Abstract

The emergence of global trading networks during the Iron Age demanded the development of sophisticated measuring techniques. Standardization of containers (usually storage jars) was especially important for controlling quantities of commodities and for efficient storage inside ships. The “torpedo” storage jars, manufactured in Phoenicia in the 8th century BCE, are a case in point. This paper deals with a large number of torpedo storage jars found in two shipwrecks off the coast of Ashkelon. The linear dimensions and volume of 20 jars that were retrieved from the two shipwrecks were analyzed in Egyptian units. These vessels were compared to torpedo storage jars found in contemporary land sites excavations. It was determined that a torpedo jar whose cylindrical part is ~1 cubit in height and ~1 cubit and 2 palms in circumference “guarantees” a volume of 4 hekats, meaning that their volume could have easily been estimated and that the level of standardization was high. The choice by Phoenician manufacturers to use Egyptian units was probably shaped by the Egyptian consumers.

INTRODUCTION

Ancient concepts of mathematics are difficult to trace if they are not expressed in writing, but they may be detected through the shape and volume of daily artifacts (POMMERENING 2005). The study of pottery vessels – the most widespread artifact in historical archaeology – has the potential to shed light on ancient people’s knowledge of computing and their ability to take linear measurements and estimate volumes (ZAPPASKY et al. 2007; 2009). Our hypothesis is that even in the Bronze and Iron Ages people were aware of the relationships between dimension, form and volume of vessels. To differ from modern formulae, these relationships were expressed by sets of rules known through empirical observations. These rules can be revealed when considering ancient vessels in their original units of measurements (ZAPPASKY et al. 2007; 2009).

In this paper we focus on the Iron IIB (the 8th century BCE) – a period of strong commercial activity in the eastern Mediterranean that involved the Assyrian empire, Phoenician maritime city-states, trade emporia along the Philistine coast and Egypt (FRANKENSTEIN 1979; DIAKONOFF 1992; NA’AMAN 1994; 1995; LIPIŃSKI 2006: 180–190; FANTALKIN and TAL 2009: 244–246). Our choice of this period is determined by the notion that in times of strongly-connected international trade networks, such as that of the Late Bronze Age under Egyptian hegemony or the Iron IIB under Phoenician (and possibly Assyrian) domination,2 trade storage vessels can be expected to show a high level of standardization in size and volume. This is so due to the need to control the quantities of traded commodities and to the requirements for efficiently stacking the storage-vessels in ships.

In what follows we present the results of our investigation of the most common trade storage jar of the Iron IIB, nicknamed (because of its shape) the “torpedo” storage jar. This jar was found in large numbers in dozens of excavated sites in Lebanon and Israel, mainly along the coast (e.g., Sarepta and Tyre) but also in inland sites located along trade-routes (such as Hazor and Megiddo). With the exception of Beersheva, located on an important Arabian trade route, the appearance of torpedo storage jars in the inland territorial kingdoms of the time is negligible (see distribution map in BALLARD, STAGER et al. 2002: fig. 10 and a recent update in SINGER-AVITZ 2010: 188–190).
Note the difference between the cargo of the two Phoenician shipwrecks and the Ulu-Burun Late Bronze shipwreck; the study of the latter indicated that its cargo consisted of a variety of commodities, which had been loaded at different ports.
Below we focus our investigation on the variability of the torpedo jars from the two shipwrecks, including comparison to those found at inland sites. We will show that the former are essentially homogeneous in form and volume and may reflect an attempt to reach a high degree of standardization.

**Classification of torpedo amphoras according to profile**

Twenty (of the 22) storage jars recovered from the shipwrecks were measured by us in their US storage and exhibition locations (the Wood’s Hole Oceanographic Institute; Institute for Exploration) and their 3D models constructed based on two or more photos taken at different angles for each (for the methodology, see ZAPASSKY et al. 2009). We then added to the analysis 134 torpedo cylindrical jars found at inland sites such as Hazor; for these, the 3D models were constructed according to published drawings.

We compared the torpedo storage jars according to their profile (a single profile per jar). Each profile was processed in several steps.

