It was only later that he sought to combine this theory with Einstein's quest for general covariance, leading to put forward what he considered to be "The Foundations of Physics" in general. The next section describes Hilbert's encounter with Mie's theory.

## 3. Born's Formulation of Mie's Theory

Max Born was the first among the Göttingen scientists to become interested in Mie's theory and to dedicate actual efforts to study and develop it. In fact, it was only through Born's reformulation of the theory, and perhaps through his personal mediation, that Hilbert got to adopt it as one of the central pillars of the unified foundation of physics that he was about to develop over the following years. Mie's theory connected naturally with Born's immediate scientific concerns. Between 1904 and 1907 Born had studied in Göttingen. Then, in 1908 he returned to that city to work with Minkowski on relativity and on electron theory—topics of common interest for the two at the time. A central issue

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14. The letter is in the Hilbert *Nachlass,* NSUB Göttingen, Cod Ms David Hilbert 254/2. A similar point is discussed in Mie 1917, p. 600.

15. Mie 1921, p. 61

among researchers involved in early research in electron theory was the question whether the electron was a rigid or a deformable sphere: Abraham subscribed to the former view, whereas Lorentz to the latter. Born addressed the apparent contradiction between the classical concept of rigidity and the principle of relativity, and introduced the relativistic concept of rigid movement. Addressing an issue that departed from Minkowski's main focus of interest, Born pursued in his work of 1909 the question of the ultimate nature of matter, and in particular questions concerning the physical properties of the electron.[16] The main issues addressed in Mie's theory thus directly appealed to Born's current basic interests.[17]

The first notice we have of Mie's theory being discussed in Göttingen is in December 17, 1912, when Born presented it at the meeting of the Göttingen Mathematical Society (GMG).[18] By that time Hilbert was deeply immersed in his research on the kinetic theory and on radiation theory (Corry 1998). On the face of it, then, the questions addressed by Mie in his article must have strongly attracted Hilbert's attention. What exactly was discussed there and what were the reactions to it, we don't really know. We do know, however, that the lecture notes of the courses Hilbert taught in the winter semester of 1912-13 ("Molecular Theory of Matter"), or in the following semester ("Electron Theory") —in spite of their obvious, direct connection with the issue— show no evidence of a sudden interest in Mie's theory or in the point of view developed in it (Corry 1999). Apparently, whatever Hilbert learnt from Mie's theory in this early occasion, it did not offer him any new element of direct interest. Possibly, this was connected to the fact that Mie's strong electromagnetic reductionism was contrary to Hilbert's current views, that also favored reductionism, but from a mechanistic perspective. Born, on the contrary, seems to have been immediately attracted by Mie's theory, since he continued to work on it by himself. On November 25, 1913, Born lectured again at the GMG on Mie's theory.[19] On December 16, he presented to the same forum his own contribution to the theory, dealing with the form of the energy laws in it. This time, he does seem to have caught Hilbert's attention.

16. For a discussion of the connections between Born and Minkowski in this context, Corry 1997a, § 5.

17. In fact, their domains of interest overlapped also in later opportunities. See Born 1938.

18. See the announcement in the *Jahresbericht der DMV* (*JDMV*) Vol. 22 (1913), 27. There is no direct evidence of the contents of Born's lecture at this opportunity.

19. See the announcement in *JDMV* Vol. 22 (1913), 207.

Born's lecture (1914), soon thereafter submitted by Hilbert for publication, was intended as a clarification of the mathematical structure of Mie's electrodynamics. Born was perhaps the physicist whom Hilbert's views on physics influenced most strongly of all, at least in what concerns the way physical theories have to be formulated and analyzed. The kind of clarification he was aiming at was therefore very close to Hilbert's line of interest from the beginning. Born stressed above all the role of the variational argument underlying Mie's theory, as well as the similarity of the latter with the classical, analytical approach to mechanics. In this way, Born's presentation connected Mie's ideas directly to Hilbert's general views on physics, and in particular to the views he had put forward in his lectures on the axiomatic method (Corry 1997). This presentation had thus a much more direct appeal to Hilbert than Mie's own one. As a matter of fact, Born's talk was introduced to the GMG by Hilbert himself.

A second innovative aspect of Born's formulation concerns his general approach and his use of notation. Born's point of view was more general than Mie's, and his presentation was tensorial in spirit, although he did not explicitly use this word. This approach allowed him to connect the specific discussion undertaken here with a much broader physical framework, which included a treatment of elasticity parallel to that of electrodynamics. Rather than speaking of the electromagnetic ether and its properties, like Mie had done, Born referred to a general four-dimensional continuum of the coordinates $x_1$, $x_2$, $x_3$, $x_4$, and to the deformations affecting it. The latter are expressed in terms of the projections $u_1$, $u_2$, $u_3$, $u_4$ (on a system of four orthogonal axes) of the displacements (*Verrückungen*) of the points of the continuum. The four basic electromagnetic magnitudes referred to by Mie, $d$, $h$, $\rho$ and $j$, appear in Born's article as no more than particular functions of the four coordinates. Born's discussion of the energy conservation principle using this formulation prepared the way, as we will see below, to Hilbert's connection between this theory and Einstein's general relativity. It is noteworthy, that precisely one week before Born's lecture at the GMG, this forum heard, perhaps for the first time, a report on Einstein and Grossmann's *Entwurf* theory.[20] The published version of Born's lecture does not record any mention of a direct relation between the latter and Mie's theory, but Born's tensor-like formulation suggests that such a connection may at least have been suggested by the participants.

