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# Exploring cruising using agent-based and analytical models of parking

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#### Exploring cruising using agent-based and analytical models of parking

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This article proposes two models to analyse parking search: an analytical model called PARKANALYST and a geosimulation model, termed PARKAGENT, which explicitly accounts for street network and drivers' parking-related decisions. We employ both models to analyse the impact of occupancy rate and demand-to-supply ratio on cruising for parking and to compare the models' outcomes. We estimate the main characteristics of parking dynamics, and find that the spatial effects influence system dynamics starting from an occupancy rate of 85% while become really important for analysing parking when the occupancy rate is above 92–93%.

**Keywords:** agent-based modelling; transportation modelling; parking modelling; parking search; cruising for parking

#### 1. Introduction

Cruising for parking occurs in virtually all cities around the world. Yet, little is known about the exact conditions under which cruising occurs and how patterns of supply of, and demand for, parking influence the scale of phenomenon. Building on earlier work (Benenson *et al.* 2008, Benenson and Martens 2008, Martens and Benenson 2008, Martens *et al.* 2010), this article presents two models that, taken together, can serve as a basis for exploring parking dynamics in full. The first model, PARKANALYST represents a simple analytical view of parking that focuses on the temporal dynamics of cruising for parking. The second model takes an explicit geosimulation view (Benenson and Torrens 2004) of the parking process and employs a new version of PARKAGENT, an agent-based model of parking dynamics in the city, first presented in Benenson *et al.* (2008). Following the literature on cruising for parking (Shoup 2004, 2006, Arnott 2006), we employ both models to analyse this phenomenon. Based on a careful comparison of the models we identify under which conditions an explicit spatial representation of parking search and choice is necessary for capturing the essentials of parking dynamics under congested conditions.

This article is organised as follows. First, we provide a brief overview of existing approaches to modelling parking in the city and assess to what extent these approaches are able to deal with (i) the inherently spatial nature of the parking process, and (ii) driver's reaction to the changing local situation during parking search, as both strongly shape

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overall parking dynamics (Section 2). Section 3 presents PARKANALYST, in which an average driver searches for a parking place (pp) in averaged, over space, circumstances. PARKANALYST accounts for duration of parking search explicitly, while for spatial effects implicitly only. Section 4 presents a second version of PARKAGENT, an agent-based model in which drivers' view of space is simulated explicitly. In Section 5, we validate PARKAGENT based on a comparison of model outcomes with field data on parking distance. In Section 6, we employ PARKANALYST and PARKAGENT to analyse cruising for parking in an abstract rectangular, two-way, street network. We compare the results of the two models regarding the relation between occupancy rate and demand-to-supply ratio on the one hand, and parking search time, parking distance and parking failure on the other hand demonstrate a high level of conformance between both models for occupancy rates below 85%. We subsequently employ PARKAGENT for estimating spatial effects of parking dynamics in the city for scenarios of increasing occupancy rates. This article ends with a conclusion and brief discussion of the results.

#### 2. Parking models

Various types of models have been developed to simulate and analyse drivers' parking behaviour in urban settings. An elaborate review can be found in Young *et al.* (1991) and Young (2000). For our purposes, the models of parking can be distinguished in terms of their level of aggregation, as well as the extent to which space is represented in the model. In terms of aggregation, a distinction can be made between models that consider groups of drivers and those which explicitly consider individual drivers. In terms of space, models can be distinguished that consider space only implicitly in the stage of model formulation and models that explicitly simulate drivers' movements in space.

One side of the parking modelling spectrum represents spatially implicit and aggregate models. Dynamic models of this kind are mostly associated with the economic view of the parking processes (e.g. Verhoef et al. 1995, Arnott and Rowse 1999, Arnott 2006, Shoup 2006). The most important contribution of these economic models lies in the systematic analysis of the interrelationship between parking conditions and parking policy. This results in the identification of sets of conditions and policies that optimise averaged parking utilisation over an area based on peak hour traffic flows, departure time, modal split and so on (Anderson and de Palma 2004, Petiot 2004, Wang et al. 2004, Arnott 2006, Calthrop and Proost 2006, D'Acierno et al. 2006, Zhang et al. 2007). Being necessary for the analytical investigation, the assumptions of perfectly rational and utility maximising behaviour, as well as limited attention to the spatial and stochastic nature of parking search, limit extrapolation of the models' conclusions towards real-world situations. Shoup's model, for instance, does not include space as it eliminates walking distance to the destination (Shoup 2006). Hence, Shoup can conclude that if prices of on-street and offstreet parking are the same, the equilibrium cruising time is zero. However, if off-street parking is relatively sparsely scattered, and destinations are scattered over space, the decision to cruise for on-street parking depends highly on the walking distance between the closest off-street parking facility (assuming it is always available) and the destination.

The other side of the modelling spectrum – that of spatially explicit simulations of drivers' parking search and choice – has started in the second half of the 1990s and is still in its infancy. The models we are aware of deal with intentionally restricted situations of

search and choice, e.g. parking search within an off-street parking lot (Harris and Dessouky 1997) or several adjacent street segments (Saltzman 1997). These explicit simulations consider parking behaviour of drivers as a set of sequential events, in which drivers respond to the actual traffic situation. In principle, these dynamic models are capable of capturing the self-organisation of the cruising phenomenon with a changing balance between parking demand and supply (see Shoup (2006) for a spectacular presentation of the problem), but it would require a substantial extension of the spatial dimensions of the models and an essential generalisation of driver's behavioural rules.

The only attempt in this direction we are aware of is presented in a paper by Thompson and Richardson (1998). They consider driver's parking search for, and choice between, onstreet and off-street alternatives within a small (20 street segments of about 50 m length), but realistic, grid network of two-way streets. The model is developed to follow one driver searching for parking within a fixed parking environment (i.e. no other drivers park or depart during this search). Nonetheless, this article clearly demonstrates that optimal parking search behaviour is hardly possible. Namely, the information available to the driver during parking search and choice is local in nature and the driver is unaware of the pps beyond her view and long-term experience does not necessarily lead to better choices, because of the high variation in parking occupancy rates. The outcome of parking processes in a spatially explicit model may thus be removed quite far from those obtained in economic models based on perfectly rational drivers.

Recently, Li *et al.* (2011) have presented a multi-class equilibrium model for investigating heterogeneous drivers' responses to route guidance and parking information systems. The model is spatially explicit and stochastic in that it accounts for the exact representation of road network connectivity and stochastic variation in traffic flow, parking availability and attractiveness of parking options. However, the model does not represent the parking process as resulting from the interaction between the decisions of multiple drivers, but instead exploits empirical formulae for estimating search time that is calibrated based on the field data. This is indeed sufficient for analysing the impacts of information systems on traffic flows and parking performance, but cannot capture the emergence and development of the cruising phenomenon as a collective outcome of drivers' parking search behaviour.

