Variational Restoration of Images with Space-Variant Blur

Leah Bar

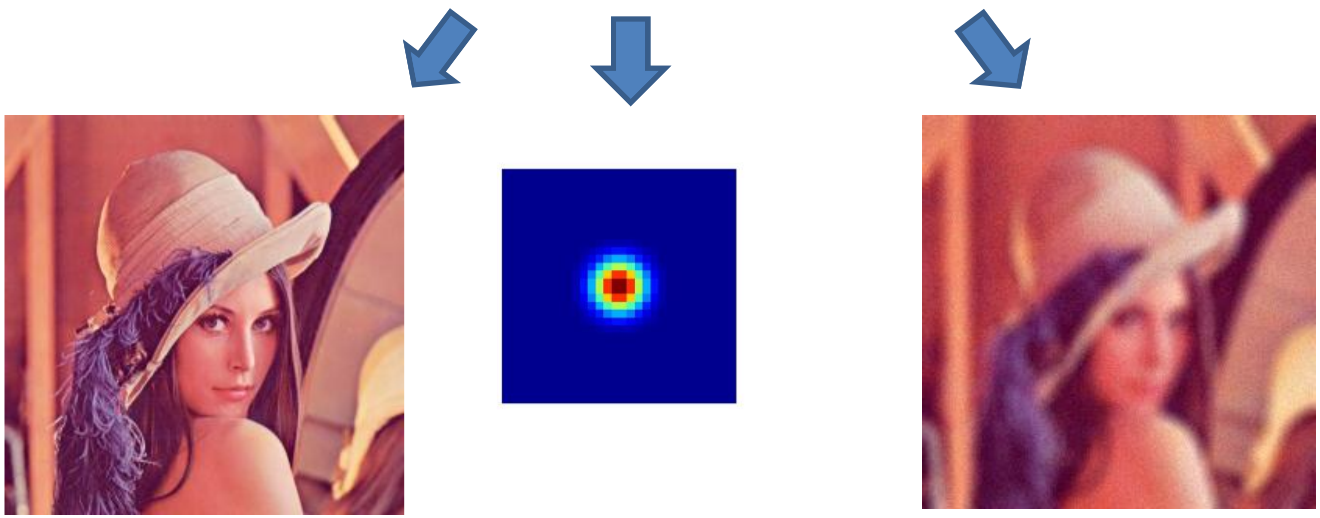
Department of Electrical Engineering

University of Minnesota

Space-Invariant Model

- Image is degraded by blur and random (noise) processes.
- Blur is assumed as a linear shift invariant process with additive noise.

$$f * h + n = g$$



Space Invariant Model – Variational Approach

Total Variation

$$F(f) = \int_{\Omega} (g - h * f)^2 \, dx + \alpha \int_{\Omega} |\nabla f| \, dx$$

fitting term regularization

Rudin, Osher, Fetami 1992

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$$F(f) = \int_\Omega (g - h * f)^2 dx + \alpha \int_\Omega |\nabla f| dx$$

fitting term   regularization

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Ω: image domain K: edge set f: recovered image g: observed image h: blur kernel

Mumford-Shah

$$F(f,K) = \int_\Omega (g - h * f)^2 dx + \beta \int_{\Omega \setminus K} |\nabla f|^2 dx + \alpha \int_K d\sigma$$

gradients within segments   total edge length

Mumford and Shah, 1985

Bar, Sochen, Kiryati 2004

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gradients within segments total edge length

