

Total Variation Blind Deconvolution

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Tony F. Chan and Chiu-Kwong Wong, IEEE
Trans. Image Processing Vol 7. 1998

Outline

- Blind Deconvolution formulation
- Numerical methods solving PDE's
- Choice of regularization parameters
- Results
- Summary

Image Restoration With Known PSF

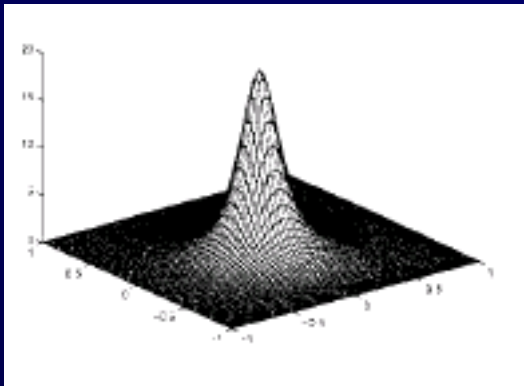
Blurring model: $z = h * u + \eta$

$$\min_u f(u) = \min_u \left\{ \underbrace{\frac{1}{2} \|h * u - z\|_{L^2(\Omega)}^2}_{\text{Denoising}} + \underbrace{\alpha \int_{\Omega} |\nabla u| dx dy}_{\text{TV smoothing}} \right\}$$

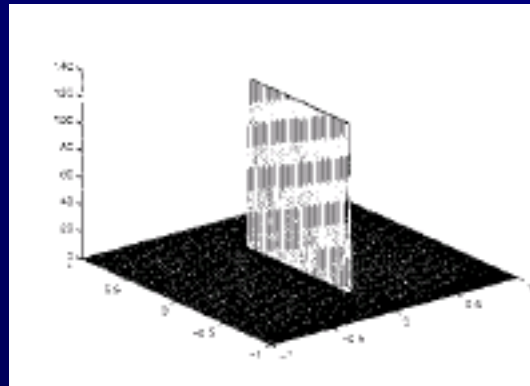
Rudin, Osher, Fetami 1992

Blind Deconvolution

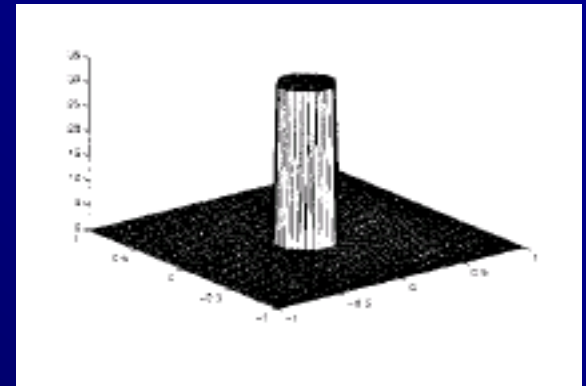
- Space invariant image restoration (of u and h) without any a priori knowledge of the PSF
- Assuming PSF is piecewise smooth



Atmospheric turbulence



Motion blur



Out of focus

Blind Deconvolution Formulation

$$\min_{u,h} f(u,h) = \min_{u,h} \frac{1}{2} \|h * u - z\|_{L^2(\Omega)}^2 + \alpha_1 \int_{\Omega} |\nabla u| dx dy + \alpha_2 \int_{\Omega} |\nabla h| dx dy$$

Euler-Lagrange optimization yields:

$$\frac{\delta L}{\delta h} = (u * h - z) * u(-x, -y) - \alpha_2 \nabla \cdot \left(\frac{\nabla h}{|\nabla h|} \right) = 0$$

$$\frac{\delta L}{\delta u} = (u * h - z) * h(-x, -y) - \alpha_1 \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) = 0$$