The above brief review leads to the conclusion that, while providing deep insight into the characteristics of parking dynamics, most of the existing models cannot be used to formally assess the spatial effects of parking search on overall dynamics. The models that potentially do provide these opportunities – spatially explicit and disaggregate models – are still underdeveloped and none of them can be employed to systematically assess the impact of many drivers simultaneously searching for on-street and off-street parking, and simultaneously entering and leaving pps, in a realistic urban environment. The models of parking search that are presented in this article aim at overcoming this gap.

#### 3. PARKANALYST – a simple analytical model of parking

PARKANALYST is an analytical model of parking, which considers the situation in which every driver is confronted with averaged parking conditions. It regards explicitly parking search time, but only implicitly accounts for space.

#### 3.1. Simplified view of parking search

To formalise the process of parking search, we consider a one-way and sufficiently long ring-like street divided into units which length is equal to the length of a pp, and assume that the area at the right side of a street consists of pps of a car's length. Let the street length be L, measured in pp, and assume that a car passes v pp during one time step. The model considers the system in discrete time (we use time step of 30 s in applications). The cars arrive to the area at an average rate of a(t) cars per time step t. A car's destination is the pp that represents a 'real' destination. A destination is chosen, before the trip, randomly and uniformly from the total set of pps. The car 'lands' at the 'search distance' r pp from the destination and the driver drives along the street towards the destination with a constant speed v pp per time unit, aiming to park at the destination or as close as possible to it. Parked cars are selected randomly and uniformly over the parking area to leave a pp, and d(t) cars, at average, leave the system per time step t.

We assume that the driver's ability to find a pp does not depend on the search time accumulated till then, as is largely true for the real-world situation. This means that the queue-like system (see, e.g. Cohen 1969) we investigate does not work on a standard first-in first-out basis, but rather on equal probability to be served for all cars at any given moment.

To reflect the intention to park as close to the destination as possible, we assume that driving and parking on the way towards the destination and driving and parking after passing the destination, in case the driver failed to park before, differ. When driving towards the destination, a driver's decision to park at a moment t depends on the density of free pps observed during driving before t. After passing the destination the driver is willing to park at any available pp.

In the model we assume that a driver, driving towards the destination from the place of entering the system, registers all free and occupied pps. Then, depending on the instantaneous distance to the destination, the driver estimates the expected number, F, of free pps on the remaining route to the destination. Based on F, when passing a free pp, the driver decides whether to park or continue driving towards the destination in order to park closer to it. We follow here the assumption of Benenson et al. (2008) and assume that the decision depends on the value of F and consider below the piecewise linear dependence of probability to continue driving on F. That is, the driver:

if  $F > F_2$ , if  $F < F_1$ ,

if  $F_1 < F < F_2$ .

- continues driving towards the destination
- parks immediately
- continues driving with probability  $p = (F F_1)/(F_2 F_1)$

In what follows we employ the values of  $F_1 = 1$  and  $F_2 = 3$ .

Let the time of driving to the destination be *m* time steps (where m=r/v), and maximum search time after passing the destination *n* time steps. We assume that the driver leaves the area if she fails to find a pp during m+n time steps.

#### **3.2.** PARKANALYST equations

Let us denote as D(t, t-k) the number of cars that entered the system at a moment t-k, and are still willing, at t, to drive towards the destination and not to park, and as P(t, t-k) the number of cars that entered the system at t-k and are willing, at t, to park.

Let C(t) be the overall number of cars searching for a pp, N(t) be the number of free pps in the system, s(t) be the fraction of drivers among those driving towards the destination who decide, at t, to start searching for parking and f(t) be the fraction of cars that want to park but fail to find a pp, between time steps t and t+1.

The dynamics of D(t, t-k), P(t, t-k), N(t) and C(t) for k = 0, 1, 2, ... is given by the following simple system of equations:

$$D(t, t) = a(t),$$

$$D(t + 1, t) = D(t, t) * (1 - s(t)),$$

$$P(t + 1, t) = D(t, t) * s(t),$$

$$D(t + 1, t - 1) = D(t, t - 1) * (1 - s(t)),$$

$$P(t + 1, t - 1) = P(t, t - 1) * f(t) + D(t, t - 1) * s(t),$$
...
$$P(t + 1, t - (m - 1)) = P(t, t - (m - 1)) * f(t) + D(t, t - (m - 1)),$$

$$P(t + 1, t - m) = P(t, t - m) * f(t),$$
...
$$P(t + 1, t - (n + m - 1)) = P(t, t - (n + m - 1)) * f(t),$$

$$N(t + 1) = \max\left\{0, d(t) + N(t) - \sum_{0}^{n+m-1} P(t, t - k) * (1 - f(t))\right\},$$

$$C(t + 1) = \max\left\{0, C(t) - \sum_{0}^{n+m-1} P(t, t - k) * (1 - f(t)) - P(t, t - (n + m - 1))\right\},$$

where a(t) is the number of cars starting parking search between t - 1 and t, and d(t) the number of cars leaving a pp between t and t + 1.

Note that P(t+1, t-(n+m-1)) is the number of cars that failed to find a pp during n+m time steps.

To illustrate the way of constructing the equations, let us consider the fifth line in the set of equations: the number P(t+1, t-1) of cars that entered the system at t-1 and is still willing at t+1 to park equals to the number of cars that entered the system at t-1, were searching for parking at t and failed -P(t, t-1)\*f(t), plus those which entered the system at t-1, to start searching for parking -D(t, t-1)\*s(t). The two last lines in the set of equations just sum up the numbers of the cars searching for parking and the numbers of free places.

To complete the model, one has to express the fraction of drivers s(t) who decide, at t, to start searching for parking, and the fraction f(t) of cars that want to park but fail to find a pp as dependent on C(t) and N(t) and average car speed v. To obtain estimates of s(t) and f(t) we split the street into chunks of a constant length of v pp and assume that the number of free pps per chunk follows a Poisson distribution with the average  $\mu(t) = N(t)v/L$  and that the number of cars searching for parking on a chunk follows a Poisson distribution with the average  $\varepsilon(t) = C(t)v/L$ , respectively. The formulae for approximate estimates of s(t) and f(t) based on the  $\mu(t)$  and  $\varepsilon(t)$  are presented in the appendix.

PARKANALYST describes temporal dynamics of parking search within a spatially averaged environment. To obtain spatial characteristics of the process explicitly, including the distribution of the distance between pp and destination, and to understand the effects of space on parking dynamics, we have developed the PARKAGENT geosimulation model.

