# Image Deblurring in the Presence of Salt-and-Pepper noise

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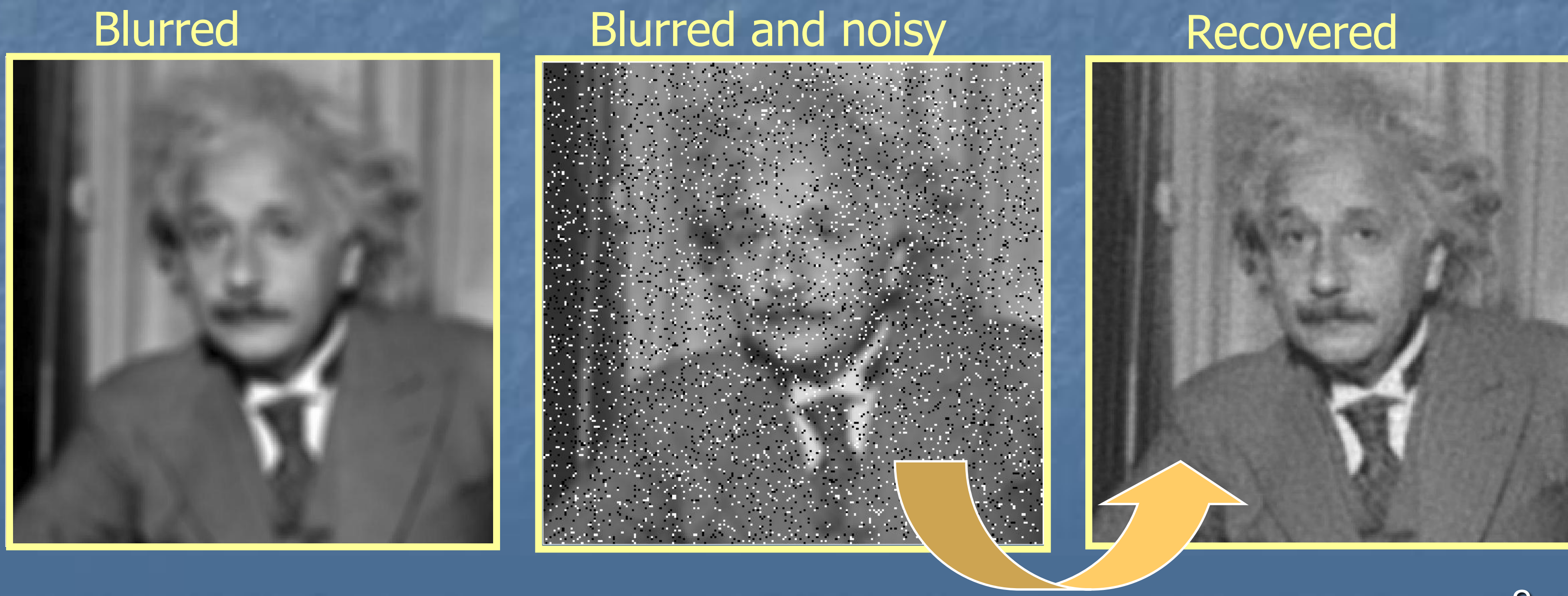
# Goal: Restoration of Blurred and Noisy Images

- Basic model:
  - Linear and shift invariant blur kernel (known)
  - Uniformly distributed impulsive (salt and pepper) noise



# Total Variation Restoration

## TV restoration in the presence of Gaussian noise

$$\min_{f} F(f) = \int_{\Omega} (g - h * f)^2 dx + \alpha \int_{\Omega} |\nabla f| \, dx$$

(Rudin, Osher, Fetami, 1992)

blurred by a pill-box kernel of radius 3 (7 × 7 kernel) with Gaussian noise

TV restoration

# Total Variation Restoration

TV restoration in the presence of salt-and-pepper noise

$$\min_{f} F(f) = \int_{\Omega} (g - h * f)^2 dx + \alpha \int_{\Omega} |\nabla f| dx$$