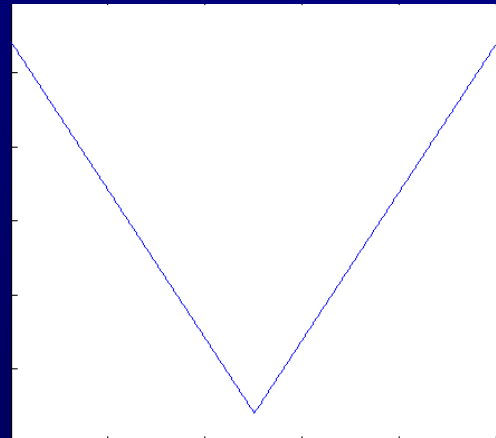
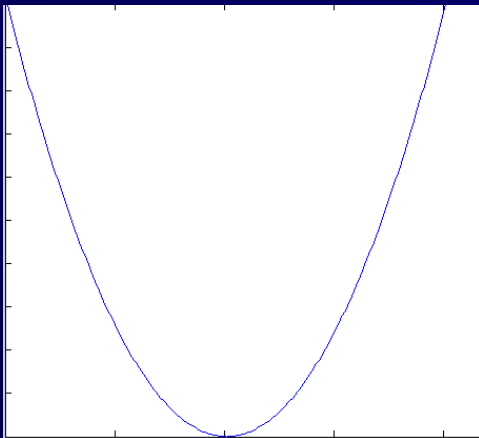
Convexity



Does the problem have global or local minima?

$$f(u, h) = \iint_{\Omega} (h * u - z)^2 + \alpha_1 |\nabla u| + \alpha_2 |\nabla h| dx dy$$

$f(u, h)$ is not jointly convex, but for a given u , $f(u, \cdot)$ is convex with respect to h and vice versa.



$$\min_{u,h} f(u,h) = \min_{u,h} \frac{1}{2} \|h * u - z\|_{L^2(\Omega)}^2 +$$
$$\alpha_1 \int_{\Omega} |\nabla u| dx dy + \alpha_2 \int_{\Omega} |\nabla h| dx dy$$

Minimization problem may not have a unique solution [e.g. $(-u, -h)$ is also solution], therefore the following conditions are imposed:

$$\int_{\Omega} h(x, y) dx dy = 1$$

$$u(x, y), h(x, y) \geq 0$$

$$h(x, y) = h(-x, -y)$$

Numerical Methods

AM – Alternate Minimization

Initialization : $u^0 = z, h^0 = \delta(x, y)$

$$f(u^0, h^1) = \min_h f(u^0, \cdot)$$

$$f(u^1, h^1) = \min_u f(\cdot, h^1)$$

The convexity property yields that the function $f(u^n, h^n)$ always decreases when n increases

$$\text{Solve } h^{n+1} : u^n(-x, -y) * (u^n * h^{n+1} - z) - \alpha_2 \nabla \cdot \left(\frac{\nabla h^{n+1}}{|\nabla h^{n+1}|} \right) = 0$$

$$\text{Solve } u^{n+1} : h^{n+1}(-x, -y) * (h^{n+1} * u^{n+1} - z) - \alpha_1 \nabla \cdot \left(\frac{\nabla u^{n+1}}{|\nabla u^{n+1}|} \right) = 0$$

Linearization of PDE's

Lagged Diffusivity Fixed Point (FP), Vogel
and Oman 1996

Coefficients are lagged by one iteration and
then linear equations are solved:

$$\text{Solve } h^{n+1} : u^n(-x, -y) * (u^n * h_{i+1}^{n+1} - z) - \alpha_2 \nabla \cdot \left(\frac{\nabla h_{i+1}^{n+1}}{|\nabla h_i^{n+1}|} \right) = 0$$

$$\text{Solve } u^{n+1} : h^{n+1}(-x, -y) * (h^{n+1} * u_{i+1}^{n+1} - z) - \alpha_1 \nabla \cdot \left(\frac{\nabla u_{i+1}^{n+1}}{|\nabla u_i^{n+1}|} \right) = 0$$

Impose:

$$h^{n+1}(x, y) = \begin{cases} h^{n+1}(x, y), & h^{n+1}(x, y) > 0 \\ 0 & \textit{otherwise} \end{cases}$$

$$h(x, y) = [h(x, y) + h(-x, -y)] / 2$$

$$h^{n+1} = \frac{h^{n+1}}{\int_{\Omega} h^{n+1}(x, y) dx, dy}$$

Choice of Regularization Parameters

Consider the noise-constraint minimization problem:

$$\min_{u,h} \iint |\nabla u| + \alpha |\nabla h| dx dy \quad \text{subject to } \|h * u - z\|^2 = \sigma^2$$

