Plasma Temperature Profiles in the Magnetosphere of Neptune

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Abstract. It is generally agreed that plasma in the magnetosphere of Neptune is produced primarily in the vicinity of the orbit of Triton, near and beyond 14 R_N, and is then transported inwards toward Neptune. The observed temperature profiles of protons and nitrogen ions, however, show an initial decrease with decreasing distance from Neptune, the expected increase from adiabatic compression not setting in until distances below 8 R_N have been reached. We propose that cooling produced by collisions with atoms of a neutral hydrogen cloud that extends inward to 8 R_N may explain this behavior, at least for protons. Inclusion of inelastic scattering in addition to the more commonly invoked charge exchange makes cooling sufficiently effective so that the observed temperatures may be accounted for with plausible values of neutral hydrogen density.

Introduction

Since the encounter of the *Voyager* 2 spacecraft with Neptune in August, 1989, a coherent picture of the magnetosphere of Neptune and its plasma and particle populations has been developed, based upon the findings and interpretations of the various experimenters. A consistent model has been assembled for the thermal plasma [*Richardson et al.*, 1991; *Zhang et al.*, 1991; *Richardson*, 1993; *Richardson et al.*, 1994], for the intermediate energy particles and hot plasma [*Mauk et al.*, 1991; *Cheng et al.*, 1992; *Mauk et al.*, 1994] and for the cosmic ray energy particles [*Selesnick and Stone*, 1991a, 1991b, 1994]. This model constrains the relevant magnetosphere plasma parameters to the following set of values:

- A typical number density of thermal plasma of the order of $5 \times 10^{-2}\mathrm{cm}^{-3}$, rising to about twice or three times that value near the closest−approach distance of 1.2 R_N.
- A Triton hydrogen source strength, estimates of which range from $6.4 \times 10^{25}\mathrm{s}^{-1}$ [*Summers and Strobel*, 1991] to $2.3\times10^{26}\mathrm{s}^{-1}$[*Krasnopolsky et al.*, 1993].
- Plasma transport, assumed to be radial diffusion, with the diffusion coefficient taken to vary as a power law of the equatorial distance of field lines L,

$$D = k\left(\frac{L}{R_N}\right)^p \tag{1}$$

with parameter estimates ranging from $k = 8 \times 10^{-9}\ \mathrm{R_N^2\,s^{-1}}$ [*Selesnick and Stone*, 1994] to $k = 2.5 \times 10^{-8}\ \mathrm{R_N^2\,s^{-1}}$ [*Richardson et al.*, 1991] and $k = 6 \times 10^{-8}\ \mathrm{R_N^2\,s^{-1}}$ [*Cheng et al.*, 1992] with $p = 3$ for the low and intermediate energy particles and $k = 10^{-13}\ \mathrm{R_N^2\,s^{-1}}$ and $p = 7$ for the cosmic ray energy particles [*Selesnick and Stone*, 1991a, 1994](note that throughout this paper, we treat L as a quantity which has the dimensions of length). Alternative views of plasma transport in which azimuthal asymmetries play an essential role have been presented[*Hill and Dessler*, 1990; *Richardson et al.*, 1990; *Selesnick*, 1990], but a clear signature of such an asymmetry has not been detected in the data[*Richardson*, 1993].
- A neutral atomic hydrogen background density of about 100 cm^{-3}, based on the model calculations of *Smyth and Marconi* [1993] and *Cheng*

and Decker [1993], together with an as yet poorly determined density of neutral atomic nitrogen.

The above values are consistent with one another and provide a magnetosphere model in which a tenuous plasma diffuses inwards fairly rapidly from its source at or near Triton, as predicted by *Delitsky et al.* [1989] before the *Voyager* 2 encounter. Much of the plasma physics of the Triton torus in the magnetosphere of Neptune may, therefore, be regarded as understood. We point out in this paper, however, that significant discrepancies still remain between the expectations of the model and the observed dependence of plasma ion temperatures on distance from Neptune. We show that these discrepancies can be resolved if the neutral cloud is more extensive than hitherto assumed. Estimates of the required hydrogen density are consistent with other available evidence.