#### 4. The PARKAGENT spatially explicit model of parking

PARKAGENT 1, a spatially explicit, agent-based model of parking search and choice in the city, was introduced in Benenson *et al.* (2008). The model links a geosimulation approach (Benenson and Torrens 2004) to a full-fledged GIS database, which are in use for an increasing number of cities around the world. In this way, PARKAGENT enables the representation of driver's parking behaviour in a real-life or artificial city, as well as indepth analysis of the overall consequences of driver's inherently local view of the parking situation. Here we present the second version of the PARKAGENT.

PARKAGENT 2 is completely re-written as a C#.NET ArcGIS<sup>TM</sup> application. The performance of PARKAGENT 2 is very high and it can be applied to a city area of any size. In addition, PARKAGENT 2 essentially extends, in comparison to PARKAGENT 1, a set of possible drivers' parking search behaviours and is capable of simulating not only on-street, but also off-street parking. The components of the road and parking infrastructure in PARKAGENT are constructed on the typical infrastructure GIS of a city and stored as a Personal Geodatabase of ArcGIS.

#### 4.1. Representation of the road and parking infrastructure in PARKAGENT

PARKAGENT is built on three GIS layers: *street network*, *buildings* and *off-street pps*. The attributes of the layers' features include, amongst other, driving and parking permissions for street segments, number and types of destinations in buildings, and capacity and price of off-street parking facilities. The layers can be acquired from standard urban GIS or constructed artificially (Figure 1).



Figure 1. A PARKAGENT view of the area in the centre of Tel Aviv (a) and of an abstract rectangular city (b). The small points on the centerline in case of the one-way road, see zoom in (a) or on parallel lines of small points on both sides of the centerline in case of the two-way road, see zoom in (b), denote the road cells; two outer rows of larger points represent pps.

Based on these GIS layers PARKAGENT 2 constructs:

A layer of *road cells*, which is employed for simulation driving. Depending on whether the street segment is one- or two-way, one or two rows of the road cells are constructed by dividing the street segment centerline into fragments with the length of an average pp (currently set at 5 m).

A layer *of on-street parking cells* – Two lines of 'parking cells' are set parallel to the road at a given distance of the centerline (Figure 1). For each 5 m of road, a parking cell is generated. The attributes of the segment's parking permissions are transferred to the parking cells.

In this article we employ artificial layers representing an abstract rectangular city with the buildings equally distributed along the streets (Figure 1(b)). Parking lots are not included

#### 4.2. Driver agents and their behaviour

PARKAGENT is an agent-based model. This means that every driver in the system is assigned a specific origin and destination and follows the rules of driving and parking behaviour. A full description of drivers' behaviour should include: (1) driving towards the destination at a large distance from the destination (before searching for parking actually commences); (2) parking search and choice before reaching the destination and after the destination is missed; (3) parking; and (4) driving out. PARKAGENT simplifies the first stage of driver's behaviour and focuses on the other ones.

#### 4.2.1. The rules car following

Based on Carrese *et al.* (2004), and our own observations while driving with drivers and recording their activities, we assume that the inherent driving speed during the stage of parking search and choice is  $12 \text{ km h}^{-1}$ , no matter what the speed was before. The simulation runs at a very high time resolution of 1 s. Each time step, a driving car can advance zero or one road cells, decides to turn or not if at a junction, or occupy a free parking cell.

We employ sequential updating and consider all moving cars in a random order, established anew at every time step. To represent an advance, let us note that at a speed of  $12 \text{ km h}^{-1}$  a car passes  $10/3 \approx 3.33 \text{ m}$  during 1 s; this distance is shorter than the length of a pp, which is assumed, according to the Tel Aviv observations, to be 5 m. To relate between the car speed and the length of a pp, we assume that the driver advances one road cell with probability  $p = 10/3/5 \approx 0.67$ , and stays at the current road cell with probability 1 - p = 0.33. This formula can be easily generalised for other speeds (Benenson *et al.*2008).

A salient feature of PARKAGENT is a car's reaction to congestion. Before advancing, a driver checks if a cell ahead is not occupied by another car. If yes, the driver does not advance during the time step; that is, the higher the density of cars on a road link, the lower is the average speed of the cars on that link.

#### 4.2.2. Initialisation of drivers and driver's choice of the route to destination

The initiation of a driver in PARKAGENT begins with assigning a destination and desired parking duration. Then a set of all road cells at a driving distance of 300 m from

the assigned destination is selected and the driver enters the model by 'landing' randomly at one of these cells, proportionally to the data on the intensity of traffic on the road links that contain these cells. From this initial cell the car drives towards the pp that is closest to the destination, starting to search for parking from a certain distance to the destination. In this article the heuristic algorithm of driving to destination is employed. The algorithm assumes that at each junction the driver chooses the link that takes her to the junction that is closest to the destination (Benenson *et al.*, 2008).<sup>1</sup> Driver's decision to park on the way to the destination in PARKAGENT is the same as in PARKANALYST (see Section 3.2).

#### 4.2.3. Driving and parking after the destination is missed

The model driver who has passed her destination cancels the decision rule employed at the stage of driving towards the destination, and is ready to park anywhere as long as it is not too far from the destination. We assume that after passing the destination, the driver aims at parking within the 'appropriate parking area' – a circle of a certain radius with the destination at its centre (Benenson *et al.* 2008). The initial radius of the appropriate area is 100 m and it is assumed to grow linearly at a rate of  $30 \text{ m min}^{-1}$  up to 400 m air distance from the destination, thus reaching its maximum in 10 min. Reaching a junction, the driver chooses the link that would take her to a junction within the 'appropriate parking area'. The driver 'remembers' several (currently two) latest street links she has passed during parking search and avoids using these links (but uses them nonetheless if no other option is available) when arriving to a junction and deciding which street to turn to. This rule prevents the driver from making short circles when cruising for parking.

If succeeding to find a pp, the driver parks for the time interval assigned during initiation. We erase the driver from the system directly after the parking duration is completed.

As in PARKANALYST, we assume that after missing the destination each driver has a maximal search time. In both models, the car is erased from the system if failing to park during this time (in reality, she may park her car at an off-street parking facility against a fee). In what follows, in both models, we set the maximal search time equal to 10 min, which is enough, at a speed of  $12 \text{ km h}^{-1}$ , for covering the distance of 2000 m or 2000/5 = 400 pp after passing the destination.

We admit that knowledge of the local road network and parking experience in a particular area can differ between drivers, but we do not account for drivers' long-term memory in this article. Note that, as has been argued by Thompson and Richardson (1998), long-term experience does not necessarily lead to better choices.

Let us now apply PARKAGENT and PARKANALYST for studying cruising for parking in the city. Donald Shoup provides an excellent aggregate analysis of the cruising phenomenon and claims, in line with engineering guidelines, that cruising can be eliminated if prices for on-street parking are set in such a way that only 85% of all onstreet parking spaces are occupied (see Shoup 2005, Chap. 12–13). In the following section we apply PARKANALYST and PARKAGENT for estimating this *cruising threshold* – the parking occupancy level beyond which parking search times increase rapidly.

