Color Image Restoration

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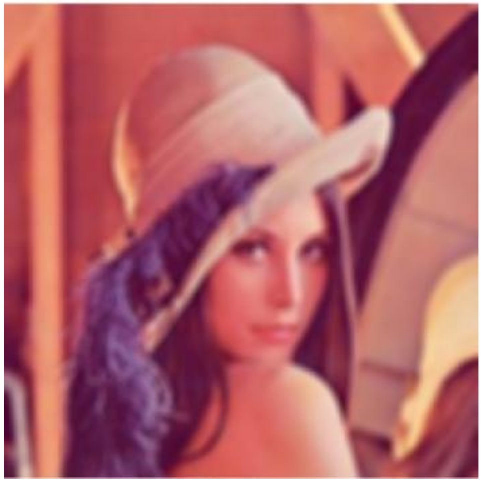
Goal: Restoration of Blurred and Noisy Color Images

Basic model:

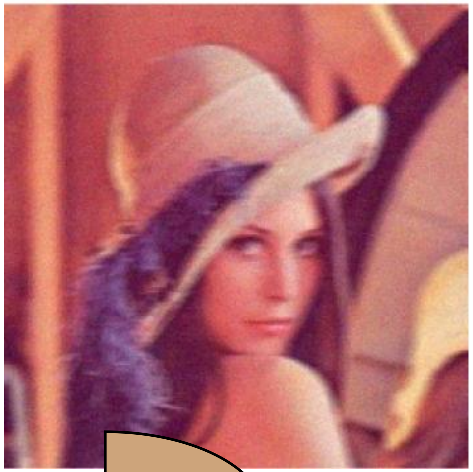
- Linear and shift invariant blur kernel (known)
- Noise: Gaussian or impulsive

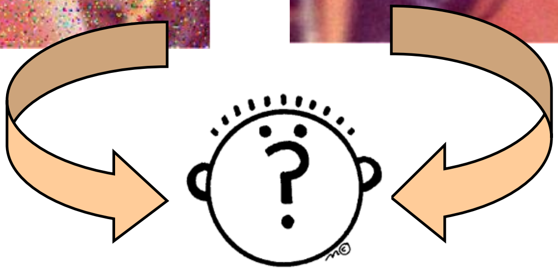
Blurred



Salt-and-pepper noise

Gaussian Noise





Gray-level Image Restoration – Variational Approach

$$F(f) = \int_\Omega \phi(g - h * f)dx \quad + \quad \lambda \int_\Omega \rho(|\nabla f|)dx$$

fidelity term | smoothness term

Ω: image domain f: recovered image g: observed image h: blur kernel

$$\phi(s) = \begin{cases} s^2 & \text{Gaussian noise} \\ |s| & \text{Impulsive noise} \end{cases}$$

$$\rho(s) = \begin{cases} s^2 & \text{L}^2\text{, Tikhonov (1977)} \\ |s| & \text{Total Variation (1992)} \\ \varsigma^2 \ln(1 + s^2/\varsigma^2) & \text{Perona-Malik (1990)} \end{cases}$$

Robust function ⇨ better edge preservation

Gray-level Image Restoration – L^2 and Total Variation

$$F(f) = \int_{\Omega} (g - h * f)^2 \, dx + \lambda \int_{\Omega} |\nabla f| \, dx$$

(Rudin, Osher, Fetami, 1992)

blurred by a pill-box kernel of radius 3 with Gaussian noise

TV restoration

Gray-level Image Restoration – L^1 and Mumford-Shah

$$F(f,K)=\int_{\Omega}\sqrt{(g-h*f)^2+\eta^2}\,dx+\beta\int_{\Omega\setminus K}|\nabla f|^2dx+\alpha\int_K dH^1$$

data fidelity — gradients within segments — total edge length

(Bar, Sochen, Kiryati, Scale-Space '05)

