Cancellation of internal forces

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reduced matrix element as a special combination of a bra, an operator, and a ket. In reality it is a single number that happens, for descriptive convenience, to be labelled in a way that parallels that for an ordinary matrix element.

It should be mentioned that the form of the Wigner–Eckart theorem used by Brink and Satchler[6] contains a Clebsch–Gordan coefficient rather than a 3-j symbol. Unexpected factors of the type of $(2S+1)^{-1/2}$ in Eq. (3) do not arise. The dominance of Edmond's form in the literature makes this simplification little more than a curiosity, though it absolves Brink and Satchler from any numerical confusion arising from the boldface notation.

Apart from the fact that italicized tensors are more logical for reduced matrix elements than the boldface forms, the conscious change in font that the writer makes in passing from the tensors themselves to their reduced matrix elements should alert him to the delicate nature of the situation. Without this distinction in the typeface, the reader too becomes uncomfortable at the very least, and he loses an even greater sense of security the moment double tensors are introduced.

Further books on angular momentum theory are in various stages of production.[9] Perhaps, with luck, a few will be on the side of the angels.

[1] G. Racah, Phys. Rev. **62**, 438 (1942).
[2] M. Weissbluth, *Atoms and Molecules* (Academic, New York, 1978).
[3] B. W. Shore and D. H. Menzel, *Principles of Atomic Spectra* (Wiley, New York, 1968).
[4] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic, New York, 1963).
[5] B. G. Wybourne, *Spectroscopic Properties of Rare Earths* (Interscience, New York, 1965).
[6] D. M. Brink and G. R. Satchler, *Angular Momentum* (Oxford University, New York, 1968).
[7] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University, Princeton, NJ, 1957).
[8] M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957); M. Mizushima, *Quantum Mechanics of Atomic Spectra and Atomic Structure* (Benjamin, New York, 1970); B. G. Wybourne, *Classical Groups for Physicists* (Wiley-Interscience, New York, 1974); B. R. Judd, *Operator Techniques in Atomic Spectroscopy* (McGraw-Hill, New York, 1963). Wigner's position is ambiguous, since he uses boldface symbols for the components of vectors and tensors. See E. P. Wigner, *Group Theory* (Academic, New York, 1959).
[9] J. C. Morrison and I. Lindgren are planning a text on the theory of atomic spectra, while Odabasi's updating of the work by E. U. Condon and G. H. Shortley with this title has been advertised (Cambridge University, New York). An announcement has also been made of a two-volume work by L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics*, and *The Racah–Wigner Algebra in Quantum Theory* (Addison-Wesley, New York, 1980).

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In a recent note, McClain[1] suggests that some textbooks are in error when proving that the rate of change of the momentum $\mathbf{P}$ of a system is equal to the resultant $\mathbf{F}_{\text{ex}}$ of all external forces acting on the system. If $\mathbf{P} = \Sigma_j \mathbf{p}_j$ then

$$d\mathbf{P}/dt = \sum_j d\mathbf{p}_j/dt = \sum_j \mathbf{f}_j, \quad (1)$$

where $\mathbf{f}_j$ is the resultant of all forces, internal and external, acting on the jth particle. McClain objects to the simple dropping of all internal forces from $\Sigma_j \mathbf{f}_j$ (as usually argued, by Newton's third law, the internal forces cancel in pairs). He states, by analogy with the horse and cart parable, that it is wrong to suppose that the force executed by the ith particle on the jth cancels the force exerted by the jth particle on the ith.

There seems to be a flaw in the line of thought of McClain, and I suggest that the textbooks are right, after all. One should note the conceptual difference between the resultant of a number of forces and the sum of given forces. The resultant is defined as a single force, the action of which is equivalent to that of several forces on a given body. The sum of forces is just the mathematical operation of adding them up in a particular way (namely, the vectorial summation) and this can be performed, if so desired, on any given set of forces. The expression $\Sigma_j \mathbf{f}_j$ in Eq. (1) is just the summation of all the forces involved, and in this sum the internal forces do cancel in pairs, yielding $\Sigma_j \mathbf{f}_{j,\text{ex}} = \mathbf{F}_{\text{ex}}$, where $\mathbf{f}_{j,\text{ex}}$ is the resultant of the external forces acting on the jth particle. The use of Newton's third law is thus not erroneous at all in this concern. The situation mentioned by McClain, whereby two particles exert equal and opposite forces on each other, resulting in a motion of the particles such that the motion of the center of mass is unchanged, is actually a special case. In this case, the sum (not the resultant) of the internal forces on the two particles is in fact zero, and therefore only the external forces (if any) can influence the motion of the center of mass.

Apparently, most of the textbook authors were aware of this subtle difference between the more general concept of the sum of forces and the more restricted one of the resultant. For example, Resnick and Halliday[2] write regarding $\Sigma_j \mathbf{f}_j$:

> Among all these forces will be the internal forces exerted by the particles on each other. However, from Newton's third law, these internal forces occur in equal and opposite pairs, so that they contribute nothing to the sum.

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[1] J. W. McClain, Am. J. Phys. **47**, 1005 (1979).
[2] R. Resnick and D. Halliday, *Physics, Part I* (Wiley, New York, 1966), p. 189.