We will base this study on our experimental data collected over the ca 1 km<sup>2</sup> area in the centre of Tel Aviv (Benenson *et al.* 2008), which serve for establishing model scenarios and enable validation of PARKAGENT.

#### 5. Validation of PARKAGENT

PARKAGENT can be validated at the micro- and macro-levels. At the micro-level, drivers' trajectories can be recorded with the help of GPS and, then, the rules of car following, parking search and parking choice can be validated. This approach demands knowledge of the drivers' destinations and of the instantaneous parking pattern around the destination. Data of this kind can be partially obtained if a selection of drivers would agree to report their destination and record their trajectories when parking, and we are currently collecting data of this kind.

At the macro-level, PARKAGENT can be validated through comparison between the aggregate characteristics produced in the model and estimated in the field. In this article, we follow this line and validate PARKAGENT by comparing model results to real-world data on the distance between pp and drivers' destination. The real-world distribution of distance is obtained in a night survey, between 0:00 and 5:00 h, when parking turnover is close to zero, in the densely built residential area of  $0.7 \text{ km}^2$  in the Northern part of Tel Aviv. This area is part of a larger area with an estimated overnight street parking demand-to-supply ratio of 1.2, according to data on parking permits held by residents of the area. Subsequently, at night all pps are occupied (more details on the area can be found in Benenson *et al.* (2008)).

During the night survey the plate number and exact location of every car parked in the area were recorded and, subsequently, related to the Israeli GIS database of car ownership addresses. In this way, a list of air distances between cars' pp and the address of the car owner was obtained during two sequential nights (disjoined, according to the law, from the plate numbers of the cars in order to avoid privacy violation). Altogether, for a total of about 1000 plate numbers that were registered in both nights, 530 cars (55%) parked at a distance of less than 350–400 m from the registered address. The distance of 350–400 m was a clear threshold and cars parking at larger distances from their 'destination' (which were distributed over a 0.4–50 km interval) remained at the same distance from their registered address of the owner. Real-world distribution of distances below is constructed based on the cars that park at a distance of less than 400 m from the owners' residence.

In order to simulate the distribution of distances, we need to re-create the real-world parking dynamics of inner-cities, which is characterised by essential parking turnover during daytime and return of large numbers of residents in the evening for overnight parking. Roughly, according to our data (Benenson *et al.* 2008), about half of the residents leave the area during the morning hours. In parallel, commuters and visitors arrive, resulting in a minimal occupancy rate, observed in the morning, of close to 60%. In addition to residents leaving the area in the morning, part of the residents leaves the area for short errands and return during the day time. Field data show that the vast majority of the residents make at least one trip a week. Residents' short trips and commuters' and visitors' parking result in an increase in occupancy rate, depending on the attractiveness of the street, of up to 80–90% towards 12:00 h. This level is preserved until early evening, when, starting from 16:00 h, residents return home from work, while commuters and visitors leave the area. The central city area is characterised by the demand-to-supply ratio above one, and, thus, the occupancy rate in the evening grows to almost 100% towards 18:00–19:00 h and remains at this level until the next morning.

In light of these observations, the scenario for obtaining distribution of the distance to pps is as follows: we start with an empty area and let the residents fill it to 100%. This results in an initial distribution of distances to destination that is essentially skewed towards low distance, because drivers arriving in the early stages of the simulation find an unrealistically low occupancy rate and are thus able to park unrealistically close to their destination. To simulate the real-world distribution, we thus simulate parking dynamics for a number of consecutive days in a row. Each day we start in the morning and randomly free 40% of pps, then simulate daily departures and arrivals according to the rates obtained in the field for working days. We use the parking pattern obtained in the end of a day as initial conditions for simulating parking dynamics during the next day.

The frequencies of the evening distribution of distances are practically stabilising (differ less than 0.5% from those for the previous day) at the fifth day, and in what follows, we compare real-world distribution of distances to destination to the outcome of the simulation that was obtained for the evening of the fifth 'day'. The simulation is performed over an area of  $\sim 1 \text{ km}^2$  that includes the area of the field research (as in Figure 1(a)).

The simulation 'day' starts at 9.00 h, when 2000 of the total of 5000 cars are randomly chosen to leave the area, resulting in an occupancy rate of 60%. After that, every hour until 16:00 h, an additional 300 parking cars are randomly chosen to leave the area, while 500 cars arrive and search for a pp according to PARKAGENT rules. As a result, about 4000 places are occupied at 16:00 h. During the evening period, 16:00–20:00 h, 1000 visitors are set to leave, while 3000 residents arrive for overnight parking. Simulation time step is 1 s and per second arrivals and departures are simulated as Poisson processes. Departing cars are chosen in space randomly. For the chosen values of parameters, all pps are occupied in the model towards 19:00 h and the last 1000 residents are not able to find an on-street pp (R = 1.2). Figure 2 presents the empirical distribution of the distances to



Figure 2. Histograms of the distance to destination, field experiment and PARKAGENT output.

destination for the cars parking at a distance of 400 m or less versus the same distribution obtained with PARKAGENT.

As can be seen in Figure 2, the distribution of the distances obtained with PARKAGENT is very close to the field distributions.<sup>2</sup> The average distance between pp and destination is 93.3 m in the field observations versus 109.3 m for the model. Note that the model overestimates the fraction of cars parking at a distance below  $\sim 60$  m and underestimates the fraction of cars parking at a distance 150–300 m. We relate this in-correspondence to heterogeneity of the urban network and leave the deeper study of the issue to future studies. Investigation of the model outcome shows that among four possible parameters of the model runs – arrival and departures during midday (9:00–16:00 h), number of visitors leaving in the evening, and minimal parking occupancy in the morning – the influence of the latter is the strongest. For lower minimal occupancy rates, the average distance to destination increases. For example, for a 50% occupancy rate at 9:00 h, the average distance between pp and destination is 92.9 m.

We consider the correspondence between the field and PARKAGENT distributions of the distance between pp and destination as confirming the validity of the model rules of parking search behaviour. In what follows we employ PARKAGENT and PARKANALYST for the analysis of cruising phenomena in the city. To understand the major features of cruising, we exclude the heterogeneity of the city from consideration and study cruising based on the abstract city grid (Figure 1(b)).