In the first step, we rotated the profile in order to adjust its orientation, as published profiles are often tilted on their side. To determine the “vertical” position of the profile, we (a) defined its center of gravity; (b) rotated it by all possible angles \( \beta \) within the interval \((-10^\circ, 10^\circ)\) at resolution \(0.1^\circ\) around the jar’s center of gravity; (c) split the rotated profile into two parts by a vertical axis drawn through its center of gravity; (d) reflected the right part in regard to this axis; (e) estimated the area of intersection \( S_{\text{intersection}}(\beta) \) for a given angle of rotation \( \beta \); and (f) calculated the index of asymmetry \( AS(\beta) = 1-2S_{\text{intersection}}(\beta)/S_{\text{profile}} \). The final rotation of the vessel was then established by the angle \( \beta_{\text{opt}} \) that provides a minimal value of \( AS(\beta) \) (Fig. 2).

In the second step, again with the help of the Rhinoceros\textsuperscript{TM} application, we scaled the profiles to the same height of 60 cm. This was done for the sake of comparison of the torpedo shapes regardless of the vessel’s relative size (Fig. 3).

We characterized the jar’s profile by the length of multiple horizontal cross-sections, equally distributed along its height. IF \( N \) is the number of these cross-sections, then each shape is represented by the \( N \)-dimensional vector of cross-section length (Fig. 4).
The higher N, the more precise the representation of the jar’s profile, and, respectively, the estimate of the jar’s volume. Due to the relatively simple form of the torpedo jars, some ten horizontal cross sections are sufficient in order to approximate the jar’s volume based on the exact profile by a sum of N truncated cones. The relative error is below 0.5%, starting from N = 50 cross-sections. In what follows we characterize the profile of the (scaled) torpedo jars by N = 75 cross-sections equally distributed along the 60 cm of its height.

**THE TORPEDO AMPHORAS’ SHAPE AND VOLUME VARIABILITY**

**Analysis of the amphoras’ shape**

To analyze the variability of the torpedo jars’ shapes we employed two methods – Hierarchical Cluster Analysis (HCA) and Principal Component Analysis (PCA) (KREMELBERG 2011) – as applied to the jars represented by the multi-dimensional vector of the cross-section lengths. We used 73 of the 75 components of the vector for this purpose, excluding the first (topmost) and the last (bottom) cross-sections as essentially depending on the draftsman’s habits. The HCA was performed with the *Euclidean distance* as the measure of the objects’ similarity and between-groups linkage as the method of clustering. In other words, the dissimilarity D(X, Y) between the shapes X and Y is measured as squared Euclid distance, D(X, Y) = (∑(x_i – y_i)^2) where x_i, y_i denote the length of the i-th horizontal cross section of X and Y (Fig. 5).

The results of the HCA and PCA analysis of all 156 cylindrical jars are presented in Fig. 6. The dendrogram consists of one large cluster (on the left) and many smaller ones. We proceeded with the analysis of the 78 jars of the former.

A closer look at the largest cluster (Fig. 7) according to the results of both the HCA and PCA reveals that it consists of four sub-clusters. As can be seen from the dendrogram (Fig. 7a) and from the scatterplot of the two first principal components of the PCA that comprise 82% of the overall variance (Fig. 7b), the largest of the four sub-clusters contains all jars recovered from the shipwrecks. It is important to note that the shape of the torpedo jars of the largest sub-cluster is the closest to a simple
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Fig. 5 Measuring dissimilarity between two torpedo amphoras

Fig. 6 Dendrogram of all 156 cylindrical jars with six clusters selected: a) and the scatterplot; b) of the two first principal components (91.1% of the overall variance). The clusters 1–6 that are revealed by the HCA are marked in the PCA scatterplot by different symbols. A typical jar is presented for each cluster.
cylinder (Fig. 7b). Table 1 shows all 78 jars presented in Fig. 7.

To sum-up, 51 of the 156 cylindrical torpedo jars comprise, according to their profile, a compact sub-cluster that includes all the 20 directly measured jars from the shipwrecks. The shape and volume of the latter can be characterized in more details.

Shape and volume of the torpedo storage jars from the shipwrecks

Let us characterize the 20 (of the 22) torpedo storage jars which were recovered from the shipwrecks (BALLARD, STAGER et al. 2002) – 13 from the Tanit (Fig. 1) and seven from the Elissa (examination of the jars was made possible courtesy of the Wood’s Hole Oceanographic Institute, Institute for Exploration and the Ashkelon Excavations, the Leon Levy Expedition).