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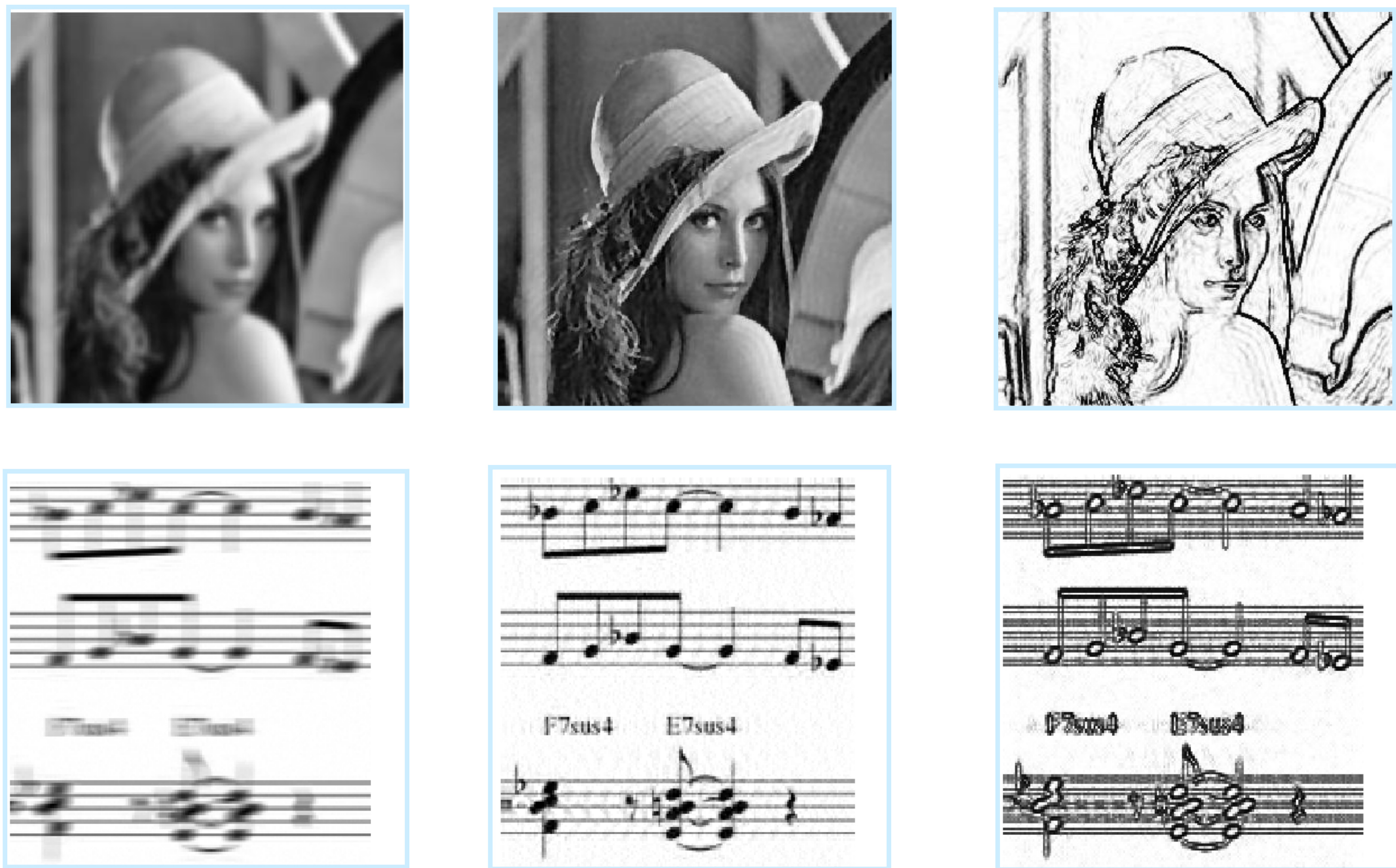
Γ-convergence Ambrosio and Tortorelli, 1990

$$F_\varepsilon(f,v) = \int_\Omega (g - h*f)^2 dx + \beta \int_\Omega v^2 |\nabla f|^2 dx + \alpha \int_\Omega \left(\varepsilon |\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon} \right) dx$$

v(x)~0 at edges *v(x)~1* otherwise (in segments)

Deconvolution with Mumford-Shah Regularization

blurred | suggested restoration | computed edges (v)



Space-Variant Image Restoration

- Assuming a piecewise point spread function
- Segmentation of sub-regions
- Space-variant deblurring

Out of Focus Blur

f_{fg} (closer)

f_{bg}

$$g = \left(f_{fg} * h\right) \cdot \left(1 - \chi_{bg} * h\right) + f_{bg} \cdot \left(\chi_{bg} * h\right)$$

$$g = f_{fg} \cdot \chi_{obj} + \left(f_{bg} * h\right) \cdot \left(1 - \chi_{obj}\right)$$

Marshal et al, “Occlusion Edge Blur”, J. Opt. Soc. Am. 1996

Motion Blur

f_{bg}

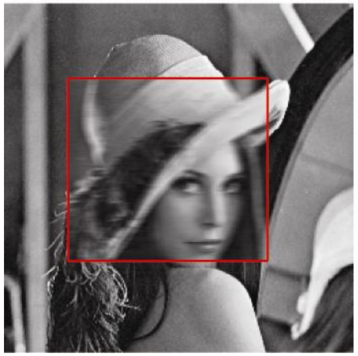
f_{fg} (closer)

$$g = \left(f_{fg} \cdot \chi_{obj} \right) * h + f_{bg} \left(1 - h * \chi_{obj} \right)$$

L. Bar, B. Berkels, G. Sapiro , M. Rumpf, ICCV 2007

Non-Blind Piecewise Deblurring

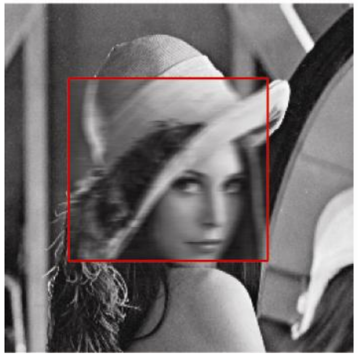
L. Bar, N. Sochen, N. Kiryati , SSVM 2007


Space-variant blur


Failure of region-wise deblurring

Non-Blind Piecewise Deblurring

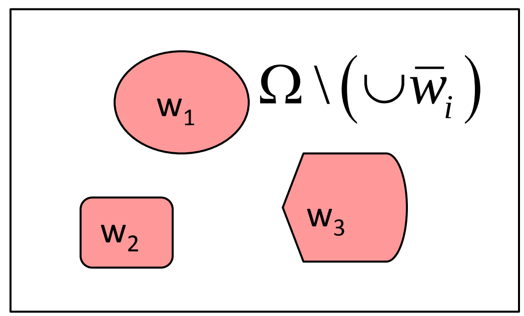
L. Bar, N. Sochen, N. Kiryati, SSVM 2007