#### 6. Studying cruising for parking with PARKANALYST and PARKAGENT

Cruising for parking in central city areas is a common phenomenon (Shoup 2005), in part because market forces have left parking space relatively unscathed (Hau 2006). Drivers prefer to park close to their destinations and pay as little for parking as possible. Hence, if off-street parking is expensive in comparison to on-street parking or located far away from the destination, and the supply of on-street parking is insufficient, drivers tend to search for a vacant parking space for a while before deciding to park farther away from the destination or in a for-pay parking facility. Obviously, this situation will not always prevail and cruising may therefore be of little significance, especially outside downtown areas. For instance, van Ommeren et al. (2010), using a nation-wide sample of car trips for the Netherlands (excluding trips ending in employer-paid or residential parking), find that average cruising time is less than a minute on average. This suggests that cruising time is negligible in many areas and for many parts of the day. Likewise, drivers' costs related to cruising for parking, which includes private time cost of parking search, additional walking time and the cost of uncertainty involved in searching, may also be limited in many circumstances, as suggested by van Ommeren et al. (2011) in a study for residential parking in central Amsterdam. Yet, when parking demand is high, parking supply is limited, and parking policies are sub-optimal, as is the case in many downtown areas around the world, cruising for parking is likely to be a significant phenomenon with significant costs and negative externalities for drivers and society. Hence, a deeper understanding of the conditions that determine the extent of cruising is relevant from both a scientific and a societal perspective.

People with different travel motives may cruise for parking. Typically, three types of cruising drivers are distinguished: commercial parkers, work-related parkers (commuter parkers) and residential parkers. The case of residential parking, which we are investigating in this article, refers to parking in the evening, when residents, with permission for overnight on-street parking in an area, return home from work.

In what follows, the model driver who *has passed her destination without finding a pp* is considered to be cruising for parking. To fit to common sense understanding of parking, we distinguish between the total parking search time that starts from the moment the driver decides to park (300 m before reaching the destination for all model experiments below) and cruising time that is counted from the moment the driver passes the destination. The total search time for a driver who drove to the destination and parked in front of the destination is larger than for a driver who parked before reaching the destination, say, 100 m distance from it. However, despite longer parking search, the former is definitely more successful than the latter. To avoid this discrepancy, we consider cruising time for both these drivers as zero and focus of the search time after the destination is missed.

#### 6.1. General view of the determinants of cruising for on-street parking

In line with Shoup (2006) and with parking regulations in many cities around the world, we assume that on-street parking is free for residents and drivers try to avoid parking in for-pay off-street facilities. Hence, if on-street parking supply is limited, drivers will have a tendency to cruise to find a vacant on-street pp. We assume that all drivers behave in the same way (i.e. no driver heterogeneity) and are willing to search for a maximum amount of time to find an on-street pp. Note that in reality, willingness-to-cruise may depend on factors like parking duration, drivers' income and trip purpose (see, e.g. Shoup 2006, van Ommeren *et al.* 2010). In case drivers fail to find an on-street pp, they refer to an off-street parking facility in the area, which is assumed to be always available. Drivers' parking search and choice behaviour is guided by the rules described in Section 4.

Both PARKANALYST and PARKAGENT are multi-parametric models and we thus have to establish the scenarios for their comparison. The scenarios we use are hypothetical, but resemble the situation in the centre of Tel Aviv that we used above for model validation (Benenson *et al.* 2008). This situation is characteristic of large cities with a substantial residential population in the urban core and a comparable approach to residential parking. Here, we discuss the key parameters, while we consider more specific scenario settings in the following section.

Critical, for scenarios of residential parking, is the ratio R of the number of residents who want to park in an area at night and the number of pps in that area, which we call below residents' *night* demand-to-supply ratio. The value of R strongly shapes the number of drivers searching for overnight parking in the area. The second basic parameter is the fraction of visitors among parked cars who leave in the same period as residents return home for overnight parking. In what follows, we explore the impact of R by studying the model for R = 1.1 and R = 1.2 and the impact of the fraction of departing visitors from the total number of parked cars at the beginning of the simulation ranging from 0% to 20%, on cruising for parking by residents. We assume that no visitors are entering the area during the period in which residents are returning home. In this case, with an overnight demand-to-supply ratio R above one, the occupancy rate within a given area will grow during the evening hours till (nearly) 100%. After that, only the departure of visitors determines the number of cars that find a pp, the number of cars that cruise for parking and the number of cars that fail to park during the maximal possible cruising time.

#### 6.2. Cruising scenarios

In addition to the key parameters discussed above (demand-to-supply ratio and fraction of departing visitors), the scenario settings for both PARKANALYST and PARKAGENT are as follows:

- Simulation period: 16:00–20:00 h for studying cruising for parking.
- On-street parking capacity K of the area:  $5000 \text{ pps.}^3$
- Number of residents who aim to park in the area is either 5500 (for R = 5500/5000 = 1.1) or 6000 (for R = 6000/5000 = 1.2).
- The fraction  $G_{\text{initial}}$  of initially occupied pps at 16:00 h equals to 0.8, i.e., 4000 of the 5000 pps are occupied; occupied pps are randomly chosen from the total set.
- Between 16:00 and 20:00 h the number D of visitors leaving the area varies between 0 and 800, i.e., the departure rate d for the entire period of observation varies between 0% and 20% of all parked cars at 16:00 h. This results in 0–200 cars leaving the area per hour.
- Number A of residents' cars arriving to the area between 16:00 and 20:00 h is calculated based on the demand-to-supply ratio R, initial occupancy rate  $G_{\text{initial}}$  and the number of departing visitors D. To ensure that all 6000 area's residents tried to park before 20:00 h we calculate the number of arriving cars as:

$$A = K * (R - G_{\text{initial}}) + D.$$
<sup>(2)</sup>

Arrivals and departures are considered in PARKAGENT as Poisson processes. Given this stochastic nature, we have carried out multiple simulation runs. The results presented below are the outcome of 10 repetitions, which have also made it possible to estimate the standard deviation of the results (see Figures 3–6 and 8). The average numbers of cars arriving and departing per time step are calculated as A/S and D/S, where S is the number of the time steps during the period of 16:00–20:00 h.

To employ PARKAGENT we need additional assumptions regarding spatial distribution of destinations and parking supply. In this article, we consider an abstract grid of two-way streets of 100 m length each, for which we consider destinations equally distributed over 600 buildings in the area, 6 per street, each being a destination of 10 drivers. The length of a pp is 5 m. Model time unit is chosen as 30 s for PARKANALYST, and as 1 s for PARKAGENT.

We characterise the results by

- $T_{\text{cruising}}$ , which represents the average search time for the cars that found a pp while cruising, i.e. after passing their destination and until finding a pp,
- $P_t$ , which represents the share of cars that have entered the system and cruise for more than t s. We present  $P_{600}$ , the percentage of cars that failed to find a pp during the maximum possible search time (termed 'parking failure' below),



Figure 3. Occupancy rate G as dependent on t in PARKANALYST and PARKAGENT, for R=1.1, 1.2, d=5% and 20%.