20. See the announcement in the *JDMV,* Vol. 22 (1913), p. 207. Unfortunately, also the contents of this lecture are not documented.

We can only guess what effect the tensorial aspect of Born's presentation may have actually had in attracting Hilbert's attention at this stage. What is clear is that its opening sentences could not have failed to do so. Born stressed the importance of Mie's contribution by comparing it to ealier works in the theory of the electron that had been based on Lorentz's ideas: whereas for the latter it had always been necessary to assume specific, *physical* hypotheses concerning the nature of matter itself, Mie had attempted to derive the existence of electrons as knots in the ether based only on *mathematical* consideration applied to a modified version of the Maxwell equations. As Born was well-aware, in his lectures on the axiomatization of physical theories Hilbert had recurrently stressed the centrality of this methodological principle, namely, the need to separate the analysis of the logico-mathematical structure of the theory from any specific assumption on the ultimate nature of nature. Minkowski had also followed this principle in his work on electrodynamics (Corry 1997a), and it had certainly influenced Born in his own work.

Born explained the central ideas of Mie's theory by analogy with Lagrangian mechanics. The equations of motion of a mass system, he said, can be derived using the Hamiltonian principle, by stipulating that the integral

$$\int_{t_2}^{t_1}(T-U)dt$$

has to attain a minimal value. Here $T$ - $U$ is the Lagrangian function, which is a function of the position $q$ and of the velocity $\dot{q}$ of the system:

$$T-U=\Phi(\dot{q},q)$$

The equations obtained from the variational principle are thus well-known:

$$\frac{d}{dt}\frac{\partial\Phi}{\partial\dot{q}}-\frac{\partial\Phi}{\partial q}=0 \qquad (1)$$

In mechanics, Born explained, one has the relatively simple case of a quasi-elastic system, in which the function $\Phi$ has the form $\Phi=\frac{a}{2}\dot{q}^2+\frac{b}{2}q^2$. One can also have, however, a more general case in which $\Phi$ is taken to be any arbitrary function satisfying the basic differential equation ( 1). The relation of Mie's theory to classical electrodynamics Born saw as parallel to that between these two possibilities in mechanics:

> Mie's equations play the same role for electrodynamics than Lagrange's second-order equations do for the mechanics of systems of points: they provide a formal scheme that, through a suitable choice of the function Φ, can be made to fit the special properties of the given system. Very much like in earlier times the aim of the mechanistic explanation of nature was pursued by assuming a Lagrangian function Φ that describes the interactions among atoms, and from which all physical and chemical properties of matter could be derived, so has Mie set forward the task of choosing a specific "World-function" Φ, in such a way that, starting from that function and from the basic differential equation it satisfies, not only the very existence of the electrons and of the atoms might be derived, but also the totality of their interactions will emerge. I would like to consider this requirement of Mie as embodying the mathematical contents of that program that has set down as the main task of physics the erection of an "electromagnetic world-view." (Born 1914, pp. 24-25)

Born was alluding here to several issues that were highly appealing to Hilbert's sensibilities. First, the analogous conception of mechanics and electrodynamics in terms of a variational derivation. At least since Hilbert lectured on the axiomatization of physics 1905, Born had repeatedly heard him postulating the possibility and the need to unify physical theories in these terms: the crucial step in any case would be the choice of the suitable Lagrangian function (Corry 1997, 170-175). Like Minkowski and like Hilbert, but unlike most other physicists, Born called this Lagrangian "World-function." Second, Born knew that Hilbert's sympathy for the mechanical reductionism was subsidiary to the mathematical simplicity that should support it. If it turned out that an electromagnetic reductionism would be simpler in mathematical terms than the mechanistic one, then Hilbert would be inclined to follow the former rather than the latter. Finally, and connected to the second, the last sentence of Born's quotation seems to allude to the famous concluding passage of Minkowski's "Space and Time":

> The validity without exception of the world-postulate, I like to think, is the true nucleus of an electromagnetic image of the world, which, discovered by Lorentz, and further revealed by Einstein, now lies open in the full light of day. (Minkowski 1909, p. 444)[21]

Born suggested that a consistent pursuit of the line of thought adopted by Hilbert and Minkowski —in which the mathematical and logical structure of the theory matters above all and in which any commitment to specific physical underlying assumptions should be avoided as much as possible— would naturally lead to pay close attention to Mie's theory.