Space-variant blur

Failure of region-wise deblurring

Let: w_i = sub-region set
h_i = kernels set
h_b = background kernel

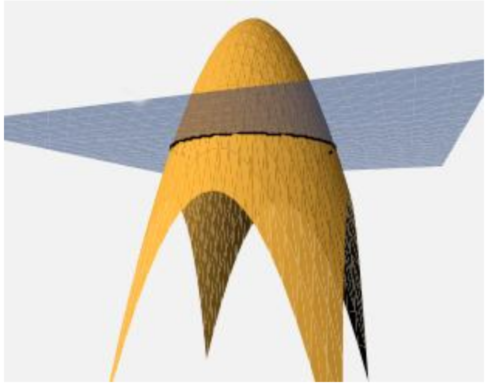


$$F(f) = \frac{1}{2} \sum_i \int_{w_i} \left(h_i * f - g \right)^2 dx + \frac{1}{2} \int_{\Omega \setminus \left(\cup \overline{w_i} \right)} \left(h_b * f - g \right)^2 dx + R^{MS}$$

Fitting term

Mumford-Shah regularization

The contour is implicitly represented by the zero level-set of a function ϕ



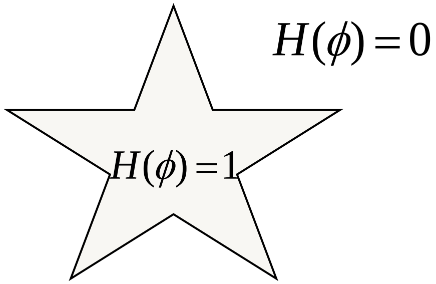
$$C = \partial w = \{(x,y) \in \Omega : \phi(x,y) = 0\}$$

$$\text{inside}(C) = \omega = \{(x,y) \in \Omega : \phi(x,y) > 0\}$$

$$\text{outside}(C) = \Omega \setminus \varpi = \{(x,y) \in \Omega : \phi(x,y) < 0\}$$

Heaviside function

$$H(s) = \begin{cases} 1 & s > 0 \\ 0 & else \end{cases}$$



$$\text{inside}(C) = \omega = \{(x,y) \in \Omega : H(\phi) = 1\}$$

$$\text{outside}(C) = \Omega \setminus \varpi = \{(x,y) \in \Omega : H(\phi) = 0\}$$

w_i: sub domain f: recovered image g: observed image h_i: blur kernels

$$\frac{1}{2}\sum_i \int_{w_i} (h_i * f - g)^2 \, dx + \frac{1}{2}\int_{\Omega \backslash (\cup \overline{w_i})} (h_b * f - g)^2 \, dx + R^{MS}$$

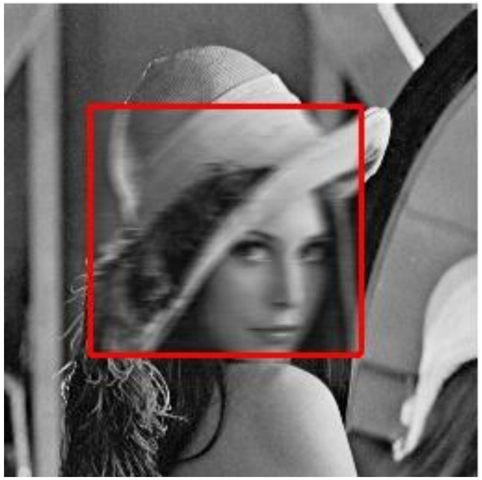
Using level-set and Γ-convergence formulation

$$F_\varepsilon^{MS}(f, v) = \frac{1}{2}\sum_i \int_\Omega (h_i * f - g)^2 H(\phi_i) dx + \frac{1}{2}\int_\Omega (h_b * f - g)^2 \left[1 - \sum_i H(\phi_i)\right] dx$$

$$+ \beta \int_\Omega v^2 |\nabla f|^2 dx + \alpha \int_\Omega \left(\varepsilon |\nabla v|^2 + \frac{(v-1)^2}{4\varepsilon} \right) dx$$

Non-Blind Piecewise Deblurring - Results

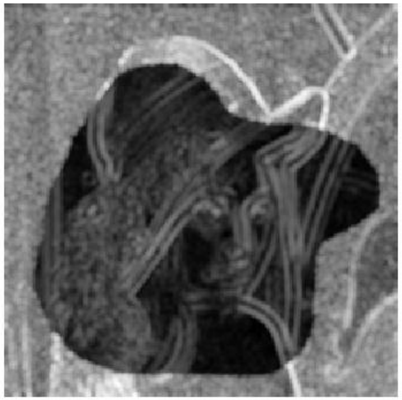





Semi-Blind Space Variant Restoration

Blur detection function

$$E(x) := \left| \log\left(\left|\nabla^2 g(x)\right| * B_r \right) \right|$$



Semi-Blind Space Variant Restoration

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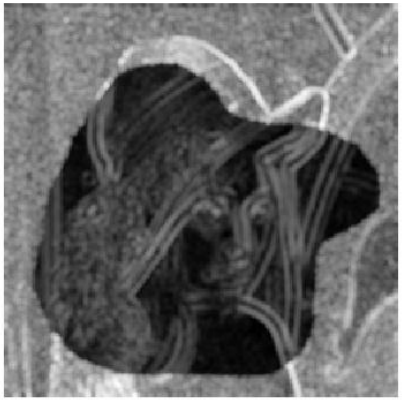