- $D_{\text{destination}}$ , which represents the average distance between the selected pp and the final destination of the drivers returning home (i.e. the cars parked before 16:00 h are not taken into consideration),
- $D_r$ , which represents the share of cars that entered the system and park at an air distance larger than r. We present  $D_{100}$ , the percentage of cars parked at a distance of 100 m and more, and  $D_{200}$ , the percentage of cars parked at a distance of 200 m and more.

In what follows, we investigate the role of spatial factors in parking dynamics by comparing the outcomes of PARKANALYST with those of PARKAGENT for the artificial, grid-shaped, city.

#### 6.3. Cruising time in PARKAGENT and PARKANALYST

Figure 3 shows that, no matter what is R, parking occupancy rate during the investigated time period grows linearly and similarly in PARKANALYST and PARKAGENT, until all (for d=0) or almost all (for d>0) pps are occupied. For the deterministic PARKANALYST model, the curves for different D fully coincide, while for stochastic PARKAGENT the results slightly vary. Maximal variation of the PARKAGENT results and maximal difference between the results of PARKANALYST and PARKAGENT are observed when the occupancy rate G approaches, but remains below 100%. However, even then the differences between the different simulation outputs remain limited. Based on this similarity, we use the occupancy rate G as a *state variable* and present model outputs as dependent on G.

Despite similar dynamics of the occupancy rate in PARKANALYST and PARKAGENT, the estimates of cruising time in the models differ quite substantially.



Figure 4. Cruising time of the cars that succeeded to find a pp: (a) as dependent on time of the simulation for R = 1.1, 1.2, d = 5%, 20%; (b) as dependent on occupancy rate, in which case the PARKANALYST curves for different values of R and d almost coincide. The diamond at the 100% value (x-axis) denotes that for d > 0, maximal occupancy rate in PARKANALYST remains very close to, but below 100%.

Figure 4 shows the change in average search time ( $T_{cruising}$ ) and Figure 5 shows the percentage of cars failing to find a pp ( $P_{600}$ ), in relation to elapsed time and parking occupancy rate, for R = 1.1 and R = 1.2 and d = 5% and 20%. The average search time and the percentage of fails are similar for PARKANALYST and PARKAGENT until the percentage of occupied places reaches the level of 85%. With further increase in *G*, average cruising time and the fractions of parking 'failures' grow in both models, but differently, until the system becomes completely saturated and PARKANALYST and PARKAGENT outputs become similar again. Note that all pps are eventually occupied for d = 0, while for d > 0 the system eventually reaches equilibrium when the system parameters are defined by the departure rate only (Figures 4 and 5).

Note that, for PARKANALYST, average cruising time and parking failures is fully defined by occupancy rate until values very close to 100% (Figures 4(b) and 5(b)). For PARKAGENT, average cruising time does not depend on *R* or *d* (Figure 4), while the

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Figure 5. Share of cars that failed to find a pp during 10 min search ( $P_{600}$ ) for R = 1.1, 1.2 and d = 5%, 20%: (a) as dependent on time of the simulation and (b) as dependent on occupancy rate, in which case the PARKANALYST curves for different values of R and d almost coincide. The diamond at the 100% value (x-axis) denotes that for d > 0, maximal occupancy rate in PARKANALYST remains very close to, but below 100%.

percentage of cars that failed to park decreases with the increase in d (Figure 5(b)). Note that stochastic variations of the PARKAGENT results essentially mask this dependence.

After the parking occupancy rate G passes the 85% level in PARKAGENT, and until the system reaches the equilibrium with a (close to) 100% occupancy rate, the differences between PARKAGENT and PARKANALYST in terms of search time and percentage of failure steadily grow, achieving maximum at  $G \sim 97\%$  for search time and at  $G \sim 98\%$  for percentage of failures. PARKANALYST' outputs catch up those of PARKAGENT for values of G that are very close to 100%. As we will see below, these differences between PARKANALYST and PARKAGENT are due to the explicit account of space in the latter.

The cruising effect becomes, indeed, meaningful after the occupancy rate exceeds 85%, as reflected in the increase in average cruising times, just as accepted by traffic engineers

and applied by Shoup (2005) (Figure 4). However, the effect becomes important at an essentially higher occupancy rate, which, according to PARKAGENT, is reached at 92–93% occupancy rate. From these rates onwards, average cruising time is above 1 min and the percentage of failures becomes non-zero. Further decrease in the percentage of free pps has strong non-linear negative effects on cruising time and driver's parking success. For example, a 95% occupancy rate entails a longer than 2 min average search time and a parking failure share of more than 5%.

#### 6.4. Distance to destination in PARKAGENT

PARKAGENT enables estimating the growth of distance between pp and final destination with the growth of the occupancy rate (Figure 6).

As one can see in Figure 6, the average distance grows from  $\sim 50$  to  $\sim 80$  m with the growth of the occupancy rate from 80% to 90%. However, the 92–93% occupancy rate can be accepted as a practical threshold, after which the average distance to the destination and, especially, the fraction of cars that park at a distance above 200 m increases relatively rapidly. As long as the percentage of occupied places remains below 92–93%, the average distance between pp and destination remains below 100 m, i.e. less than 2 min walk at a speed of  $3.5-4 \text{ km h}^{-1}$ , and the fraction of the drivers that park at a distance above 200 m is below 10%.

## 6.5. When does the space matter? Differences between PARKANALYST and PARKAGENT

It can be easily noted from Figures 4 and 5 that the lack of free pps becomes important in PARKAGENT at an essentially lower occupancy rate than in PARKANALYST. The fraction of cars that fail to find a pp during 10 min cruising time ( $P_{600}$ ) can serve an indicator: for PARKANALYST,  $P_{600}$  passes the level of 0.1% after the occupancy rate G exceeds 0.993, while for the PARKAGENT this happens at  $G \approx 0.930$ .

To understand the reason for such a difference, let us consider the development of the parking pattern in PARKANALYST and PARKAGENT for the scenario in which no parked cars leave the area (d=0) and a demand-to-supply ratio R=1.2. To compare the results obtained with PARKAGENT to those obtained with PARKANALYST let us note that in the latter model, drivers 'decide' whether to park based on the average density of free pps as calculated for the entire modelled area. Furthermore, free pps are assumed to be randomly distributed over space. In what follows, we thus compare the emerging pattern of occupied pps with a growth in G as generated by PARKAGENT, with the random pattern of occupied pps characteristic for this G, as used in the runs of PARKANALYST.