21. For a reinterpretation of the meaning of this passage, see Corry 1997a, p. 298.

In the body of his treatment, and according to the tensor-like spirit of the presentation, Born introduced the notation

$$\frac{\partial u_\alpha}{\partial x_\beta} = a_{\alpha\beta},$$

and demanded that all the properties of the continuum might be deduced alone from the projections of the displacements $u_\alpha$ and their derivatives $a_{\alpha\beta}$. In this way, the variational principle is applied to an integral of the form

$$\int \Phi(a_{11}, a_{12}, a_{13}, a_{14}; a_{22}, \ldots, a_{44}; u_1, \ldots u_4) dx_1 dx_2 dx_3 dx_4.$$

If, in addition, one introduces the notation

$$\frac{\partial \Phi}{\partial a_{\alpha\beta}} = X_{\alpha\beta}, \quad \frac{\partial \Phi}{\partial u_\alpha} = X_\alpha$$

then the variational principle leads to equilibrium equations that can be expressed as

$$\sum_\gamma \frac{\partial X_{\beta\gamma}}{\partial x_\gamma} - X_\beta = 0.$$

Born proceeded now to derive the energy momentum conservation principle as a generalization of energy conservation in mechanics, and using the fact that his Lagrangian did not explicitly depend on time. Born started from the general assumption that $\Phi$ depends on the magnitudes $a_{\alpha\beta}$ exclusively through the differences

$$a_{\alpha\beta} - a_{\beta\alpha} = \frac{\partial u_\alpha}{\partial x_\beta} - \frac{\partial u_\beta}{\partial x_\alpha}. \tag{2}$$

These differences can be interpreted as the components of the infinitesimal rotation of a volume element of the continuum in the four-dimensional world. Born showed that in Mie's theory, these components appear as the coordinates of the six-vector (***M***, -*i**e***), where ***M*** represents the magnetic induction and ***e*** the intensity of the electric field. The values of the rotational components are obtained from the determinant

$$(a_{\alpha\beta} - a_{\beta\alpha}) = \begin{vmatrix} 0 & -M_z & M_y & ie_x \\ M_z & 0 & -M_x & ie_y \\ -M_y & M_x & 0 & ie_z \\ -ie_x & -ie_y & -ie_z & 0 \end{vmatrix}$$

If Φ is independent of the four coordinates $x_1, x_2, x_3, x_4,$ then the energy momentum conservation principle is valid in the theory and it can be reformulated as follows:

$$\frac{\partial \Phi}{\partial x_\alpha} = \sum_\gamma \frac{\partial}{\partial x_\gamma}\left(\sum_\beta X_{\beta\gamma} a_{\beta\alpha}\right).$$

If one defines a 4×4 matrix $T$

$$T_{\alpha\beta} = \Phi\delta_{\alpha\beta} - \sum_\gamma a_{\gamma\alpha} X_{\gamma\beta}$$

( 3)

then the principle takes the form

$$\text{Div } T = 0.$$

( 4)

This general result can be specialized to the case of Mie's theory, given that its Lagrangian is assumed to be independent of the four coordinates $x_i$, This assumption, Born stated (1914, p. 32), "is the true mathematical reason for the validity of the energy momentum conservation principle" in the theory. On the other hand, Born also relied on the dependence of the Lagrangian function on the $a_{\alpha\beta}$ via the expressions ( 2). He thus defined a new 4×4 matrix $S$, $S = T + \omega$, where

$$\omega_{\alpha\beta} = \sum_\gamma a_{\alpha\gamma} X_{\gamma\beta} - u_\alpha X_\beta .$$

Born showed now easily that Div $\omega = 0$, from which he obtained, finally,

$$\text{Div } S = 0.$$

( 5)

As will be seen below, in Hilbert's 1915 lecture on general relativity this matrix $S$ is alluded to as "Mie's stress energy tensor", and it plays a central role in the theory. In defining it, Born was introducing a magnitude which is not dependent only on the field's strength yet satisfies the energy equation.

It must be stressed that in the body of Born's article gravitation is barely mentioned, thus suggesting his awareness to the problematic status of this phenomenon in the framework of Mie's theory. Born declared that the theory, in the variational formulation he was proposing here, was an extension of Lagrange's "magnificent program":

the theory attempts to find the appropriate world-function from which all the electromagnetic properties of the electrons and the atoms might be derived. All properties, that is, except gravitation, which, as Born explained in a footnote added at this point (1914, pp. 31-32), was left outside the scope of the article.

It is likely that Born had discussed these ideas with Hilbert way before the actual lecture was delivered at the GMG. As a matter of fact, we have direct evidence of Hilbert's interest on Mie's work, dating from before that lecture. On October 22, 1913, Mie wrote a letter to Hilbert expressing his satisfaction for the interest that the latter had manifested (in an earlier letter, which has not been preserved,) in his recent work.[22] Thus, it was probably not necessary for Born, at this stage, to phrase his introduction with the specific task in mind of convincing Hilbert of the importance of Mie's theory and of the power of its concomitant electromagnetic world-view. But it seems clear that under the formulation embodied in Born's presentation and for the reasons alluded in his introduction, Hilbert himself could not have failed to recognize the direct allure of Mie's theory to his own current concerns. Still, some time was needed until Hilbert came to adopt fully the view of physical reality presupposed by Mie's theory. In his lectures on electromagnetic oscillations, during the winter semester of 1913-14, we find clear indications that Hilbert had begun to think seriously about this theory, but until his talk of November 1915 on the fundamental equations of physics he never mentioned Mie's theory explicitly either in his published work or in the manuscript of the lectures that have been preserved.