Using Lagrange multiplier notation,

$$f(u, h) = \iint |\nabla u| + \alpha |\nabla h| dx dy + \frac{\lambda}{2} (\|h * u - z\|^2 - \sigma^2)$$

On the other hand

$$f(u, h) = \iint \alpha_1 |\nabla u| + \alpha_2 |\nabla h| dx dy + \frac{1}{2} \|h * u - z\|^2$$

$$\Rightarrow \alpha_1 = \frac{2}{\lambda} \quad \alpha_2 = \frac{2\alpha}{\lambda}$$

Choice of Parameters

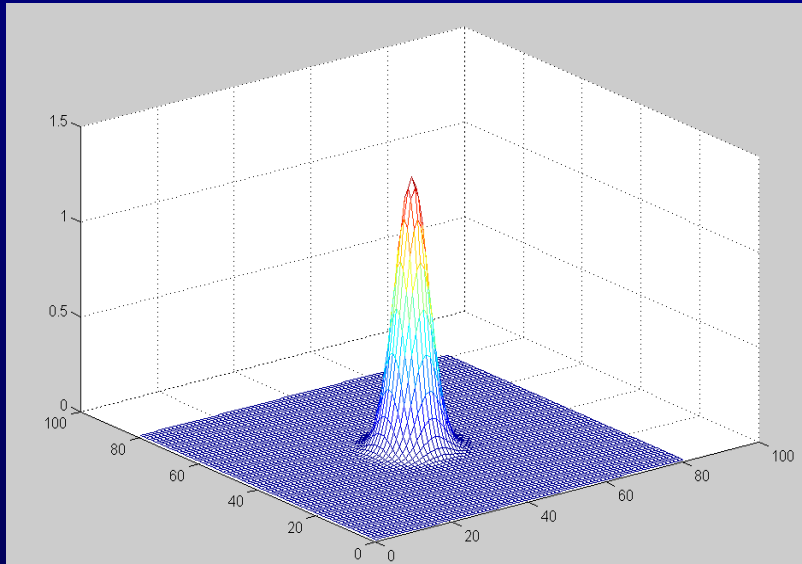
α_1 depends on the noise level

$$\begin{aligned} SNR \downarrow &\Rightarrow \sigma \uparrow \Rightarrow \lambda \downarrow \\ \alpha_1 &= \frac{2}{\lambda}, \quad = C \sigma \end{aligned}$$