Find a contour C and four constants c_1, c_2, c_3, c_4 such that

$$F(C,\vec{c}) = \frac{\lambda_1}{2} \int_{inside(C)\cap K} (E-c_1)^2 dx + \frac{\lambda_2}{2} \int_{outside(C)\cap K} (E-c_2)^2 dx$$

$$+ \frac{\lambda_3}{2} \int_{inside(C)\setminus K} (E-c_3)^2 dx + \frac{\lambda_4}{2} \int_{outside(C)\setminus K} (E-c_4)^2 dx$$

$$+ \oint_C F_{GAC}\left(\left| \nabla E \right| \right) ds$$

$$F_{GAC} = \frac{\mu}{1+\left(V_T^2 \, |\nabla E|^2 \right)/\gamma} + \nu$$

Semi-Blind Space Variant Restoration

observed

recovered

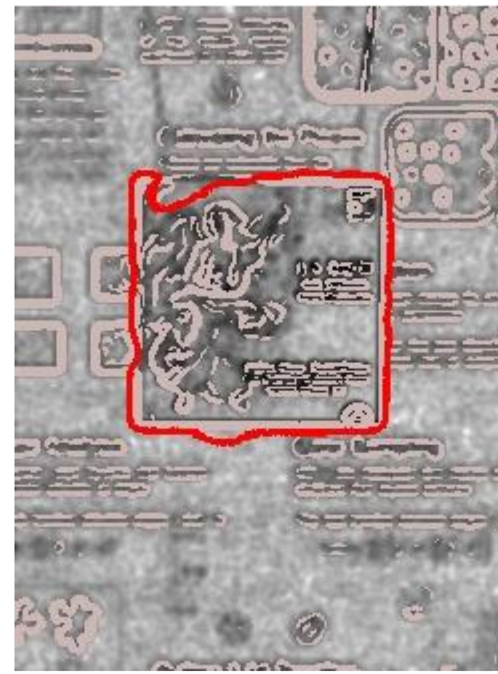
blur detection function

Semi-Blind Space Variant Restoration

observed

recovered

blur detection function



Motion Estimation and Restoration of Blurred Video

L. Bar, B. Berkels, G. Sapiro , M. Rumpf, ICCV 2007

$t=t_0$

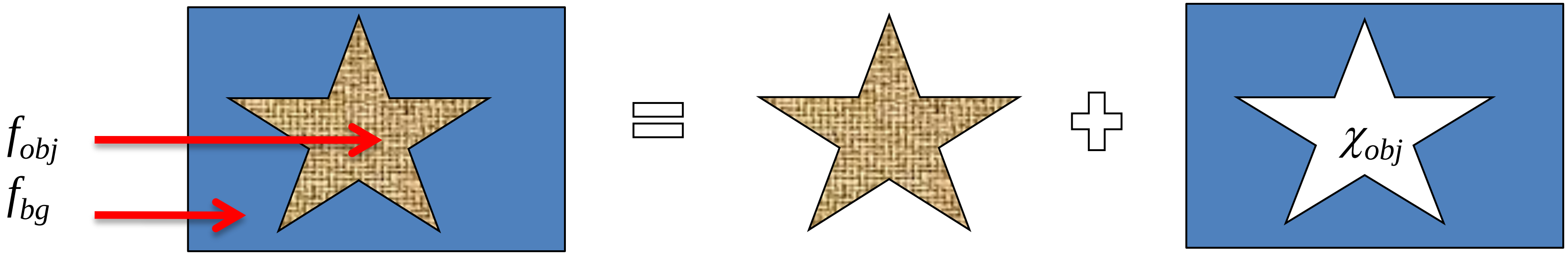
$t=t_1$

Deconvolution ⟺ Motion Estimation

Given 2 or more video frames: estimate the

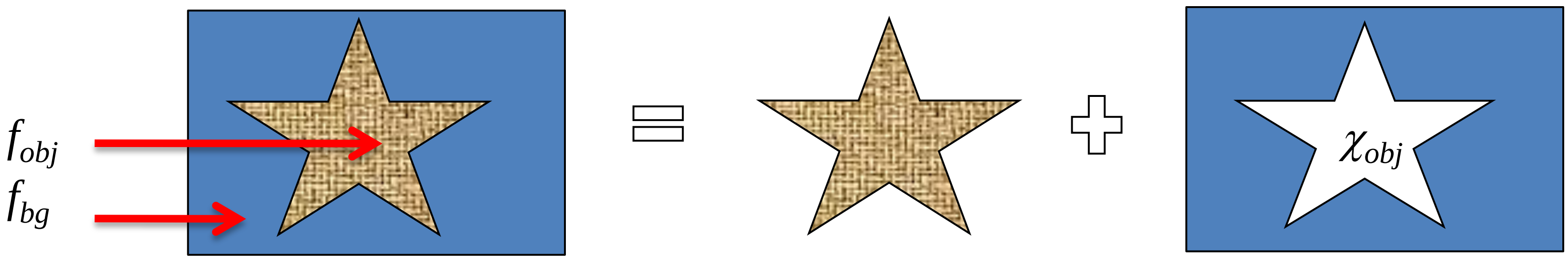
- velocity of the moving object
- sharp version of the object
- object characteristic function (segmentation)