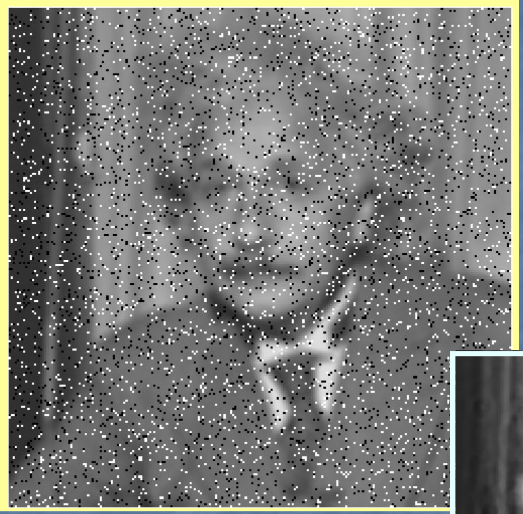
(Rudin, Osher, Fetami, 1992)

blurred by a pill-box kernel of radius 3
(7 × 7 kernel) with impulsive noise

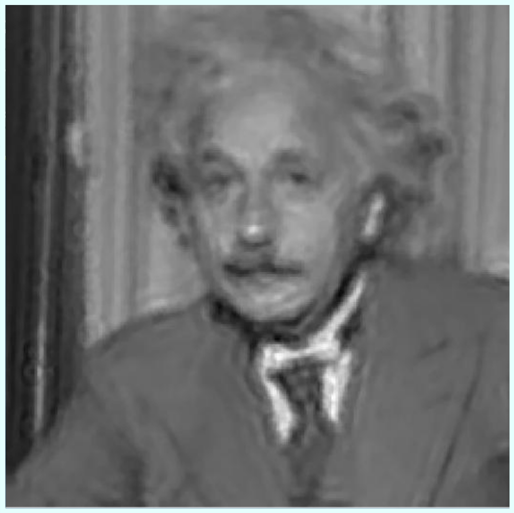
TV restoration

# Sequential Restoration

Blurred (pill-box with radius 4)
and noisy (10%) image



1. Median filter 3x3 window
2. TV restoration
3. Noise remains!



1. Median filter 5x5 window
2. TV restoration
3. Nonlinear distortion!

# Unified Restoration and Denoising

$$F(f) = \int_\Omega \phi(g - h * f)\,dx \quad + \quad \lambda \int_\Omega \rho(|\nabla f|)\,dx$$

fidelity term smoothness term

Ω: image domain *f*: recovered image *g*: observed image *h*: blur kernel

Need to choose $\phi$ and $\rho$!

# Unified Restoration and Denoising

- Fidelity term
  - Salt and pepper: outliers
  - Robust function: modified $L^1$ norm
- Smoothness term
  - Image prior: piecewise smoothness
  - Regularizer: Mumford-Shah (M-S) segmentation terms

$$F(f,K)=\int_{\Omega}\sqrt{(g-h*f)^2+\eta^2}\,dx+\beta\int_{\Omega\setminus K}|\nabla f|^2\,dx+\alpha\int_K d\sigma$$

data fidelity — gradients within segments — total edge length

$\Omega$: image domain  $K$: edge set  $f$: recovered image  $g$: observed image  $h$: blur kernel

- M-S functional: difficult to minimize (free-discontinuity problem).
- Solution is via the $\Gamma$-convergence framework

# Unified Restoration and Denoising

$\Gamma$-convergence approximation (Ambrosio and Tortorelli, 1990)

Strategy: approximate the solution by approximation of the problem

$$F_\varepsilon(f,v) \xrightarrow[\Gamma]{\varepsilon\to 0} F(f,K)$$

$$\arg\min \; F_\varepsilon(f,v) \xrightarrow{\varepsilon\to 0} \arg\min \; F(f,K)$$

$$F(f,K) = \int_\Omega \sqrt{(g-h*f)^2+\eta^2}\,dx + \beta \int_{\Omega\backslash K} |\nabla f|^2 dx + \alpha \int_K d\sigma$$

data fidelity — gradients within segments — total edge length

$$F_\varepsilon(f,v) = \int_\Omega \sqrt{(g-h*f)^2+\eta^2}\,dx + \beta\int_\Omega v^2 |\nabla f|^2 dx + \alpha \int_\Omega \left( \varepsilon |\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon} \right) dx$$