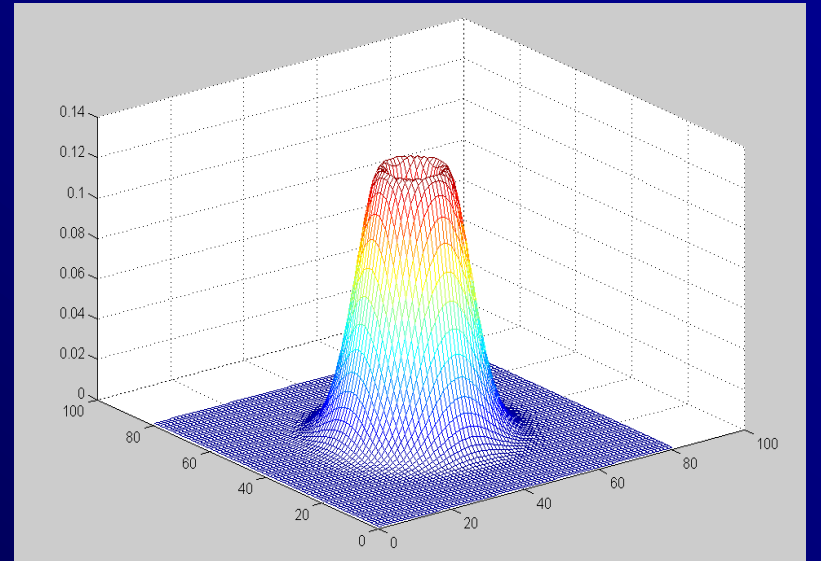
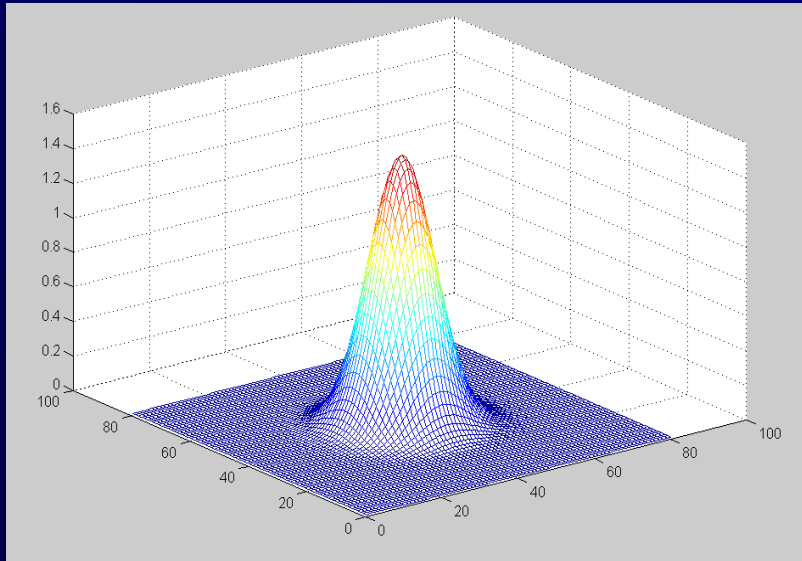
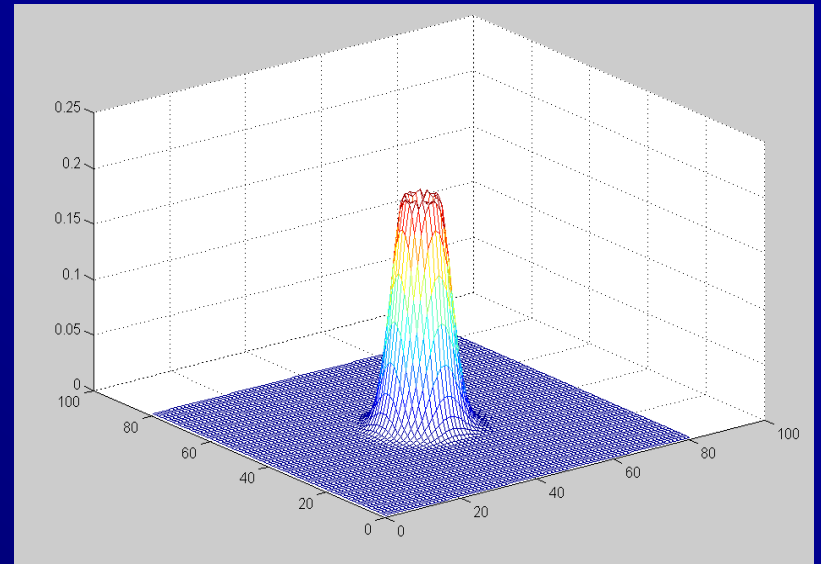
α_2 controls the support of the spread of PSF
(desired deblurring)

$$\alpha_2 \uparrow \Rightarrow \int |\nabla h| dx dy \downarrow, \int h dx dy = 1 \Rightarrow psf \text{ spread out}$$