## 4. Hilbert's Communication and Mie's Theory

Although we do not know with certainty when Hilbert finally adopted Mie's theory as a possible basis for a unified foundation of physics in general, we do know that Born was instrumental in the process leading to it. The second pillar of Hilbert's theory was provided by Einstein's work on general relativity, about which Hilbert had at least some idea by the end of 1913. Einstein was invited to discuss the current state of his theory in Göttingen and he visited there between June 29 and July 7, 1915. Unfortunately, the exact content of his lectures in Göttingen is unknown to us,[23] and yet it is clear that he consid-

22. Mie's letter is in Hilbert's *Nachlass*, NSUB Göttingen - Cod Ms David Hilbert 254/1.

ered his visit to have been a complete success. He felt that his theory had been understood to the details and he was deeply impressed by Hilbert's personality (Hermann 1968, p. 30). Hilbert, in turn, was likewise impressed by the younger Einstein (Pyenson 1985, p. 193).

Einstein's trip to Göttingen came after more than two years of intense struggle with the attempt to formulate a generalized theory of relativity. He had initially abandoned the demand of general covariance as part of his theory, after coming to the conclusion that generally covariant field equations would necessarily lack any physical interest, because they would contradict the principle of causality. The ground for this conclusion was the so-called "hole argument", which he introduced first in the *Entwurf* paper of 1913, and later articulated most clearly in a summary of the latter, presented in October 1914 to the Berlin Academy of Sciences (Einstein 1914). Quite certainly, Einstein's lectures in Göttingen did not depart significantly from what he had presented in this summary.

Einstein's quest for a relativistic theory of gravitation was eventually crowned with success only after he abandoned completely the 'hole argument', and adopted general covariance again as a leading principle of that theory. Einstein's confidence on the validity of the argument, however, did not begin to erode until mid-October 1915. He thus embarked in the effort that led him to present four consecutive papers at the weekly meetings of the Berlin Academy, starting on November 4. The fourth paper, presented on November 25, contained his final version of the generally covariant field equations of gravitation (Norton 1984, 138-152). Over this crucial month of November, Einstein and Hilbert engaged in an intensive correspondence in which they reported to each other, in "real time", about their current progress in developing their respective results. They also continued to correspond with each other after presenting their respective works. A detailed analysis of the interchange of ideas between Einstein and Hilbert, and of their possible mutual influence is, of course, an enormously interesting and important topic, and it will be addressed in a sequel to the present article.[24] In the rest of this section I will be focusing on the connection between Mie's and Hilbert's theory.

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23. I have made some efforts to gather documents related to this visit, so far without much success. What I did find in Hilbert's *Nachlass* in Göttingen, nevertheless, are the handwritten notes taken by an unidentified person at he first of Einstein's lectures (Staats- und Universitäts Bibliothek, Göttingen, Cod Ms D Hilbert 724). These notes have now been published in, Kox et al (eds.) 1996, App. B, pp. 586-590.

24. A preliminary discussion of this topic appears in Corry, Renn & Stachel 1997.

Hilbert's communication appeared in print on March 1916 in the proceedings of Göttingen Academy of Science under the title: "The Foundations of Physics". This printed version, however, differs substantially from what he actually presented in his talk, as we learn from a document I discovered only recently in Hilbert's *Nachlass* and which sheds much light upon this entire story: the proof galleys of the communication, dated December 6, 1915.[25] The most significant differences between the two versions are not marked on the proofs themselves, and they were probably introduced somewhat later, that is, after December 6. It is worth noticing, moreover, that Hilbert's article was republished again in 1924 in the *Mathematische Annalen* with some additional, interesting changes, and yet once again with additional editorial comments in 1932, in the third volume of Hilbert's collected works. Typically, Hilbert did never mention any of the major changes he introduced between the various versions. In (1924, p. 1), for instance, Hilbert explained that he was basically reprinting what had appeared in the past in two parts, with only minor editorial changes. For lack of space the detailed analysis of these interesting changes will also remain for a future opportunity.

Hilbert's theory took from Einstein the account of the structure of spacetime in terms of the metric tensor. Mie's theory served as a basis for explaining the structure of matter in terms of the electromagnetic fields. To these two elements Hilbert applied powerful mathematical tools taken from the calculus of variations and from Riemannian geometry.