*v(x)*: smooth function *v(x)~0* at edges *v(x)~1* otherwise (in segments)

# Minimization of the Functional

$$F_\varepsilon(f,v) = \int_\Omega \sqrt{(g-h*f)^2+\eta^2}\,dx + \beta\int_\Omega v^2|\nabla f|^2 dx + \alpha\int_\Omega \left(\varepsilon|\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon}\right)dx$$

Iterate

Minimize with respect to $v$ (edge detection)

$$\frac{\delta F_\varepsilon}{\delta v} = 2\beta v|\nabla f|^2 + \alpha\frac{v-1}{2\varepsilon} - 2\varepsilon\alpha\nabla^2 v = 0$$

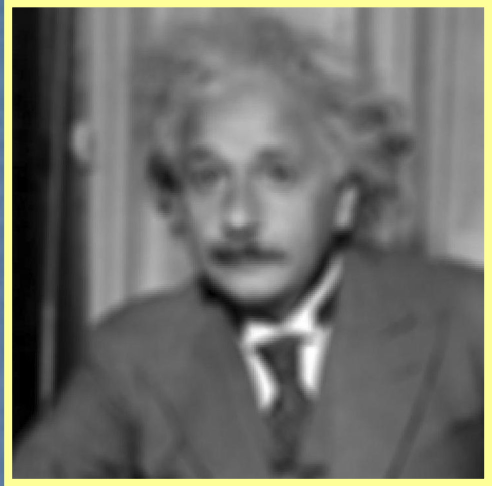
Minimize with respect to $f$ (image restoration)

$$\frac{\delta F_\varepsilon}{\delta f} = \frac{(g-h*f)}{\sqrt{(g-h*f)^2+\eta^2}} * h(-x,-y) - 2\beta\nabla\cdot\left(v^2\nabla f\right) = 0$$