blurred by a pill-box kernel of radius 3
with 30% salt-and-pepper noise

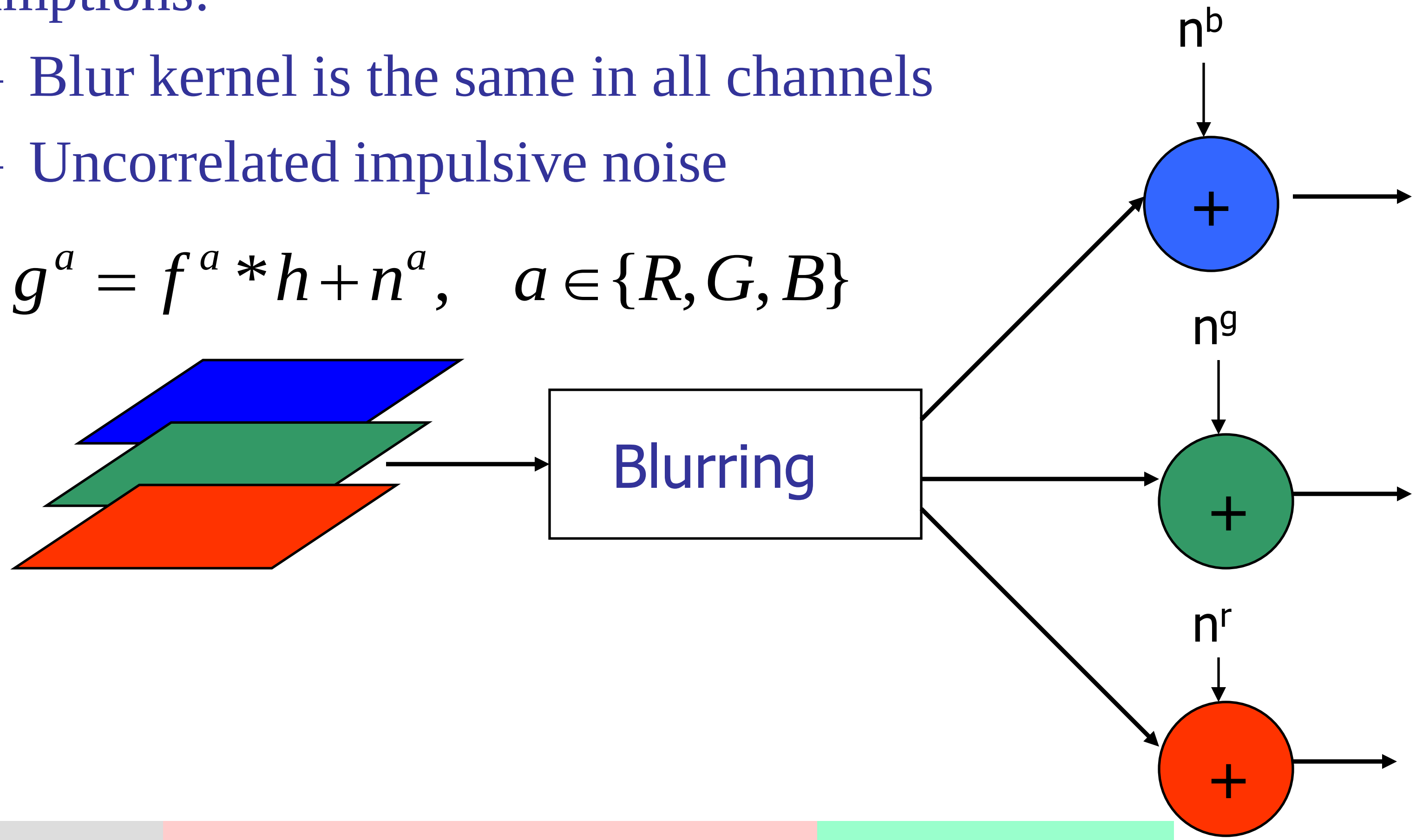
recovered

Color Image Restoration

Assumptions:

- Blur kernel is the same in all channels
- Uncorrelated impulsive noise

$$g^a = f^a * h + n^a, \quad a \in \{R, G, B\}$$

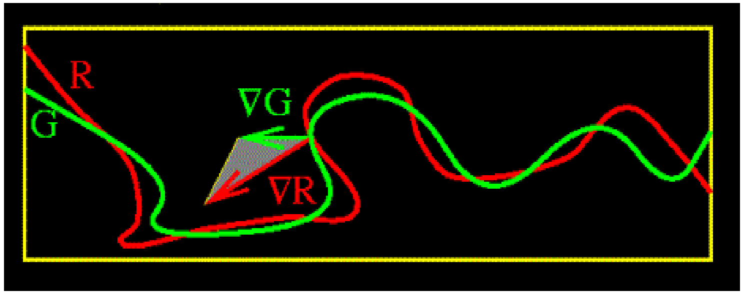


$$F(\vec{f}) = \int_\Omega \sum_{a \in RGB} \phi(g^a - h * f^a)\, dx + \lambda \int_\Omega \rho(\vec{f})\, dx$$

Color Image Restoration

Naive strategy: channel-by-channel restoration

Problem: Channel-edges are not necessarily aligned



Blurred and noisy (30%)

L^1+TV

L^1+MS

Color Interaction in Image Restoration

Problem: Channel-edges are not necessarily aligned

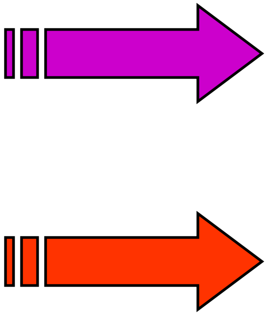
Solution: Color-coupling in regularization term

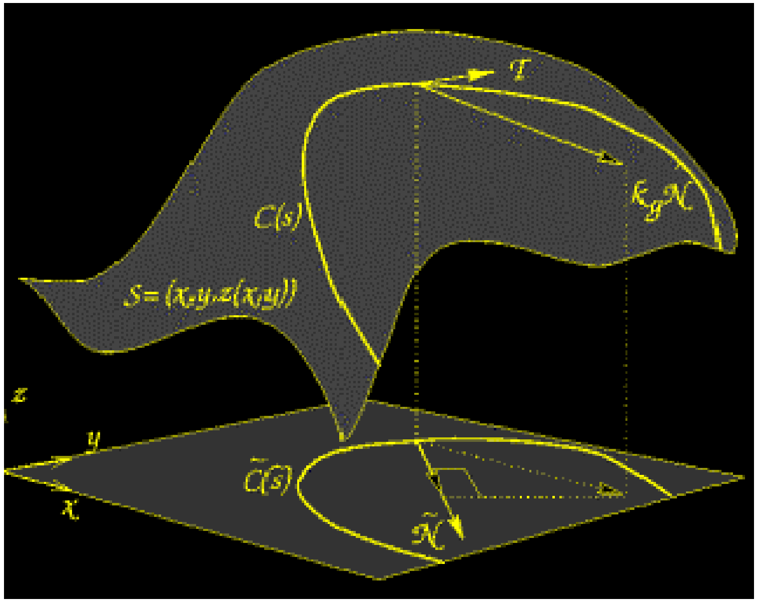
- Beltrami flow
- Coupled Mumford-Shah
- Coupled Total Variation Mumford-Shah

Beltrami Flow – Geometric Framework

- Combine spatial and color measures to a hybrid space

(Sochen, Kimmel, Malladi, '98)





- Image is a 2D surface Σ, $ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon$

embedded in higher dimensional (Riemannian) manifold M, $ds^2 = h_{ij} dy^i dy^j$

- Induced metric: $g_{\mu\upsilon} = h_{ij} \partial_\mu y^i \partial_\upsilon y^j$

Beltrami Flow – Geometric Framework

Smoothing: minimizing the area of the image surface

$$\psi^{BEL} = \int_{\Omega} \rho^{BEL}(\vec{f}) = \int_{\Omega} \sqrt{\det(G)}\, dx \rightarrow \min$$