The first axiom of Hilbert's theory of gravitation (which he called: "Axiom I: Mie's axiom of the world-function") is based on a variational argument. The axiom is formulated for a scalar Hamiltonian function[26] $H(g_{\mu\nu}, g_{\mu\nu l}, g_{\mu\nu lk}, q_s, q_{sl})$, whose parameters are the ten gravitational potentials $g_{\mu\nu}$, together with their first and second derivatives

$$g_{\mu\nu l} = \frac{\partial g_{\mu\nu}}{\partial \omega_l},\ g_{\mu\nu lk} = \frac{\partial^2 g_{\mu\nu}}{\partial \omega_l \partial \omega_k} \qquad (l,k = 1,2,3,4)$$

25. *Nachlass* David Hilbert, NSUB Göttingen, Cod Ms 634.

26. At this point a terminological clarification may be in order. In present-day terms, this function would be more properly called a Lagrangian function, while the term "Hamiltonian" usually refers to functions involving momenta and representing the total energy of the system considered. See, e.g., Lanczos 1970, Chapt. IX. For the purposes of the present article and for the sake of historical precision, however, it seems more convenient to abide by the original terminology.

and the four electromagnetic potentials $q_s$, together with their first derivatives $q_{sl}$. The gravitational potentials $g_{\mu\nu}$ are the components of a symmetric tensor and, like in Einstein's theory, they constitute the metric tensor of a four-dimensional manifold. The electromagnetic potentials behave like vectors with respect to the four world-parameters $\omega_l$ ($l$ = 1,2,3,4). The Hamiltonian is used to derive the basic equations of the theory, starting from the assumption that, under infinitesimal variations of its parameters, the variation of the integral

$$\int H\sqrt{g}\,d\omega$$

(where $g = |g_{\mu\nu}|$, and $d\omega = d\omega_1 d\omega_2 d\omega_3 d\omega_4$) vanishes for any of the potentials. In fact, instead of the covariant magnitudes $g_{\mu\nu}$ and their derivatives, Hilbert used consistently the contravariant tensor $g^{\mu\nu}$ and their derivatives throughout the argument.[27] The second basic axiom of the theory (Axiom II: axiom of general invariance) postulates that $H$ is invariant under arbitrary transformations of the coordinates $\omega_l$.

At this stage, Hilbert added two interesting footnotes that explain the relation of the two first axioms with Mie's and with Einstein's works respectively. First, he said, Mie himself had not included the electromagnetic potentials and their derivatives in the world-function, but rather this had been a contribution of Born. Thus, from this point on, it is clear that Hilbert will be referring to Born's version of the theory, rather than to Mie's original one. Still, what characterizes Mie's theory, Hilbert explained, is the very introduction of the world-function as part of the Hamiltonian. Second, Mie had postulated the demand of orthogonal, rather than general covariance. But whereas in Einstein's work the Hamiltonian principle plays only a secondary role, Axiom II expresses in the simplest way his demand for general covariance.

Besides the two basic axioms, the core of Hilbert's derivation is based on a central mathematical result (Theorem I), which Hilbert initially described as the *Leitmotiv* of the theory. According to this theorem the number of equations that can be obtained from the variational integral is in fact smaller than the fourteen that one would expect to attain on the face of it. More specifically, in the first printed version of the theory Hilbert formulated the theorem as follows:

> Theorem I. Let $J$ be a scalar expression of $n$ magnitudes and their derivatives that is invariant under arbitrary transformations of the four world-parameters, and let the Lagrange variational equations corresponding to the $n$ magnitudes be derived from the integral

27. In a course taught at Göttingen in 1916-17, Hilbert explicitly explained that this is done for reasons of convenience. See Hilbert 1916-17, 109.

$$\delta \int J \sqrt{g} d\omega = 0.$$

> Then, in the system of *n* differential equations on *n* variables obtained in this way, four of these equations are always a consequence of the other *n* - 4, in the sense that four linearly independent combinations of the *n* differential equations and their total derivatives are always identically satisfied. (Hilbert 1916, 397)

The variational principle introduced above yields ten equations for the gravitational potentials and four for the electromagnetic ones:

$$\frac{\partial \sqrt{g} H}{\partial g^{\mu\nu}} - \sum_k \frac{\partial}{\partial \omega_k} \frac{\partial \sqrt{g} H}{\partial g_k^{\mu\nu}} + \sum_{k,l} \frac{\partial^2}{\partial \omega_k \partial \omega_l} \frac{\partial \sqrt{g} H}{\partial g_{kl}^{\mu\nu}} = 0 \quad (\mu, \nu = 1,2,3,4) \tag{6}$$

$$\frac{\partial \sqrt{g} H}{\partial q_h} - \sum_k \frac{\partial}{\partial \omega_k} \frac{\partial \sqrt{g} H}{\partial q_{hk}} = 0 \qquad (h = 1,2,3,4) \tag{7}$$

Hilbert denoted the left-hand sides of these equations as $[\sqrt{g} H]_{\mu\nu}$ and $[\sqrt{g} H]_h$, and called them the fundamental equations of gravitation and of electrodynamics respectively. Theorem I was obviously conceived with the intention of being applied to these equations, thus leading to the claim that four of them are in fact consequences of the other ten. In particular, Hilbert concluded, the four equations $[\sqrt{g} H]_h = 0$, are a consequence of the ten gravitational ones, $[\sqrt{g} H]_{\mu\nu} = 0$. This latter conclusion amounted, Hilbert suggested, to nothing less than a definitive explanation of the intimate interconnection between the two kind of physical phenomena involved:

> Based on the above theorem we can advance the following claim: *in the already indicated sense the electrodynamic phenomena are an effect of gravitation.* By recognizing this, I discern the simple and very surprising solution of the problem of Riemann, who was the first to search for a theoretical connection between gravitation and light. (Hilbert 1916, 397-398)[28]

28. Hilbert was presumably referring here to a short paper on gravitation and light taken from Riemann's *Nachlass* (Riemann *Werke*, 532-538). A similar assessment appears in Hilbert 1916-17, 168.