# Results

pill-box kernel (9x9), radius 4, 10% noise

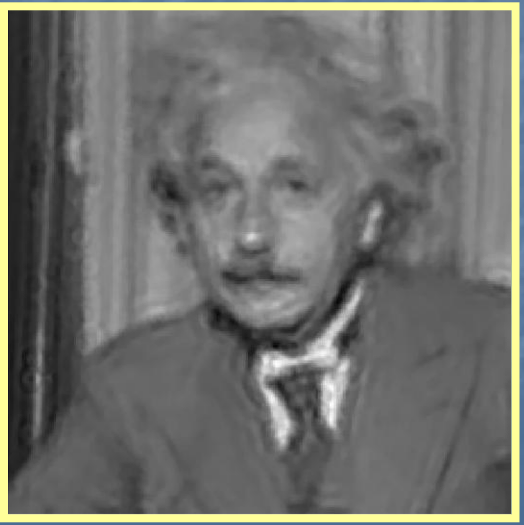
blurred



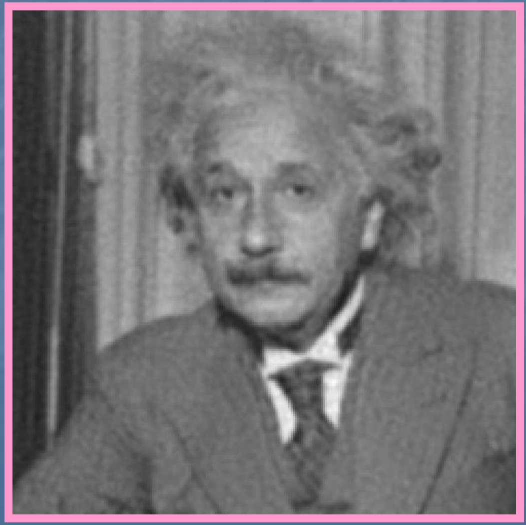
blurred and noisy

3x3 median + TV

5x5 median + TV



suggested



# Results pill-box kernel (7x7), radius 3, 1% noise

blurred and noisy

recovered

# Results pill-box kernel (7x7), radius 3, 10% noise

blurred and noisy

recovered

# Results

pill-box kernel (7x7), radius 3, 30% noise

blurred and noisy

recovered

# Results

motion blur kernel, $\theta=25^o$, len=8, 10% noise

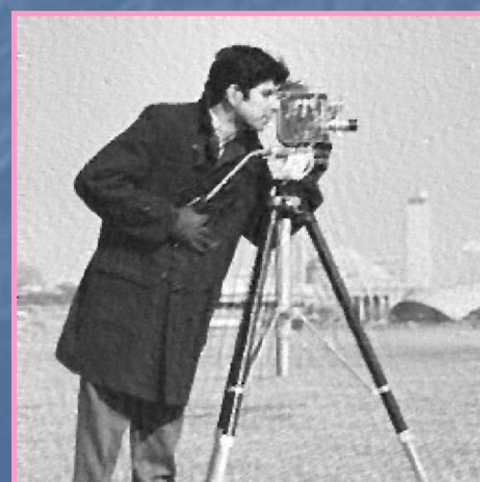
blurred

blurred and noisy

3x3 median+TV

5x5 median+TV

suggested



# Frequently Asked Questions

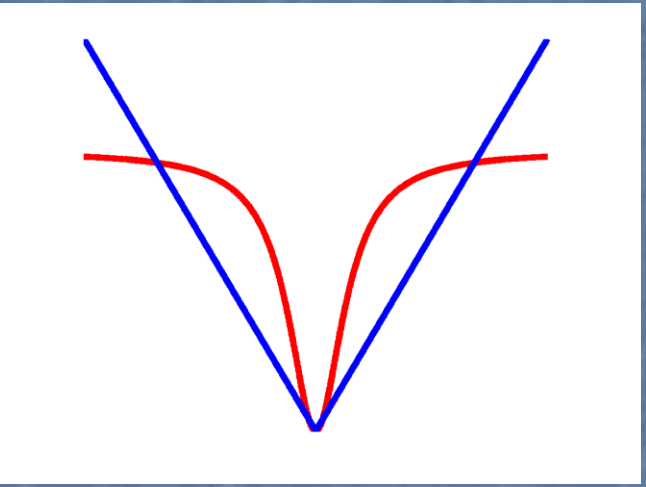
- Is Mumford-Shah (M-S) regularization a robust statistics method?
- Is M-S regularization an anisotropic diffusion process?
- Is M-S better than Total Variation?

# The Geman-McClure Robust Function

- Geman-McClure is more robust than $L^1$ (TV)

$$\rho(s) = \frac{s^2}{1+s^2}$$

Geman and McClure, 1987



- Geman-McClure is superior to convex functions in image classification problems

Aubert et al. 2004

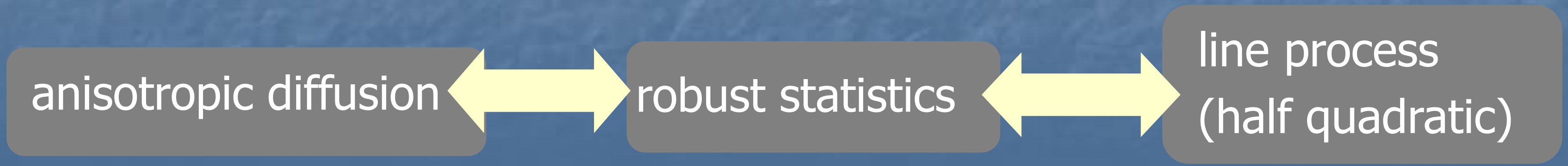
# Robust Statistics, Anisotropic Diffusion and Line Process

Relations between:

- robust statistics — Hampel et al., 1986
- anisotropic diffusion — Perona & Malik, 1987
- line process (half-quadratic) — Geman & Reynolds, 1993; Charbonnier et al., 1997

were shown by

- Black and Rangarajan, IJCV, 1996
- Black, Sapiro, Marimont and Heeger, IEEE T-IP, 1998