Blur Formation



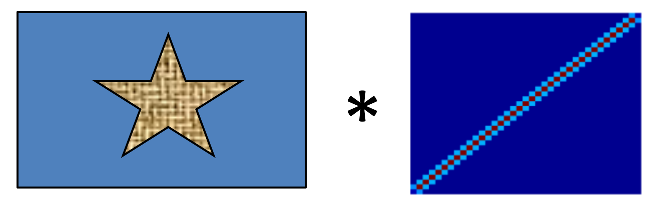
$$f(t,x) = f_{obj}(x - vt)\chi_{obj}(x - vt) + f_{bg}\left[1 - \chi_{obj}(x - vt)\right]$$

Blur Formation



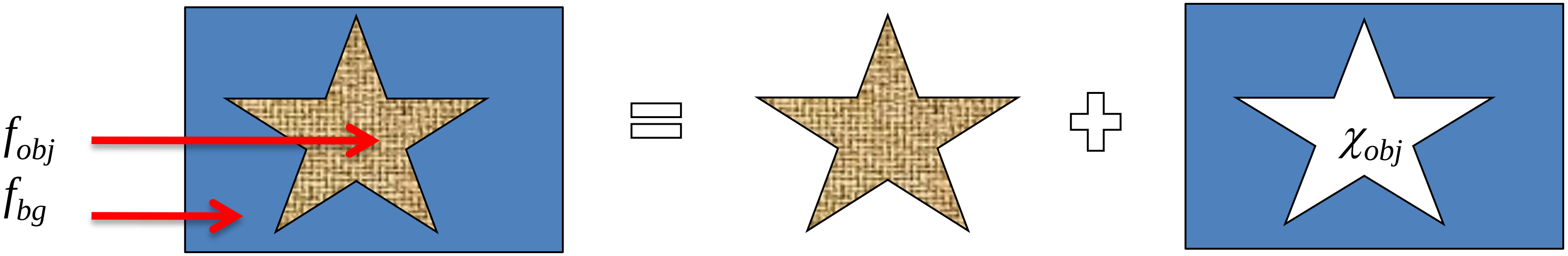
$$f(t,x) = f_{obj}(x - vt)\chi_{obj}(x - vt) + f_{bg}\left[1 - \chi_{obj}(x - vt)\right]$$

$$g_i(x) = \frac{1}{T}\int_{t_i - T/2}^{t_i + T/2} f(x - sv)\,ds = f * h_v(x)$$



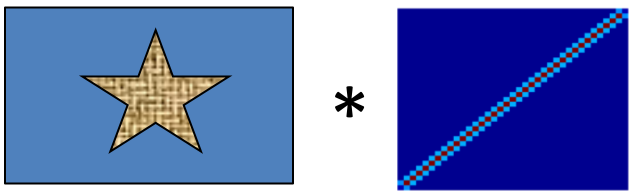
$$h_v(x) = \frac{1}{T\|v\|}\delta\left(v \cdot x^{\perp}\right) H\left(T\|v\| - 2\|x\|\right)$$

Blur Formation



$$f(t,x) = f_{obj}(x-vt)\chi_{obj}(x-vt) + f_{bg}\left[1-\chi_{obj}(x-vt)\right]$$

$$g_i(x) = \frac{1}{T}\int_{t_i-T/2}^{t_i+T/2} f(x-sv)ds = f * h_v(x)$$



$$h_v(x) = \frac{1}{T\|v\|}\delta\left(v\cdot x^{\perp}\right)H\left(T\|v\|-2\|x\|\right)$$

$$g_i(t,x) = \left(\left(f_{obj}\chi_{obj}\right)*h_v\right)(x-vt_i) + f_{bg}\left[1-(\chi_{obj}*h_v)(x-vt_i)\right]$$

Objective Functional

$$\varepsilon(\chi_{obj}, f_{obj}, v) =$$

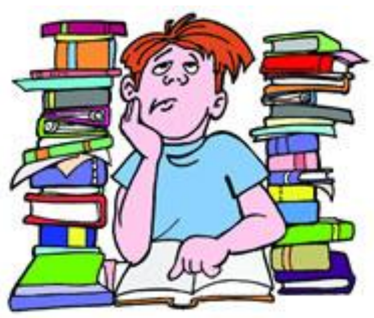
$$\sum_{i=1}^{2}\int_{\Omega}\left\{\left(\left(f_{obj}\chi_{obj}\right)*h_v\right)(x-t_i v)+f_{bg}(x)\left[1-\left(\chi_{obj}*h_v\right)(x-t_i v)\right]-g_i\right\}^2 dx+\int_{\Omega}\beta\left|\nabla f_{obj}\right|+\upsilon\int_{\Omega}\left|\partial\chi_{obj}\right|dx$$

$$\chi_{obj} = \text{Heaviside}(\phi)$$

$$\varepsilon(\phi, f_{obj}, v) =$$

$$\sum_{i=1}^{2}\int_{\Omega}\left\{\left(f_{obj}H(\phi)*h_v\right)(x-t_i v)+f_{bg}(x)\left[1-\left(H(\phi)*h_v\right)(x-t_i v)\right]-g_i\right\}^2 dx+\int_{\Omega}\beta\left|\nabla f_{obj}\right|+\upsilon\int_{\Omega}\left|\nabla H(\phi)\right|dx$$

How to optimize?