For color image metric

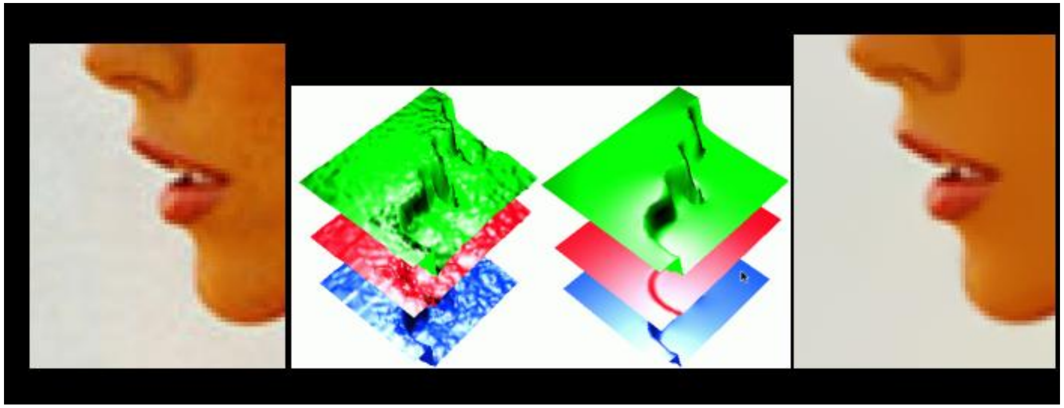
aligns colors together

$$\rho^{BEL}(\vec{f}) = \sqrt{1 + \beta^2 \sum_{a \in RGB} \left|\nabla I^a\right|^2 + \frac{1}{2} \sum_{a,b \in RGB} \left|\nabla I^a \times \nabla I^b\right|^2}$$

$$\frac{1}{\sqrt{\det(G)}} \frac{\delta F}{\delta f^a} = -\Delta_g (f^a)$$

Laplace-Beltrami operator

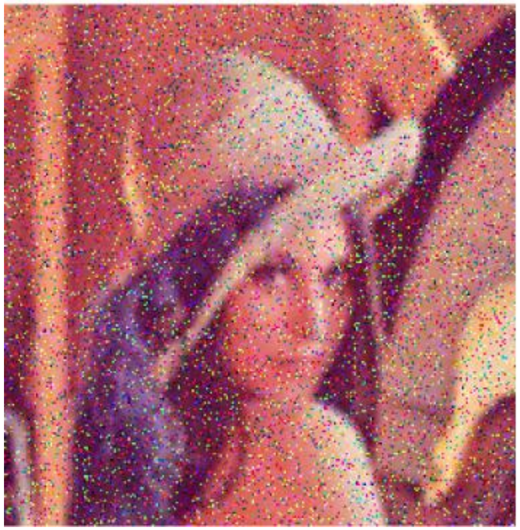
edge indicator



Color Image Restoration with Beltrami Flow

$$F(\vec{f}) = \int_{\Omega} \sum_{a \in RGB} \sqrt{(g^a - h * f^a)^2 + \eta}\, dx + \lambda \int_{\Omega} \rho^{BEL}(\vec{f})\, dx$$

$$\frac{\delta F}{\delta f^a} = \frac{g^a - h * f^a}{\sqrt{(g^a - h * f^a)^2 + \eta}} * h(-x,-y) - \lambda \Delta_g (f^a)$$



Coupled Mumford-Shah

Image as piecewise smooth function (Mumford and Shah '85)

Grey-level image:

$$\psi^{MS} = \beta \int_{\Omega / K} |\nabla f|^2 dx + \alpha \int_K dH^1$$

Approximation by Γ-convergence framework

$$\psi_\varepsilon^{MS}(f, v) = \beta \int_\Omega v^2 |\nabla f|^2 dx + \alpha \int_\Omega \left(\varepsilon |\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon} \right) dx$$

$v(x) \approx 0$ across edges $v(x) \approx 1$ within segments

Color coupling by Frobenius norm (Brook, Kimmel, Sochen, '03)

$$|\nabla \vec{f}|^2 = \sqrt{R_x^2 + R_y^2 + G_x^2 + G_y^2 + B_x^2 + B_y^2}$$

Color Image Restoration with Coupled Mumford-Shah

$$F(\vec{f},v) = \int_{\Omega} \sum_{a \in RGB} \sqrt{(g^a - h * f^a)^2 + \eta}\, dx + \psi_{\varepsilon}^{MS}(\vec{f},v)$$

$$\frac{\delta F_{\varepsilon}}{\delta v} = 2\beta v \left|\nabla \vec{f}\right|^2 + \alpha \frac{v-1}{2\varepsilon} - 2\varepsilon\alpha\nabla^2 v = 0$$

$$\frac{\delta F_{\varepsilon}}{\delta f^a} = \frac{\left(g^a - h * f^a\right)}{\sqrt{\left(g^a - h * f^a\right)^2 + \eta^2}} * h(-x,-y) - 2\beta\nabla\cdot\left(v^2 \nabla f^a\right) = 0$$