# Example: Geman-McClure Function

**Robust Smoothing**

$$R(f) = \lambda \int_{\Omega} \rho\left(\left|\nabla f\right|\right) dx$$

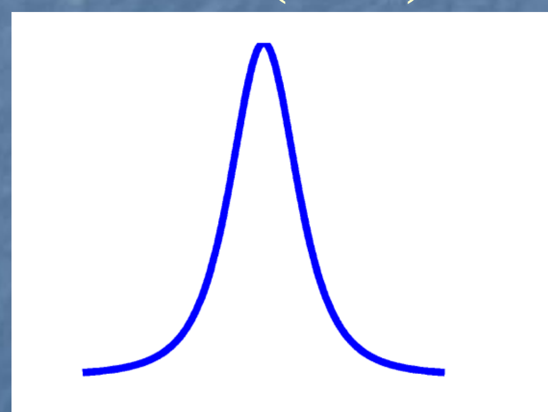
$$\rho(s) = \frac{s^2}{1+s^2}$$


robust ρ-function

**Anisotropic Diffusion**

$$\frac{\partial f}{\partial t} = \lambda \, div \left( \mathrm{A}\left(\left|\nabla f\right|\right) \nabla f \right)$$

$$\mathrm{A}(s) = \frac{2}{(1+s^2)^2}$$

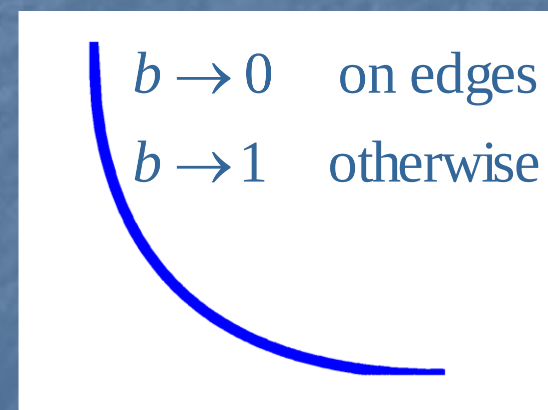


edge stopping function

**Line Process (Half-Quadratic)**

$$R(f) = \lambda \int_{\Omega} \inf_{b} \left( b \,|\nabla f|^2 + \mu(b) \right) dx$$

$$\mu(b) = \left(\sqrt{b} - 1\right)^2$$




edge penalty

# Robust Smoothing - Drawbacks

- Robust denoising with the Geman-McClure function has significant mathematical problems:
  - Instability (backward diffusion)

    Catte, Lions, Morel and Coll, SIAM J. Numer. Anal., 1992
  - No minimizer

    Chipot et al., 1997

# Relation to M-S Terms

- The Geman-McClure function in half-quadratic form

$$\frac{|\nabla f|^2}{1+|\nabla f|^2} \rightarrow b|\nabla f|^2 + \left(\sqrt{b}-1\right)^2$$

$$F_\varepsilon(f,v) = \int_\Omega \sqrt{(g-h*f)^2+\eta^2}\,dx + \beta\int_\Omega v^2|\nabla f|^2 dx + \alpha\int_\Omega\left(\varepsilon|\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon}\right)dx$$

- Appears in M-S terms with $b = v^2$

- M-S: extended line process = extended Geman-McClure

# Mumford-Shah (M-S) advantages

- M-S is more robust than $L^1$
- It can be shown that M-S is free of the mathematical drawbacks of Geman-McClure

# Answers to Frequently Asked Questions

- Is the M-S regularization a robust statistics method? NO (it is more general)

- Is the M-S regularization an anisotropic diffusion process? NO (more general)