Initialization

Assume that there is no blur

$$\varepsilon_{init}(\phi, v) =$$

$$\sum_{i=1}^{2} \int_{\Omega} \left\{ \left(f_{obj} H(\phi) \right)(x - t_i v) + f_{bg}(x)\left[1 - H(\phi)(x - t_i v)\right] - g_i \right\}^2 dx + \upsilon \int_{\Omega} \left| \nabla H(\phi) \right| dx$$

$$f_{obj} = g_1, \quad \phi^0 = \text{cone}, \quad k = 0$$

do{

$$\phi^{k+1} = \phi^k - \tau^{\phi} \nabla_{\phi} \varepsilon_{init}[\phi^k, v^k] * G_{\sigma}$$

$$v^{k+1} = v^k - \tau^{v} \nabla_{v} \varepsilon_{init}[\phi^{k+1}, v^k] * G_{\sigma}$$

$$k = k + 1$$

}while $\left(\left\| \phi^{k+1} - \phi^k \right\|, \left\| v^{k+1} - v^k \right\| > \delta \right)$

$$\Rightarrow \phi^{init}, v^{init}$$

Main Loop

$$\varepsilon(\phi, f_{obj}, v) =$$

$$\sum_{i=1}^{2}\int_{\Omega}\left\{\left(f_{obj}H(\phi)*h_v\right)(x-t_i v)+f_{bg}(x)\left[1-\left(H(\phi)*h_v\right)(x-t_i v)\right]-g_i\right\}^2 dx+\int_{\Omega}\beta\left|\nabla f_{obj}\right|+\upsilon\int_{\Omega}\left|\nabla H(\phi)\right|dx$$

$$f_{obj}^{0}=g_1,\quad \phi^{init},\quad v^{init}\quad k=0$$

do{

$$\phi^{k+1}=\phi^{k}-\tau^{\phi}\nabla_{\phi}\varepsilon[\phi^{k},v^{k},f_{obj}^{k}]*G_{\sigma}$$

$$v^{k+1}=v^{k}-\tau^{v}\nabla_{v}\varepsilon[\phi^{k+1},v^{k},f_{obj}^{k}]*G_{\sigma}$$

$$f_{obj}^{k+1}=f_{obj}^{k}-\tau^{f}\nabla_{f}\varepsilon[\phi^{k+1},v^{k+1},f_{obj}^{k}]*G_{\sigma}$$

$$k=k+1$$

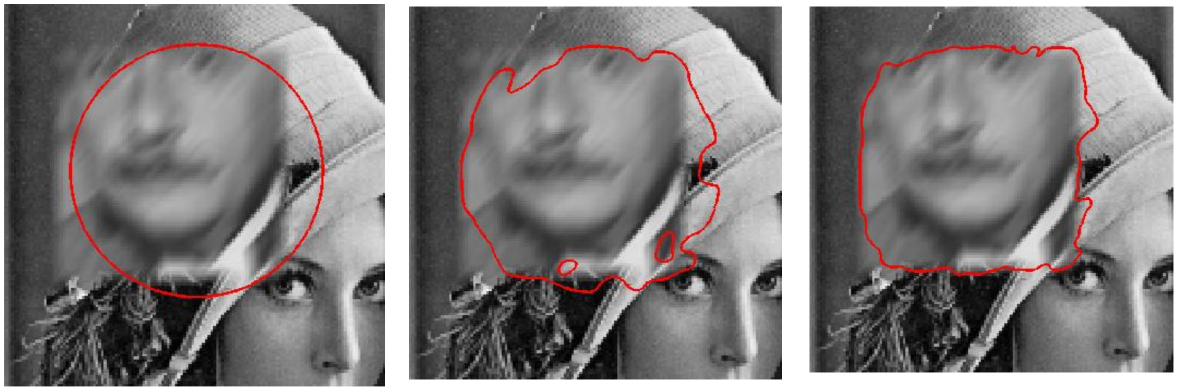
}while $\left(\left\|\phi^{k+1}-\phi^{k}\right\|,\left\|v^{k+1}-v^{k}\right\|,\left\|f_{obj}^{k+1}-f_{obj}^{k}\right\|>\delta\right)$