Figure 7 presents the spatial patterns of the occupied pps obtained in PARKAGENT and two numerical characteristics of these patterns. The first is the correlation between the fraction of occupied pps on a road link and the average fraction of occupied pps on road links that are connected to it, estimated as Moran I coefficient of spatial autocorrelation (Anselin 1995). The second numerical characteristic gives the distribution of the number of free pps on a link for different values of *G* in PARKAGENT and in the random pattern. The scenario analysis starts with an 80% occupancy rate, and a random distribution of



Figure 6. Percentage of cars that park at a distance above 100 and 200 m (a) and average distance between pp and destination (b), for R = 1.1, 1.2, d = 5%, 20%.

occupied pps. The distribution of the occupied places remains random with an insignificant Moran I as G increases to 85%, for both PARKAGENT as well as the random pattern (Figure 7(a)). However, with a further increase in G, the difference between PARKAGENT and the random pattern increases and the Moran I value becomes highly significant (Figure 7(b)–(d)). This phenomenon is the consequence of the high demand-to-supply ratio R. Indeed, for stochastic reasons, the destinations of drivers arriving while the occupancy rate grows from 80% to, say, 85% are distributed non-uniformly. The fully occupied link marked in Figure 7(a) is a result of this stochasticity. In the case of R > 1, some drivers arriving later will still aim at a destination on this fully



Figure 7. The distribution of the number of free pps on a road link (left column) and parking spatial pattern (right column, free pps are marked by white circles, occupied pps are marked by black circles) as obtained with PARKAGENT for d=0 and R=1.2, at the moment of time when the average occupancy rate achieves (a) 85%; (b) 90%; (c) 95% and (d) 98%.



Figure 8. Average percentage of links that are fully occupied as dependent on the percentage of occupied pps.

occupied link. These drivers will park as close as possible to their destination and, thus, the places adjacent to the link will be occupied too. This tendency to park as close as possible to already occupied places entails spatial autocorrelation between the fraction of occupied pps at adjacent links. In time, a randomly initiated fully occupied patch expands into a larger area as drivers who aim at a destination within the patch will park on its periphery (Figure 7). Figure 8 presents aggregate characteristics of this process.

To sum up, PARKANALYST ignores the contiguity of the parking space and the autocorrelation that emerges when the occupancy rate is high and cruising drivers search for a pp just at the border of a fully occupied area; PARKAGENT accounts for it. As can be seen, spatial effects become strong enough to influence parking dynamics when the occupancy rate exceeds the  $\sim$ 95% threshold.

#### 7. Conclusions and discussion

In this article, we have presented two models of parking search and choice: a spatially explicit, agent-based, model termed PARKAGENT and a non-spatial model, termed PARKANALYST, which is constructed based on the behaviour of an 'average' driver within an 'averaged' environment and which does not account for the contiguity of space. PARKANALYST enables investigation of the influence of the basic parameters of the parking system, such as demand-to-supply ratio and arrival and departure rates, on the temporal aspect of parking dynamics when the occupancy rate is below ~85% and in the saturated state, when almost all pps are occupied and the process is determined by departing cars only. PARKAGENT enables direct implementation of the existing knowledge on drivers' parking behaviour in a real-world spatially heterogeneous environment. It enables investigation of the temporal aspects of parking dynamics for

the entire spectrum of model parameters and states, and, especially, enables estimating of the spatial aspects of the emerging parking pattern, such as clustering of streets segments with high occupancy rates in case parking demand is equal to, or exceeds supply. Model validation based on the distributions of the distances between pp and resident's destination, shows that PARKAGENT generates patterns that closely resemble the realworld situation.

To estimate the basic properties of parking dynamics, we apply PARKAGENT and PARKANALYST in a stylised homogeneous environment. We assume a homogeneous distribution of parking demand over space, roughly reflecting the real-life situation of residential parking in the evening, when area's residents get back home from work. We investigate model outcomes for different, but higher than one, values of overnight street demand-to-supply ratio, and for different departure fractions, reflecting visitors who park in a residential area during the daytime and leave in the evening.

As may be expected, the dynamics of the parking occupancy rate, average cruising time and fraction of failures are, first and foremost, determined by the demand-to-supply ratio: the higher the ratio, the earlier, in time, the system is saturated. However, the state of the system is perfectly reflected by the occupancy rate that shapes parking dynamics in both PARKANALYST and PARKAGENT. Knowledge of the occupancy rate is sufficient to predict average parking search time, fraction of failures and distance to destination.

The comparison between PARKAGENT and PARKANALYST emphasises the role of the contiguity of space in parking dynamics, which become essential when the occupancy rate is above 85%. From this rate onwards, the lack of spatial contiguity in PARKANALYST results in underestimating the average cruising time and the fraction of cars that fail to park, reflected in an increasing divergence between PARKANALYST's and PARKAGENT's results. The differences between the models' outcomes become essential when the occupancy rate achieves 92–93%, reach a maximum for the occupancy rates 97–98% and become similar again when the rate approaches 100%, when every pp is occupied very soon by a cruising car after being vacated.

The understanding of parking dynamics provided by the PARKANALYST and PARKAGENT models creates the background for the investigation of real-world situations, when both on- and off-street parking is allowed, the road network is nonuniform, parking space is heterogeneous (e.g. in terms of parking fees or parking permissions), destinations are not uniformly distributed over space (e.g. clustering of attractions or irregular distribution of off-street parking facilities) and drivers behave heterogeneously (e.g. in terms of their willingness to search or pay for on-street parking). The real-world situation is also characterised by the daily waves of demand and supply.

Further extension of the problem, beyond on-street parking and towards combination of free and paid on-street and off-street parking facilities should allow for explicit GISbased representation of heterogeneity of the urban road network, destinations, parking permissions or street lay out. This can be done with PARKAGENT, which thus could become a decision-support tool that can assist decision-makers to develop parking policies that reduce cruising for parking as much as possible. In parallel, PARKANALYST can be further developed towards considering two-dimensional space and including parking effects revealed by the PARKAGENT, e.g., positive spatial autocorrelation between the parking occupation rates.

#### Notes

- 1. The shortest-distance path algorithm is also implemented in PARKAGENT and can be activated instead of the heuristic algorithm. This article investigates abstract rectangular two-way road networks, for which both algorithms generate identical routes.
- 2. Note that with 2267 field observations and unlimited number of observations in PARKAGENT, very small differences between the distributions become highly significant from a statistical point of view. We, thus, ignore the issue of statistical significance when comparing the field and model distributions of the distances between parking place and destination.
- 3. This capacity corresponds to an area of about 1 km<sup>2</sup> and seems sufficient to ignore boundary effects.
- 4. For real-world conditions, the probability of three or more cars searching for parking on the same chunk is close to zero.

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#### Appendix: aggregate model of parking

Let us consider parking in a discrete time, using 30 s as the typical time unit for the parking system. Let us assume that the time of driving to the destination is m time units, maximum search time after passing the destination is n, and the driver leaves the area if failing to find a pp during this time interval.