Gaussian



Total Variation



Numerical Results

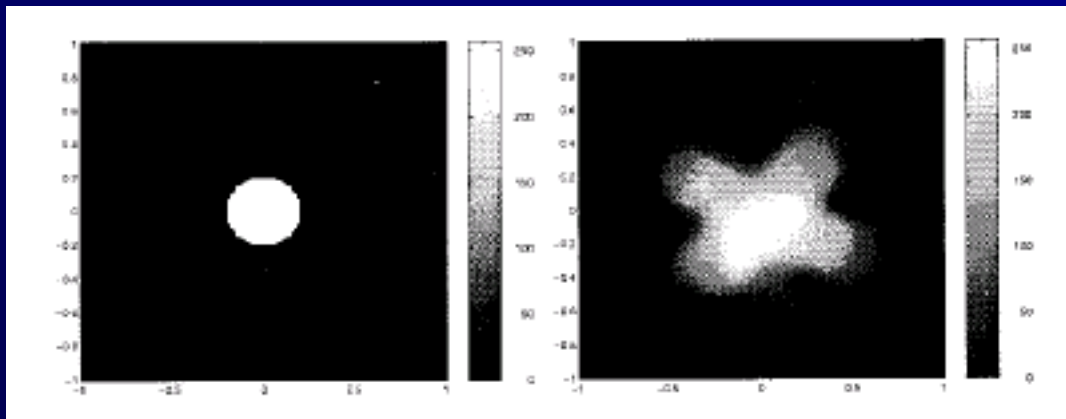
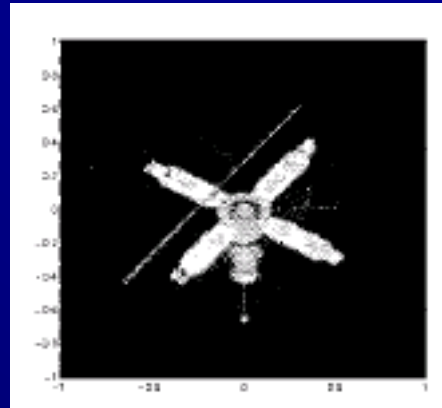
- There were 10 iterations in every FP stage.
- Within each FP iteration there is a linear system which was solved by **Conjugate Gradient** method (Hestenes and Stiefel 1952), which converges in a finite number of iterations.

$$f(\underline{x}) = \frac{1}{2} \underline{x}^T \underline{\underline{A}} \underline{x} - \underline{b}^T \underline{x} - c$$

$$\min f(\underline{x}) \Leftrightarrow \underline{\underline{A}} \underline{x} = \underline{b} \quad \forall A \text{ Symmetric Positive Definite}$$