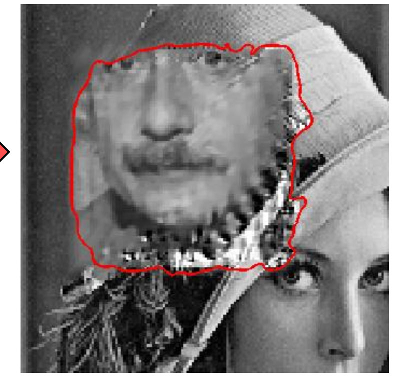
$$\Rightarrow\quad \phi\ ,v\ ,f_{obj}$$

Motion Estimation and Restoration of Blurred Video - Results

Synthetic but challenging example

Initialization

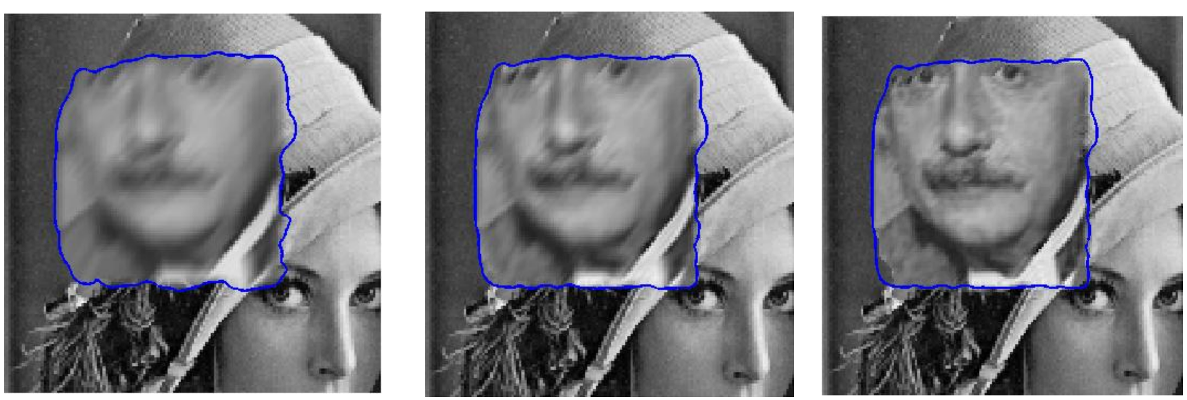




ground truth: v=[6,7]

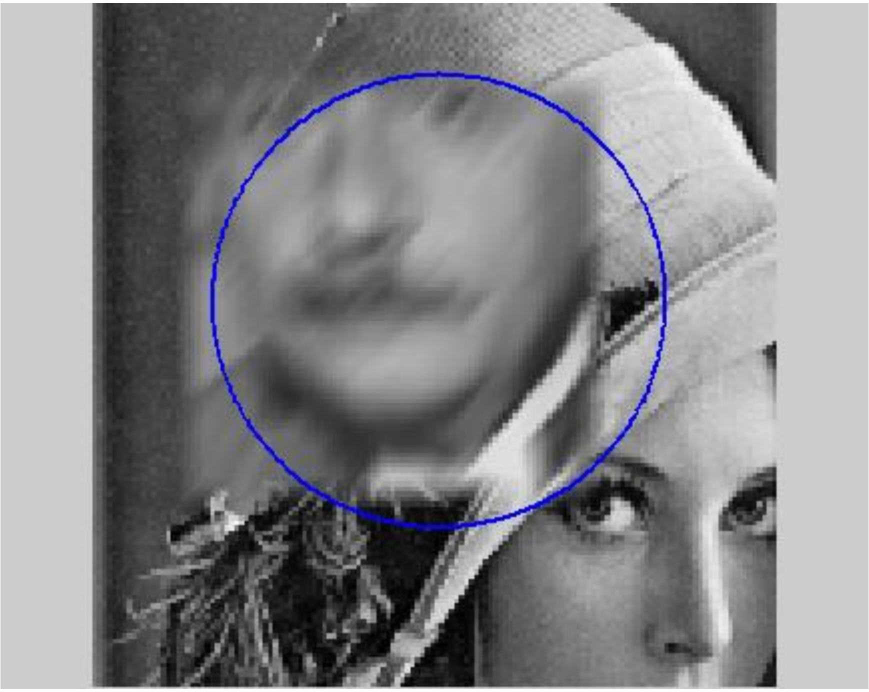
wrong model
v=[5.78,6.8]

Main loop



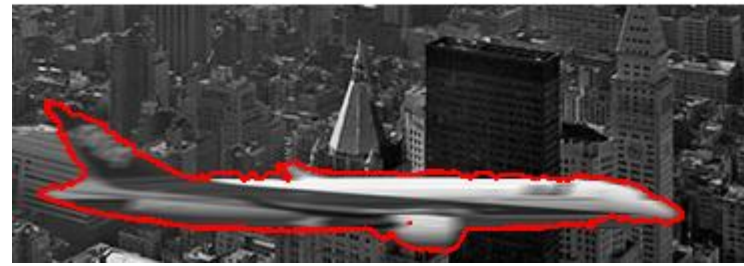
right model
v=[5.98,7.009]

Motion Estimation and Restoration of Blurred Video - Results



$v_{\text{truth}} = [6,7]$
$v_{\text{init}} = [5.78,6.8]$
$v_{\text{final}} = [5.98,7.009]$

Motion Estimation and Restoration of Blurred Video - Results



Ground truth: v=[10,0]

wrong model:
v=[9.46 ,-0.0213]

right model:
v=[9.48,-0.007]