Hilbert did not prove this theorem as part of his exposition of the theory, but he claimed that the necessary proof would appear in a different place. As it happened, however, the mathematical conclusions Hilbert drew from the theorem were erroneous: in fact, the validity of the theorem would imply that four among the equations are dependent on the other ten, but this in no way warrants that precisely the four electromagnetic ones are dependent on the gravitational ones, as Hilbert asserted here. As will be seen below, this problem did not pass unnoticed among Hilbert's colleagues and it was mentioned as part of the criticism directed towards the theory.

The ambitious title of Hilbert's paper reflects faithfully what he thought to be achieving with his theory: a formulation of the foundations of physics *in general*, rather than just of a particular kind of physical phenomena. The derivation of the field equations of gravitation appears in Hilbert's theory as a kind of by-product embedded in a broader argument with far-reaching intended consequences. It is evident from the text that Hilbert conceived this broader argument to be the foremost example of that idea he had consistently stressed in the past when speaking of the axiomatization of any individual physical discipline: the basic equations of the discipline should be deduced from a general variational argument, to which one must add *specific axioms* meant to capture the essence of the theory in question, so that the particular form of the necessary Hamiltonian function involved might be exactly determined. This time, Hilbert thought to have accomplished for physics what he had done for geometry in 1899, or at least so he declared consistently.

Hilbert's theory is based on two axioms. The first is a variational argument formulated for a scalar Hamiltonian function $H(g_{\mu\nu}, g_{\mu\nu l}, g_{\mu\nu lk}, q_s, q_{sl})$, whose parameters are the ten gravitational potentials $g_{\mu\nu}$, together with their first and second derivatives, $g_{\mu\nu l}$, $g_{\mu\nu lk}$, and the four electromagnetic potentials $q_s$, together with their first derivatives $q_{sl}$. The Hamiltonian is used to derive the basic equations of the theory, starting from the assumption that, under infinitesimal variations of its parameters, the variation of the integral

$$\int H\sqrt{g}\,d\omega \qquad (8)$$

(where $g = |g_{\mu\nu}|$, and $d\omega = d\omega_1 d\omega_2 d\omega_3 d\omega_4$) vanishes for any of the potentials. In fact, for reasons of convenience, instead of the covariant magnitudes $g_{\mu\nu}$ and their derivatives, Hilbert used consistently the contravariant tensor $g^{\mu\nu}$ and their derivatives throughout the argument. Hilbert called this axiom "Mie's axiom of the world-function".

The second basic axiom of the theory (Axiom II: axiom of general invariance) postulates that $H$ is invariant under arbitrary transformations of the coordinates $\omega_{l}$.

According to Hilbert, Mie himself had not included the electromagnetic potentials and their derivatives in the world-function, but rather this had been a contribution of Born. What characterized Mie's theory in his view, was the demand of orthogonal, rather than general covariance. In Einstein's work, on the other hand, the Hamiltonian principle plays only a secondary role, whereas Axiom II expresses in the simplest way his demand for general covariance (Hilbert 1916, p. 396, ftn.).

Besides the two basic axioms, the core of Hilbert's derivation is based on a central mathematical result (Theorem I), which Hilbert initially described as the *Leitmotiv* of the theory. According to this theorem in the system of *n* differential equations on *n* variables obtained from a variational integral such as ( 8), four of these equations are always a consequence of the other *n* - 4, in the sense that four linearly independent combinations of the *n* differential equations and their total derivatives are always identically satisfied (p. 397).

Integral ( 8) yields ten equations for the gravitational potentials and four for the electromagnetic ones:

$$\frac{\partial\sqrt{g}H}{\partial g^{\mu\nu}} - \sum_k \frac{\partial}{\partial\omega_k}\frac{\partial\sqrt{g}H}{\partial g_k^{\mu\nu}} + \sum_{k,l}\frac{\partial^2}{\partial\omega_k\partial\omega_l}\frac{\partial\sqrt{g}H}{\partial g_{kl}^{\mu\nu}} = 0 \quad (\mu,\nu = 1,2,3,4) \tag{9}$$

$$\frac{\partial\sqrt{g}H}{\partial q_h} - \sum_k \frac{\partial}{\partial\omega_k}\frac{\partial\sqrt{g}H}{\partial q_{hk}} = 0 \qquad (h = 1,2,3,4) \tag{10}$$

Hilbert denoted the left-hand sides of these equations as $[\sqrt{g}H]_{\mu\nu}$ and $[\sqrt{g}H]_h$, and called them the fundamental equations of gravitation and of electrodynamics respectively. Theorem I was obviously conceived with the intention of being applied to these equations, thus leading to the claim that four of them are in fact consequences of the other ten. The four equations $[\sqrt{g}H]_h = 0$, Hilbert concluded, are a consequence of the ten gravitational ones, $[\sqrt{g}H]_{\mu\nu} = 0$, or in other words, "*electrodynamic phenomena are an effect of gravitation*" (pp. 397-398).