Results

Test Image 127x127



Out of Focus blur

blurred image

Recovered
image

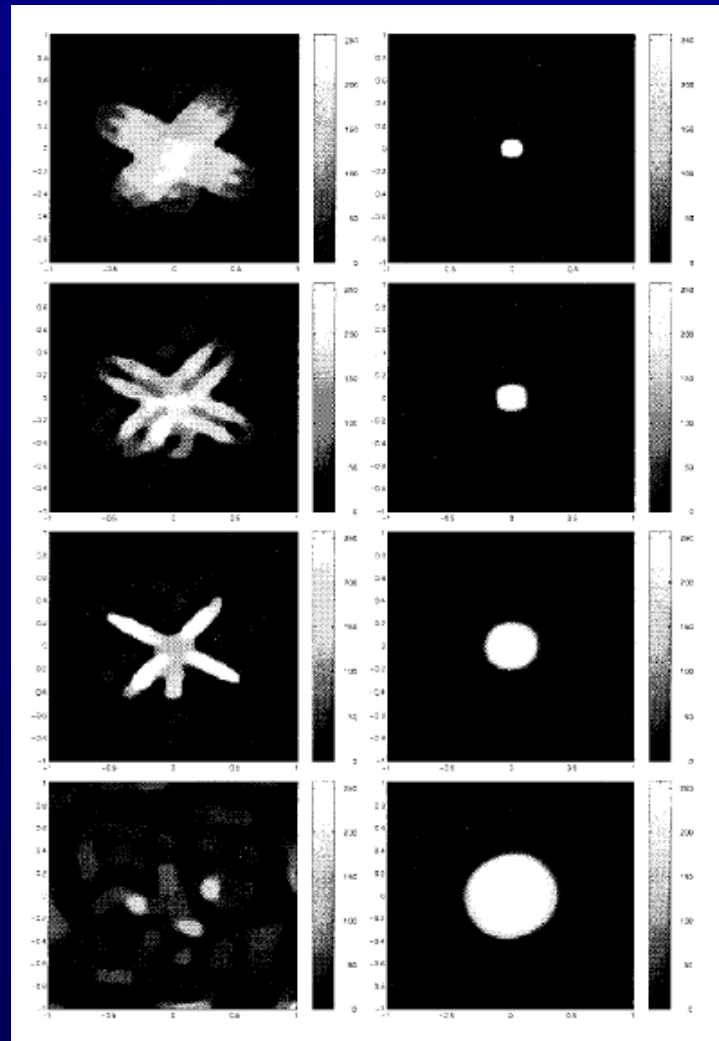
PSF

$$\alpha_2 = 10^{-7}$$

$$\alpha_2 = 10^{-6}$$

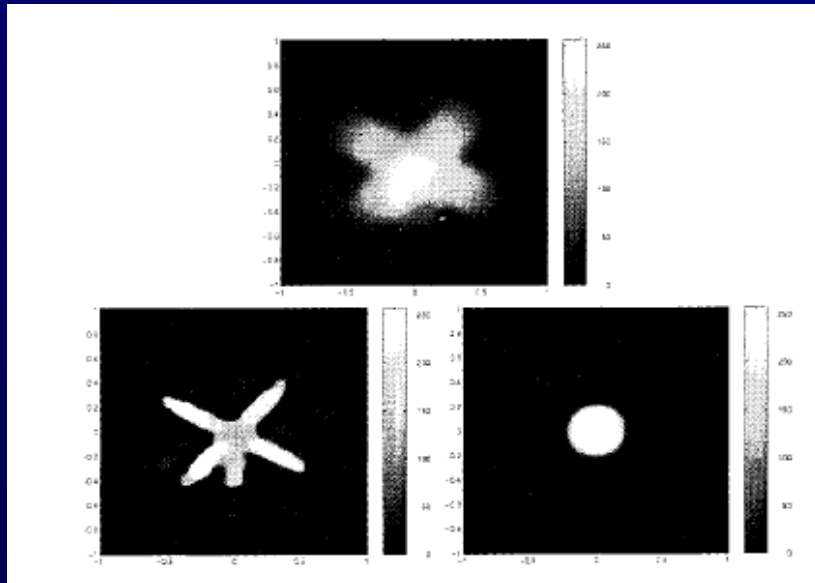
$$\alpha_2 = 10^{-5}$$

$$\alpha_2 = 10^{-4}$$

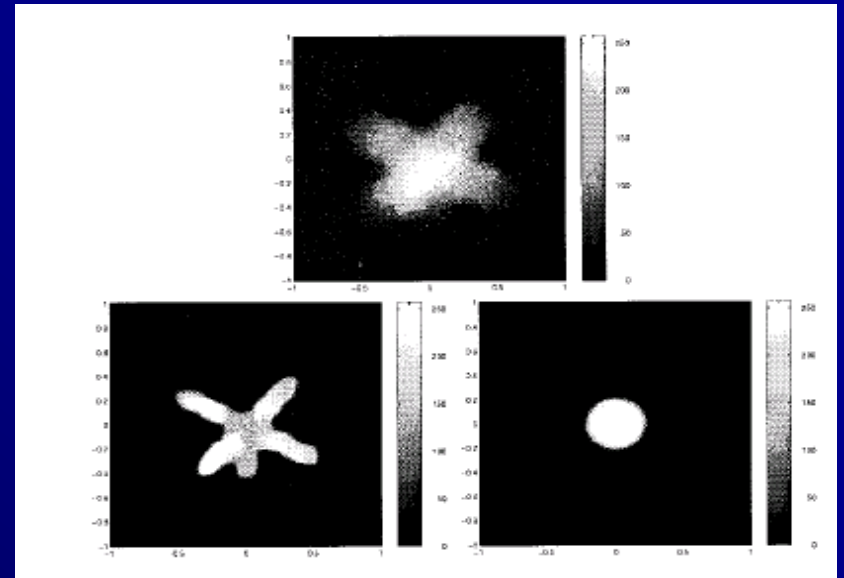


$$\alpha_1 = 2 \cdot 10^{-6}$$

Results After Three AM Iterations



$$\alpha_1 = 2 \cdot 10^{-6}, \alpha_2 = 1.5 \cdot 10^{-5}, SNR = \infty$$