Coupled Total Variation Mumford-Shah

$$\psi_{\varepsilon}^{MS}(f,v) = \beta\int_{\Omega} v^2 |\nabla f|^2 dx + \alpha\int_{\Omega}\left(\varepsilon|\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon}\right)dx$$

$$\psi_{\varepsilon}^{MSTV}(f,v) = \beta\int_{\Omega} v^2 |\nabla f| dx + \alpha\int_{\Omega}\left(\varepsilon|\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon}\right)dx$$

(Shah CVPR '96)

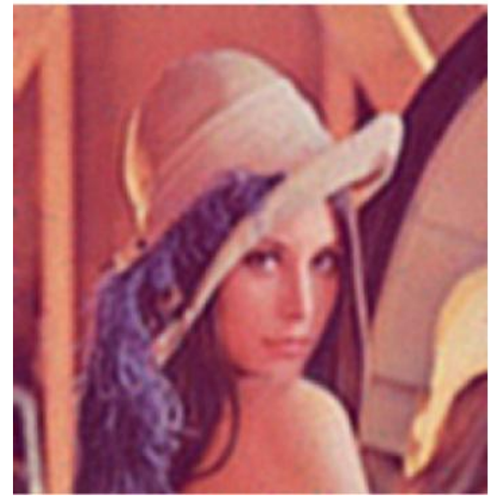
MSTV Regularization is much more robust and attracts the recovered image towards cartoon or piecewise constant function.

Comparative Results

30% salt-and-pepper noise

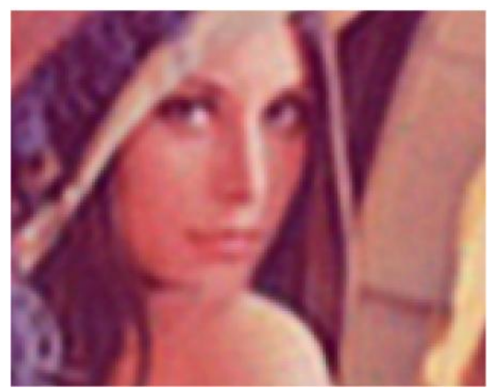
Channel-by-channel MS

Coupled MS



Beltrami

Coupled MS

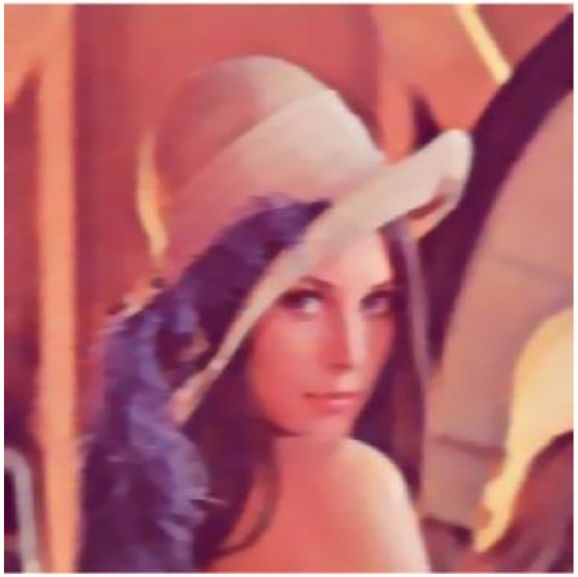


Comparative Results

Motion blur and Gaussian noise

coupled MS

Total Variation MS



Summary

- Variational color restoration is considered
- Color coupling is crucial in image restoration
- Several edges preserving regularizers were suggested
- Mumford-shah regularizer performs better in the presence of impulsive noise
- TV Mumford-shah regularizer performs better in the presence of Gaussian noise