In order to understand the rules that could be employed for producing these jars, let us translate their measurements into ancient units of length and volume which may apply to 8th century BCE Phoenician trade vessels. The two evident alternatives are the Egyptian system of length and volume units, which was used in the region for many centuries (POMMERENING 2005) and the one deployed in Assyria – the great empire of the time. We opted for the former for two reasons: First, Phoenicia was influenced by Egypt throughout its Bronze and Iron Ages, starting as early as the third millennium BCE (REDFORD 1992; STAGER 1992; 2001; DE MIROSCHENI 2002; SOWADA 2009); second, in...
other realms, e.g., pictorial representations on seals and seal impressions, Egyptian influence in the Levant continues at least until the early 7th century BCE (Keel and Uehlinger 1998: 248–265; see also, e.g., Winter 1976 for Egyptian motifs in 8th century BCE Phoenician ivories, and 2011 for 7th century BCE Egyptian situlae at coastal sites designed to provide ritual Nile water for traveling Egyptians).

Our working hypothesis, then, is that the potters aimed at Egyptian units – the royal cubit as the unit of length and the hekat as the unit of volume. These units had the same meaning and values starting at least in the early second millennium BCE. According to ~20th century BCE rods and ancient records, 1 royal cubit = 7 palms = 28 fingers ≈ 52.3 cm (Gardiner 1957; Pomerening 2005). The value of the hekat is obtained from marked vessels (Pomerening 2005), which provide 1 hekat ≈ 4.8 liters, and from Problem #44 in the Rhind papyrus (Pomerening 2005) of the ~17th century BCE, which provides the relation between a cube of the royal cubit edge-length and the hekat: 1 hekat = 1/30*cubit^3 ≈ 4.8 liters (Pomerening 2005).

The shape of the shipwreck amphoras

The original publication already noticed that visually, the jars are not only similar in shape, but also close in dimensions (Ballard, Stager et al. 2002: 159). This observation is confirmed by our measurements: The cylindrical part of the 20 jars is almost straight, with less than 5% differences between the narrowest and the widest diameters and the variation of width between the shipwreck jars is very low – at a level of 3–4% (Table 2), which can still be achieved by the expert eye of the potter (Eerkens 2000).

Low variation in the shape and dimensions of the shipwrecks' torpedo jars entails low variation in their volume.

The volumes of the shipwrecks' amphoras

We have estimated the volume of the shipwreck jars (Fig. 8a) in two ways – directly, with poly-

<table>
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<th>Cluster #</th>
<th>No of items</th>
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<tbody>
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</tr>
<tr>
<td>4</td>
<td>12</td>
<td><img src="image4" alt="Jar shapes" /></td>
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</tbody>
</table>

Table 1 All 78 amphoras presented in Fig. 7
styrene beads and numerically, based on their 3D models. For the numeric estimates we approximated the jars’ cylindrical part with a cylinder whose diameter is equal to the average of the widest and narrowest diameters. The height of the approximated cylinder was measured as a distance between the highest and lowest points of the cylinder’s intersection with the jar’s profile (Fig. 8b).

The torpedo storage jars have two parts, easily distinguishable by the naked eye – cylindrical body and oval base. In Egyptian units, the average height of the torpedo jars’ cylindrical part equals 27.61 fingers = 0.99 cubits, while its average circumference equals 35.9f ≈ 1 cubit + 2 palms. The average half-length of the torpedo jars’ vertical profile is 36.27f, i.e., also 1 cubit and 2 palms (Fig. 9).

The one cubit height and one cubit and two palms circumference of the almost straight cylinder-like shipwrecks’ jars automatically guarantees that (ignoring the side width of the vessels and their base part), their volume is very close to 4 hekats – the basic trade volume unit of that time:

\[ V = \pi R^2 H = \pi (L/2\pi)^2 H = \pi (36f/2\pi)^2*28f = 36*36*28/(4\pi)^3 = 2887f = 3.95h \]  

Measuring the torpedo jars’ volume with polystyrene beads provided values that are very similar to this estimate (Fig. 10). In other words, the additional volume of the base part is very close to the volume of the wall that should be subtracted from the estimate given in formula (1). We thus conclude that simple and directly taken outer measurements of the height and circumference of a torpedo jar’s cylindrical part could guarantee its volume.

It is noteworthy that these two measurements are not related (Fig. 11). In other words, the pot-
ter(s) produced neither jars of a constant volume nor jars of identical shape but different size; in such cases one would have expected negative or positive correlation respectively in Fig. 11 (for a positive correlation, see, e.g., our study of the Judahite Royal jars – Zapasky et al. 2009).