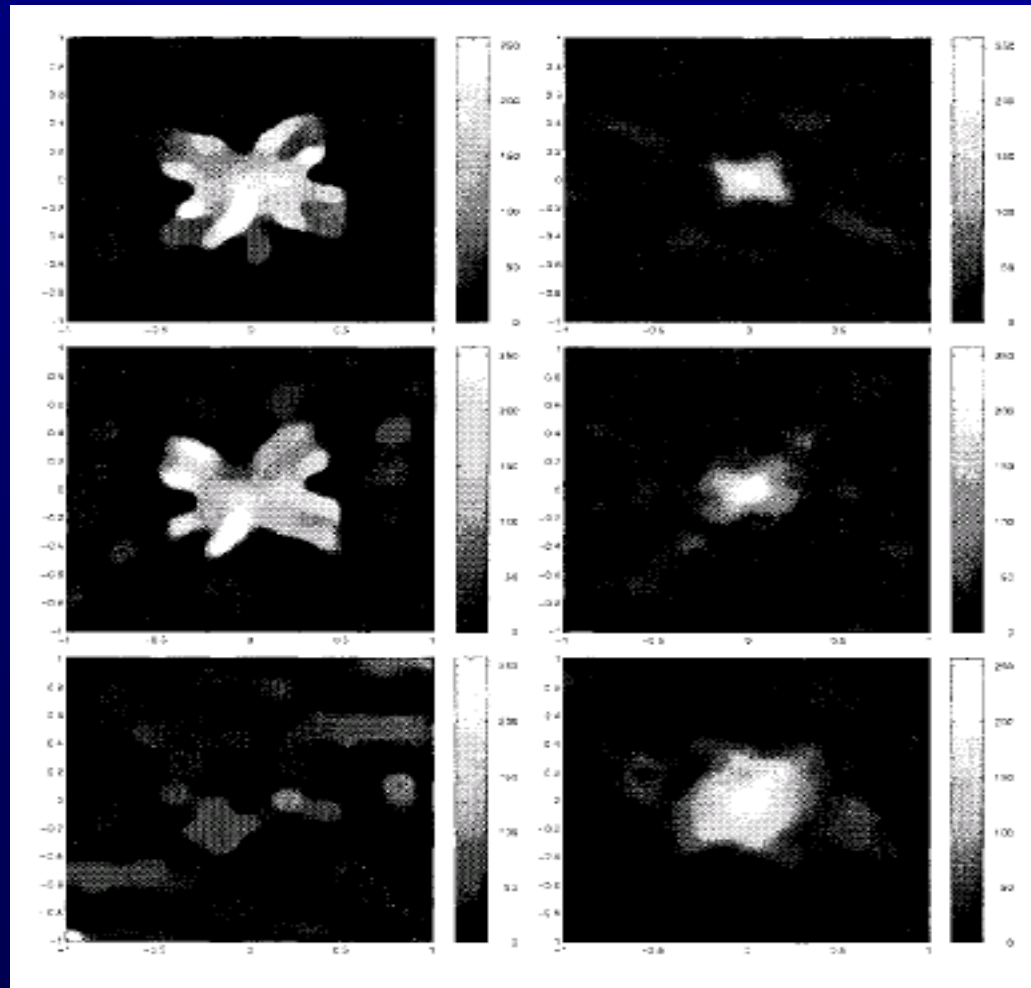
$$\alpha_1 = 2 \cdot 10^{-5}, \alpha_2 = 1.5 \cdot 10^{-5}, SNR = 5$$

TV Norm in u and L² Norm in h

$$\alpha_2 = 10^{-5}$$

$$\alpha_2 = 10^{-4}$$

$$\alpha_2 = 10^{-3}$$



Summary

- We used TV norm for regularizing u and h in the blind deconvolution problem
- AM/FP algorithm was proposed with choice of parameters heuristics
- Algorithm found to be robust and efficient, recovered images are as good as that recovered with the exact PSF