We consider the system in discrete time and space, and assume that the cars drive and park along a one-way street divided into units which length is equal to the length of a pp, and each unit has a pp to its side. We assume that a driver is willing to park at any distance from the destination after passing it. Let the street length be L (pp) and a car pass v pp during one time step. Let us consider street as split into chunks of length v.

Let D(t, t-k) be the number of cars that entered the system at t-k, and are still willing, at t, to drive towards the destination and not to park, and P(t, t-k) the number of cars that entered the system at t-k and are searching, at t, for a pp.

Let a(t) be the number of cars starting parking search between time moments t - 1 and t, and d(t) be the number of cars leaving a pp between time moments t - 1 and t.

Let C(t) be the overall number of cars searching for a pp and N(t) the number of free pps at t. Let us assume that the cars searching for free pps and the free pps themselves are located randomly along the chunks.

Let s(t) be the fraction of drivers among those driving towards a destination who decide, at t, to start searching for parking, and f(t) be the fraction of cars that want to park but fail to find a pp, between t and t+1. To estimate s(t) and f(t), let us denote as  $\varepsilon(t)$  the average number of free pps on a chunk, and as  $\mu(t)$  the average number of cars on a chunk searching for a pp.

#### Decision to search for parking

When driving towards the destination the driver's decision to start searching for parking is defined by the expected number of free pps on the chunk ahead. According to the PARKAGENT2 assumption, the driver always decides to continue driving and not to search for parking in case there are expected to be three or more pps on the chunk ahead and with probability 0.5 in case two free pps are expected. Given a Poisson distribution of the number of free pps on a chunk, the fractions of chunks with zero, one and two free pps on it are  $e^{-\varepsilon(t)}$ ,  $e^{-\varepsilon(t)*\varepsilon(t)}\varepsilon(t)$  and  $e^{-\varepsilon(t)*\varepsilon(t)^2/2}$ , respectively. That is, the overall probability s(t) that the driver will decide not to park on the chunk ahead and just pass it is:

$$s(t) = 1 - e^{-\varepsilon(t)} - e^{-\varepsilon(t)} * \varepsilon(t) - 0.5 * e^{-\varepsilon(t)} * \varepsilon(t)^2 / 2.$$
(A.1)

When missing the destination, the driver is ready to park immediately when encountering a free place on a chunk, i.e., s(t) = 0.

#### The failure or success of the decision to park

To estimate the probability f(t) of failure to park on a chunk, let us consider the chunks with n = 0, 1, 2, ... free pps and assume that in case the number of cars g on the chunk is n or less, all of them could park there, while in case g > n the excessive g - n cars would fail to park. For the Poisson distribution of the number of cars searching for parking on a chunk, the average number of cars per chunk with currently n free pps that fail to park, during the time unit, is equal to  $\sum_{k>n} [(e^{-\mu(t)}\mu(t)^k/k!)(n-k)]$ , while the average number of free places that remain, during the time unit, free on the chunk with currently n free places is  $\sum_{k< n} [(e^{-\mu(t)}\mu(t)^k/k!)(n-k)]$ . Consequently, the overall number of cars per chunk that failed to park during  $\Delta t$  is

$$f(t) = \sum_{n} \left[ e^{-\varepsilon(t)} \varepsilon(t)^n / n! \sum_{k>n} e^{-\mu(t)} \mu(t)^k (k-n) / k! \right],$$

while the average number of free places per chunk g(t) that were free and remain free during the time unit, because the number of cars on a chunk is insufficient to occupy all of them, is

$$g(t) = \sum_{n} \left[ e^{-\varepsilon(t)} \varepsilon(t)^n / n! \sum_{k < n} e^{-\mu(t)} \mu(t)^k (n-k) / k! \right].$$

Limiting ourselves to a case of less than three cars searching for a pp on a chunk,<sup>4</sup> i.e.,  $n \le 2$  we obtain the following estimate of f(t):

$$f(t) = e^{-\varepsilon} \left[ \mu + \varepsilon (\mu - 1 + e^{-\mu}) + \varepsilon^2 (\mu - 2 + e^{-\mu} (2 + \mu))/2 \right] / \mu,$$
(A.2)

where  $\varepsilon = \varepsilon(t)$ ,  $\mu = \mu(t)$ .

The dynamics of D(t, t-k), P(t, t-k), N(t) and C(t), for k = 0, 1, 2, ... can thus be presented as follows:

Initial and boundary condition:

$$N(0) = N_{\text{initial}} + d(0),$$
$$D(t, t) = a(t).$$

Equations of system dynamics:

$$\varepsilon(t) = N(t)v/L,$$
  

$$\mu(t) = C(t)v/L,$$

$$\begin{split} s(t) &= 1 - e^{-\varepsilon(t)} - e^{-\varepsilon(t)} * \varepsilon(t) - 0.5 * e^{-\varepsilon(t)} * \varepsilon(t)^2 / 2, \\ f(t) &= e^{-\varepsilon} [\mu + \varepsilon(\mu - 1 + e^{-\mu}) + \varepsilon^2(\mu - 2 + e^{-\mu}(2 + \mu)) / 2] / \mu, \\ D(t + 1, t) &= D(t, t) * (1 - s(t)), \\ P(t + 1, t) &= D(t, t) * s(t), \\ D(t + 1, t - 1) &= D(t, t - 1) * (1 - s(t)), \\ P(t + 1, t - 1) &= P(t, t - 1) * f(t) + D(t, t - 1) * s(t), \\ \dots \\ P(t + 1, t - (m - 1)) &= P(t, t - (m - 1)) * f(t) + D(t, t - (m - 1)), \\ P(t + 1, t - m) &= P(t, t - m) * f(t), \\ \dots \\ P(t + 1, t - (n + m - 1)) &= P(t, t - (n + m - 1)) * f(t), \\ N(t + 1) &= \max \left\{ 0, d(t) + N(t) - \sum_{0}^{n+m-1} P(t, t - k) * (1 - f(t)) \right\}, \\ C(t + 1) &= \max \left\{ 0, C(t) - \sum_{0}^{n+m-1} P(t, t - k) * (1 - f(t)) - P(t, t - (n + m - 1)) \right\} \right\}$$

Note that P(t+1, t-(n+m-1)) is the number of cars that failed to find a pp during a predefined maximum search period.

In this article we assume that the time step is 30 s,  $v = 12 \text{ km h}^{-1}$ , the drivers enter the system 300 m before the destination, and the maximal duration of the parking search after passing the destination is 10 min, i.e., n = 20. For these parameters the driver passes, during a time unit of 30 s, a street chunk of 100 m. It would thus take the driver 3.5 min time units to drive to the destination, i.e. m = 3. We also assume that the average length of the space interval between parking cars is 5 m and, thus, the typical length of a chunk is 20 pps.