- Is M-S better than Total Variation? YES

# Results

Blurred and noisy

TV Regularization

MS Regularization

$L^1$ fidelity

# Results

Blurred and noisy

TV Regularization

MS Regularization

$L^1$ fidelity

# Denoising

Nikolova, 2004

TV regularization

suggested

MS regularization

$L^1$ fidelity

# Denoising

Nikolova, 2004

TV regularization

suggested

MS regularization

$L^1$ fidelity

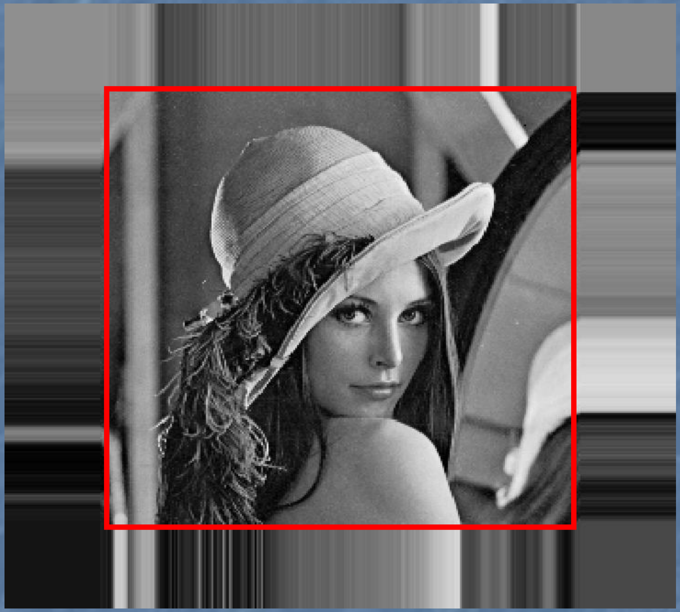
# Summary

- Unified approach for image deblurring in the presence of salt-and-pepper noise
- More general than robust statistics, etc.
- Better than $L^1$ in theory and practice
- Superior experimental results

Thank you

# Convolution Implementation

- Homogenous Neumann boundary conditions

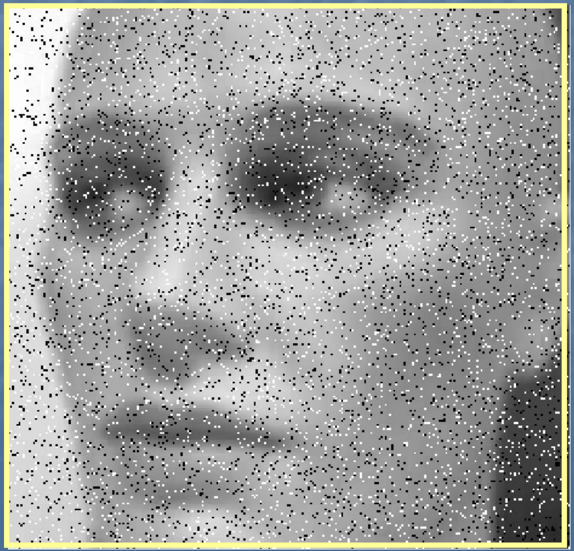


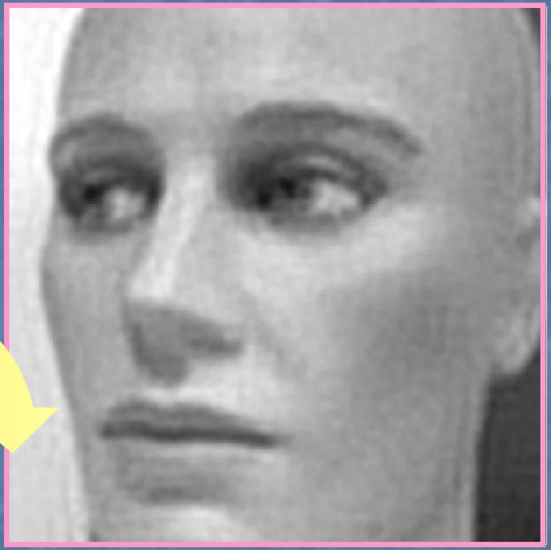
- FFT multiplication with zero padding

# Results – real out of focus image, 10% noise

Original

Noisy





Recovered
(pill-box radius=5)

# Results – Adaptive Median + TV

pill-box kernel (9x9), radius 4, 40% noise

observed

3x3 median+TV

5x5 median+TV

Adaptive median +TV

Suggested

# Results – Random Impulse Noise

# Sequential Results: MS + MS

Observed: noise: 0.1

MS denoising

MS deblurring

suggested

# Sequential Results: $L^1 + L^1$

Observed:  noise: 0.1

$L^1$ denoising

$L^1$ deblurring

suggested