The Plasma Temperature Profiles

The definitive discussion to date of plasma ion temperature profiles has been given by *Richardson* [1993]. The proton and nitrogen ion temperatures generally rise steeply with decreasing L and then drop as L decreases below $3\mathrm{R_N}$. In the model, the rise is attributed to heating by adiabatic compression as plasma is transported inwards from the source region, and the drop to various loss processes near the planet. There are, however, two problems:

1. Adiabatic compression, starting from the expected corotation energy at the source near Triton, would lead at smaller distances to a much higher temperature than is observed; Richardson was forced to assume(and to seek special reasons for) a severe reduction in azimuthal plasma flow speed near Triton, by as much as a factor of three below corotation.

2. In the region from $L = 8\,\mathrm{R_N}$ out to the end of available data at $12\,\mathrm{R_N}$, the model accounts for the average value of the temperatures but not for their variation with distance; here the proton and nitrogen ion temperatures both decrease with decreasing L, contrary to what would be expected from adiabatic compression. To illustrate this, in figures, 1 and 2 we show the proton and nitrogen time−averaged temperature profiles as a function of L. Plotting 6 minute averages of the data reduces the scatter considerably; nonetheless, scatter by about a factor of two, which may reflect either inaccuracy of measurement or intrinsic short−term plasma variability or both, remains and must be allowed for in any comparisons. Curves of the temperature varying as corotation energy and as the $L^{-\frac{8}{3}}$ dependence of adiabatic compression have been superimposed on the data to emphasize the correspondences and discrepancies.