Hilbert did not prove this theorem here, but he claimed that the necessary proof would appear in a different place. As it happened, however, the mathematical conclusions Hilbert drew from the theorem were erroneous: in fact, the validity of the theorem would imply that four among the equations are dependent on the other ten, but this in no way warrants that precisely the four electromagnetic ones are dependent on the gravitational ones, as Hilbert asserted here. Theorem I was an early version of what later came to be known as Noether's theorem (Noether 1918), but Hilbert's conclusions went way beyond what the theorem actually allows. Over the coming years, Hilbert's theory gave rise to a vivid debate among the Göttingen mathematicians, and the problematic status of his Theorem I and its implications came to be at the focus of that debate (Rowe 1998).

The main point of connection between Mie's and Hilbert's theory comes to the fore in the treatment of the concept of energy. This is also a point where we find truly significant difference between the proofs and the printed version. In each case Hilbert defined a certain magnitude that is a sum of formal expressions involving the Hamiltonian *H* with some additional differential relations among the various potentials, plus an arbitrary contravariant vector $p^l$. The expressions defined in both cases were quite different from each other, but in both cases Hilbert performed very complex mathematical derivations that led to the conclusion that the magnitude in question has zero divergence, thus justifying their choice as representing energy in the theory.

A complete formulation of the theory required additional assumptions necessary for determining the specific form of the world-function *H*. Hilbert stipulated that the Hamiltonian be composed of two parts: $H = K + L$. The first term *K* accounts for the gravitational part of the world-function. Like Einstein, Hilbert made *K* to depend on the gravitational potentials and their first and second derivatives, in order to produce a theory as close as possible to Newton's. *K* is then, in fact, the Riemann curvature scalar

$$K = \sum_{\mu,\nu} g^{\mu\nu} K_{\mu\nu}$$

, where $K_{\mu\nu}$ is the Ricci tensor.

The second term, *L*, is also an invariant, and it accounts for the electromagnetic part. For simplicity, Hilbert assumed that it depends on $q_s$, $q_{sl}$ and $g^{\mu\nu}$, but not on the derivatives of the latter. Using again a formal mathematical theorem (Theorem II) —a correct result which he did not prove here, yet claimed that proving it would be an easy task— Hilbert showed that, under the assumptions stated above, *L* must satisfy the following relation:

$$\frac{\partial L}{\partial q_{sk}} + \frac{\partial L}{\partial q_{ks}} = 0.$$

He thus concluded that the derivatives of the electromagnetic potentials appear in the equations only as part of the relation:

$$M_{ks} = q_{sk} - q_{ks},$$

( 11)

from which he deduced that, as a consequence of the basic assumptions of the theory, $L$ depends only on $g^{\mu\nu}$, $q_s$, and curl $q_s$ (but not simply on the derivatives of $q_s$ as originally assumed). Hilbert claimed that this conclusion was among the most significant results of his theory, since, as he said, it "is a necessary condition for establishing the Maxwell equations," and here it was obtained as a direct consequence of the assumption of general covariance alone. It is in passages like this, that Hilbert's reliance on Born's version, rather than on Mie's own presentation of the theory, becomes directly manifest. In fact, we saw above that Born had stressed as a main characteristic of the theory, that its Lagrangian depends only on differences of the kind ( 2), which are in fact equivalent to those appearing in ( 11).

Based on Theorem II Hilbert also deduced the form of the electromagnetic energy in the theory, which in the proofs[29] was

$$-2\sum_{\mu} \frac{\partial\sqrt{g}L}{\partial q^{\mu\nu}} g^{\mu m} = \sqrt{g}\left\{L\delta_{\nu}^{m} - \frac{\partial L}{\partial q_m} q_\nu - \sum_{s} \frac{\partial L}{\partial M_{ms}} M_{\nu s}\right\}.$$

( 12)

Hilbert now claimed that in the limiting case —$g_{\mu\nu} = 0$ (for $\mu \neq \nu$), $g_{\mu\mu} = 1$ (i.e., when no gravitational field is present)— his expression for the stress energy tensor equals that of Mie's theory. This fact led him to conclude, with evident satisfaction, that:

> Mie's electromagnetic energy tensor is none but the generally covariant tensor obtained by derivation of the invariant $L$ with respect to the gravitational potentials $g^{\mu\nu}$ in the limit. This circumstance first indicated me the necessary, close connection between Einstein's general theory of relativity and Mie's electrodynamics, and also convinced me of the correctness of the theory developed here. (p. 404)

29. In the printed version the expression was somewhat different (see on pp. 403-404), of course, but Hilbert's argument concerning it was similar.