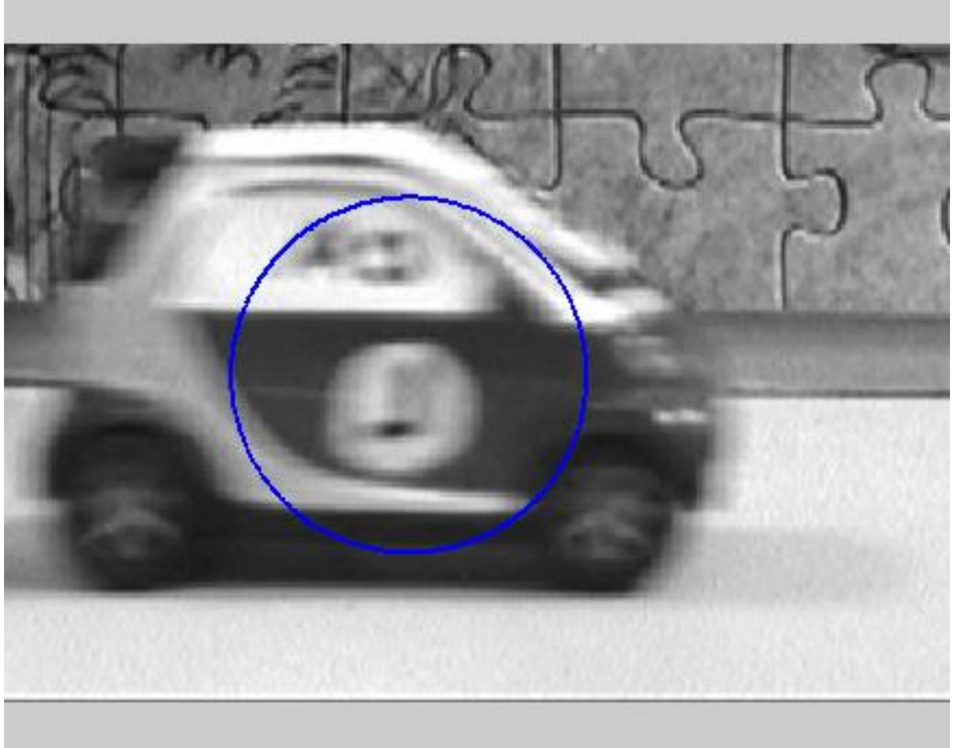
Motion Estimation and Restoration of Blurred Video - Results

v_{truth} = [10,0]
v_{init} = [9.46 ,-0.0213]
v_{final} = [9.48,-0.007]

Motion Estimation and Restoration of Blurred Video - Results

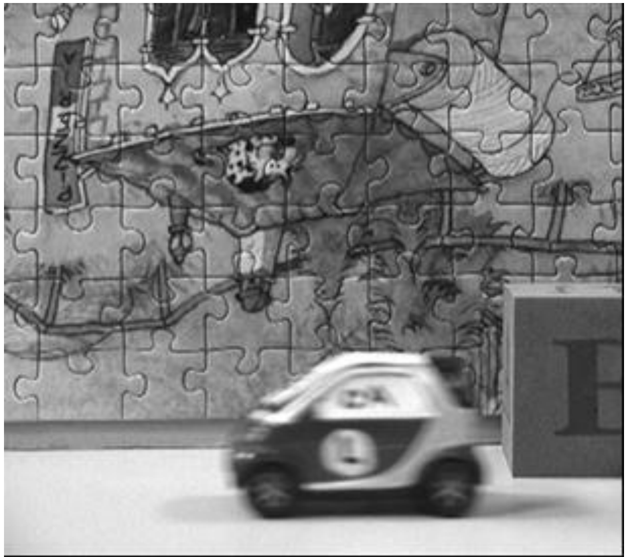
Original movie

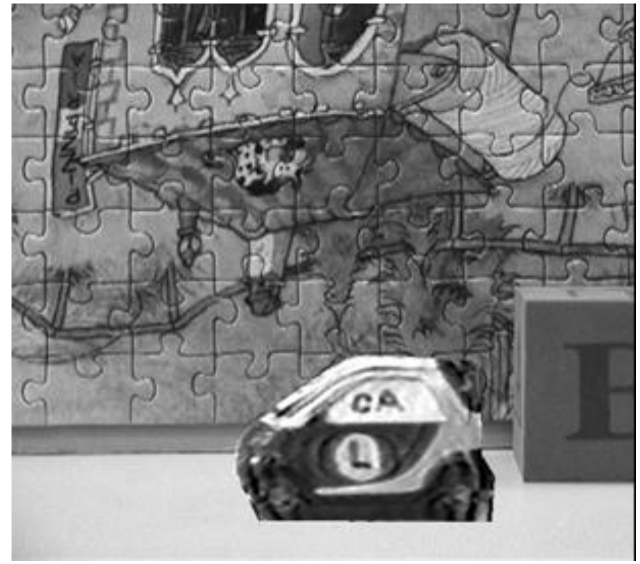
Motion Estimation and Restoration of Blurred Video - Results



v_{truth} = ?
v_{init} = [18.67, 0.0588]
v_{final} = [18.66,0.0473]

Motion Estimation and Restoration of Blurred Video - Results





Conclusions

- Space variant restoration is an ongoing and challenging research field
- A piecewise point spread function was used for out of focus and motion blur
- Promising results for synthetic/real images if using the physical model of the blur formation.

Thank you for your attention