1. Are there integers $x, y, z$ such that $3x^2 + 2 = y^2 + 6z^3$ ?

2. Show that the congruence $x^3 \equiv a \pmod{167}$ has solutions for all $a \in \mathbb{Z}$.

3. Find all solutions to each of the following congruences:
   (a) $x^2 \equiv 9 \pmod{256}$.
   (b) $x^2 \equiv -7 \pmod{128}$.
   (c) $3x^2 + 6x + 1 \equiv 0 \pmod{19}$.
   (d) $x^2 + 3x + 7 \equiv 0 \pmod{37}$.

4. How many solutions does the congruence $x^2 \equiv 121 \pmod{1800}$ have?

5. Prove that for each prime number $p$ there exist $a, b \in \mathbb{Z}$ such that
   $$-1 \equiv a^2 + b^2 \pmod{p}.$$  
   (Hint: how many values in $\mathbb{F}_p$ do the expressions $a^2$ and $-1 - b^2$ take?)

6. Evaluate each of the following symbols: $(8/11), (7/13), (5/19), (2/383), (-1/113), (-2/773), (71/73), (37/137), (30/199), (1711/1999), (-1/523)$.

7. Which prime numbers $p$ can divide integers of the form $x^2 - 5$ ?