What Hilbert meant with these claims would be rather obscure, unless we recalled that he was actually referring to Born's rendering of Mie's theory, rather than to the latter's own. In Born's formulation, the stress energy tensor of Mie's theory was given by equation ( 5). When this is specialized to the flat case, its connection with ( 12) (or to the corresponding equation that Hilbert wrote in the printed version) becomes apparent, although it still needs to be spelled out in detail.

## 5. Concluding Remarks

The foregoing sections can be summarized as follows: By the end of 1912, the question of the structure of matter had come to occupy a central place among Hilbert's scientific concerns. Mie's theory of matter, however, does not seem to have attracted his attention until Born reformulated it in terms more akin to his scientific sensibilities. Eventually, Hilbert became convinced that the theory showed good prospects for helping erect, based on it, a foundation for a unified theory that would account for all physical phenomena. Hilbert's interest in Einstein's theory came later. What startled Hilbert from Einstein's ideas, and directly motivated the consolidation of his own theory, was the possibility of embedding Mie's theory into a space-time formalism, that rendered evident a new, significant relation between gravitation and two important elements of the theory (the stress-energy tensor and the electromagnetic Lagrangian). At the same time the metric tensor was ostensibly put to the service of the explanation of the structure of matter, which was Hilbert's main focus of interest over the preceding years. Thus, inspired by Einstein's introduction of the metric tensor as a basic idea in the discussion of gravitation, Hilbert was led to consider Born's version of Mie's theory from a new perspective, under which new insights came to light that were not perspicuous in the flat case.

It is noticeable that neither in Born's nor in Hilbert's articles we find any direct or implicit reference, to Mie's *gravitational* theory. As already mentioned the latter presented considerable difficulties that Mie himself never really came to terms with. Born and Hilbert simply seem to have ignored this part of the theory in the framework of their discussions. Mie's gravitational theory was a scalar one and Born did not attempt to find a way to embed it in his own tensor-like presentation of the electromagnetic theory. Moreover, Born was most certainly aware of the criticism directed towards the theory in the Vienna meeting of 1913 or in its sequel, and he had no intention to counter this criticism when elaborating Mie's electromagnetic theory of matter.

Then, in Hilbert's article, Mie is only mentioned with reference to the electromagnetic part of the theory presented. Hilbert did not generalize Mie's scalar gravitational theory into a tensorial, generally covariant version of it, but *rather, he used Mie's electrodynamic account of matter as a basis for his own unified field theory.*

On the other hand, Hilbert's idiosyncratic, and perhaps somewhat narrow, way of approaching Einstein's ideas precluded him from seeing the whole *physical* situation involved here. Hilbert did not discuss in any detail the main physical questions that had perplexed Einstein over the preceding years, and had delayed for so long the formulation of his generally-covariant equations. Moreover, in those places where Hilbert did elaborate on the physical implication of his theory, some of his claims are quite problematic. For instance, after formulating the field equations and commenting on the relation between Einstein's and Mie's theories, Hilbert returned to the interconnection —already suggested at the beginning of his argument— between the electromagnetic and the gravitational basic equations, and in particular concerning the linear combinations between the four electromagnetic equations and their derivatives. These linear combinations Hilbert deduced to be of the following form:

$$\sum_m \left( M_{mr}\left[\sqrt{g}L\right]_m + q_r \frac{\partial}{\partial \omega_m}\left[\sqrt{g}L\right]_m \right) = 0.$$

This formula embodied, in Hilbert's view, "*the exact mathematical expression of the claim formulated above in general terms, concerning the character of electrodynamics as a phenomena derived from gravitation*" (p. 406. Italics in the original). But in fact this conclusion turned out to be quite problematic and in the future versions of the theory, Hilbert had to reconsider the significance of the relation between these two kinds of physical phenomena.

The opening passage of the printed version of Hilbert's communication (quoted above on p. 1) explains the background to the theory by giving credit first to Einstein and only then to Mie. It is remarkable that the proofs show the reverse order, as follows:

> The far reaching and original conceptions by means of which Mie produced his electrodynamics, and the tremendous problems formulated by Einstein, as well as the penetrating methods he devised for solving them, have opened new ways to the research of the foundations of physics.

In the events following the publication of his theory we find many reasons why Hilbert chose to publish in the order he actually did. But in light of the historical context described in the foregoing pages, it seems to me that the order chosen in the original version (first Mie and only then Einstein) reflects more faithfully the way in which he had actually arrived at his theory. The same can be said about the relative importance that both components must be attributed as the actual motivations behind his efforts.

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In the course of my research, Klaus and Friedrich Mie kindly put at my disposition the *Nachlass* of their granduncle Gustav Mie. Although in the present article I did not quote directly from any specific document in that interesting *Nachlass*, the possibility of examining its contents was very helpful for understanding the spirit of Mie's work. I thank them very much from their help and openness.

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NOTES

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