To sum-up, easy-to-take (e.g., with a marked string) linear measurements – 1 cubit height and 1 cubit and 2 palms circumference of the cylindrical part (and the same length of half of the vertical profile) guarantee a standard volume of 4 hekats for the Phoenician torpedo trade storage jars – the standard liquid trade unit of the time.

CONCLUSIONS

The two shipwrecks found off the shores of Ashkelon provide a rare window through which the sophisticated and specialized nature of international trade networks of the 8th century BCE are revealed. It seems that the standard torpedo amphoras of simple cylindrical form were primarily made for marine transportation in specialized pottery workshops (Ballard, Stager et al. 2002: 159–160). It seems likely that the ships were headed for Egypt, and that the consumer (Egypt) provided the stimulus for the producer (Phoenicia).

The need to control the size and volume of the jars was convenient for all parties involved; the consumers in Egypt, merchants and possibly bureaucrats behind the trade needed to know the quantity of the commodities and the builders of the ships probably designed space suitable for stacking these perfectly-shaped jars. The driving force...
behind this standardization was probably the Egyptian consumer; in other words, the Phoenician traders seem to have operated under a consumer-oriented mentality.

Needless to say, the potter need not have been aware of the mathematical rules relating to the linear dimensions and the volume of the jars and, furthermore, probably did not check if the volume of a produced jar was 4 hekats. This is evident from the fact that the height and circumference of the cylindrical part of the torpedo jars found in the shipwrecks are unrelated. In other words, the potters at the workshop could work mechanically with measurements given to them by their clients. The potters did not know that if the jar came out too long they needed to balance its circumference in order to obtain the same volume.

Finally the standardization of the Iron IIB torpedo jars raise two additional issues regarding earlier and later times, which should be investigated in the future:

Future research should study capacity standardization in earlier periods. The Late Bronze Age II was a time of strong commercial activity in the Eastern Mediterranean when large numbers of jars stacked on ships. For that period, the transport jars from the Uluburun shipwreck will provide a key comparative corpus to the Phoenician shipwreck amphoras.

Even though the shipwreck amphoras appear designed for maritime trade and adapted to Egyptian units of length and volume, they were modified substantially in the 7th century. Future studies might attempt to account for these changes by studying other aspects of producer-consumer relationships (branding, price-quantity ratio) as well as changes in the marketplace such as the rise of competition from the Aegean.

Bibliography

AZNAR, C.A.

BALLARD, R.D., STAGER, L.E., MASTER, D., YOERGER, D., MONDELL, D., WHITCOMB, L.L., SINGH, H. and PIECHOTA, D.

BASS, G.F.

BELL, L.

BIKAI, P.M.
1978 The Pottery of Tyre, Warminster.

BIKAI, P.M.

DIKUNOFF, L.M.

EERKENS, J.W.

FANTALKIN, A. and TAL, O.

FRANKENSTEIN, S.

GARDINER, A.H.
1957 Egyptian Grammar, Oxford.

GILBOA, A.

GILBOA, A., KARASIK, A., SHARON, I. and SMILANSKY, U.

KEE, O. and UEHLINGER, C.

KREMELBERG, D.

LIPIŃSKI, E.
Phoenician “Torpedo” Amphoras and Egypt: Standardization of Volume Based on Linear Dimensions

DE MIROSCHEDJI, P.

NA’AMAN, N.
1994 Esarhaddon’s Treaty with Ba’al and Assyrian Provinces along the Phoenician Coast, Rivista di Studi Fenici 22, 3–8.

NA’AMAN, N.

POMMERENING, T.
2005 Die altägyptischen Hohlmaße, Hamburg.

POSTGATE, J.N.
1974 Taxation and Conscription in the Assyrian Empire, Rome.

PULAK, C.

REDFORD, D.
1992 Egypt, Canaan and Israel in Ancient Times, Princeton.

SINGER-AVITZ, L.
2010 A Group of Phoenician Vessels from the Iron Age IIB at Tel Beersheba, Tel Aviv 37, 188–199.

SOWADA, K.N.

STAGER, L.E.


WINTER, I.J.

YARDENI, A.

ZAPASSKY, E., FINKELSTEIN, I. and BENENSON, I.
2006 Ancient Standards of Volume: Negevite Iron Age Pottery (Israel) as a Case Study in 3D Modeling, JAS 33, 1734–1743

ZAPASSKY, E., FINKELSTEIN, I. and BENENSON, I.