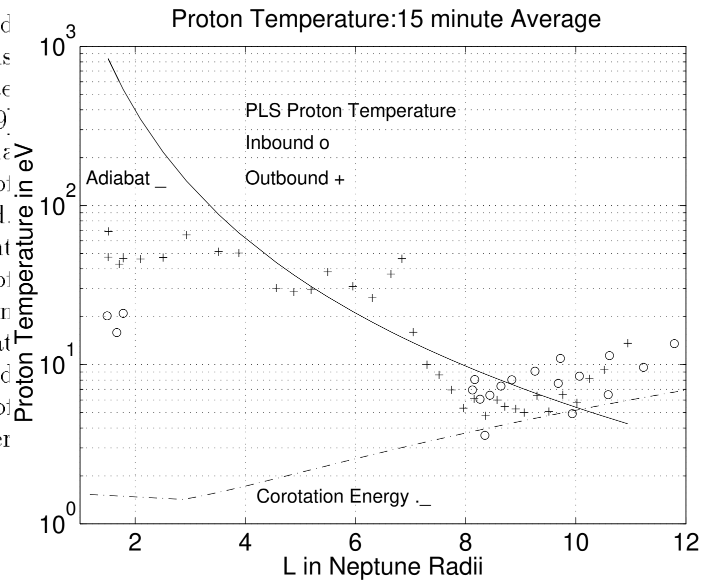

Figure 1. Proton Temperature Profiles

Figures 1 and 2 show that for $3\,\mathrm{R_N} < L < 8\,\mathrm{R_N}$ both proton and $\mathrm{N^+}$ temperatures lie close to adiabatic curves, which would extrapolate to well below corotation energy if extended to the assumed source region beyond $12\mathrm{R_N}$. As L increases beyond 8 $\mathrm{R_N}$, however, the temperatures no longer behave adiabatically but undergo instead a broad minimum and then increase with increasing L, the proton temperature staying slightly above, the nitrogen at or slightly below the corotation energy. The simplest way to explain these profiles is to assume that the ions are created near the Triton distance of 14 $\mathrm{R_N}$ and acquire the local corotational energy by pick-up, but as they are transported inward they are subject to a powerful cooling mechanism that more than counteracts adiabatic heating and leads to a net reduction of temperature. Inward of 8 $\mathrm{R_N}$ the cooling mechanism ceases, and the temperature increases adiabatically, starting from the reduced values at 8 $\mathrm{R_N}$. The possibility that the ions might be injected at full corotational energy and subsequently cooled was discussed by *Richardson* [1993], who considered cooling by charge exchange with neutral atoms and concluded that the process was ineffective; therefore he assumed, instead, severe subcorotation in the source region and left the outward temperature increase unexplained. We reexamine here the process of cooling by interaction with a neutral cloud and show that including the pre-

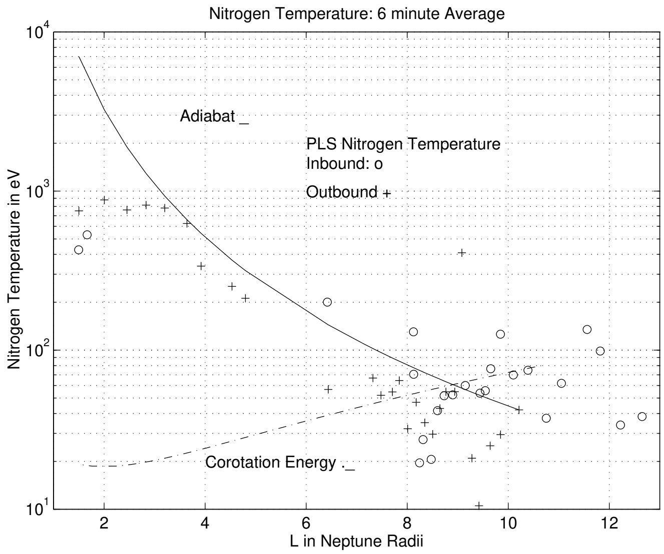

Figure 2. N^+ Temperature Profiles

viously neglected effects of inelastic scattering makes the cooling sufficiently strong to explain the observed temperature profiles.

The situation at Neptune cannot be compared with that in the Io plasma torus. There are several significant differences between the two plasma bodies:

1. In the Io torus, the plasma is fully ionized, *i.e.* neutral densities are much smaller than the plasma density. The composition of heavy ions with strong EUV features provides a strong radiation sink which is not available to the mainly proton plasma at Neptune. The nitrogen ion is a weaker radiator than the sulfur and oxygen ions at Io.

2. The density at Io is about five orders of magnitude greater and exciting collisions are much more effective there

Physical Mechanisms

The classic mechanisms by means of which the proton and heavy ion plasma can lose energy and be cooled nonadiabatically involve collisions with neutral atoms or charged particles. In a magnetosphere in which plasma is transported by radial diffusion, the process may be described by the two equations for the flux tube content of particles and of energy:

$$L^2 \frac{d}{dL}\left\{\frac{D}{L^2}\frac{d}{dL}(Y)\right\} + S = 0 \qquad (2)$$

and

$$L^2 \frac{d}{dL}\left\{\frac{D}{L^2}\frac{d}{dL}\left(YTL^{\frac{8}{3}}\right)\right\} + ST_{pk}L^{\frac{8}{3}} + \dot{Q}L^{\frac{8}{3}} = 0. \qquad (3)$$

Here Y is the flux tube particle content (written often as NL^2), D is the radial diffusion coefficient, S is the ionization rate:

$$S = n_a(\phi + \alpha_e n_e)V \qquad (4)$$

in which n_a is the number density of the neutral atoms (hydrogen or nitrogen), suitably averaged over the volume V of the flux tube, $\phi = 9.8 \times 10^{-11}\mathrm{s}^{-1}$ is the solar photoionization rate [*Giguere and Huebner*, 1978] and $\alpha_e n_e$ is the electron impact ionization rate integrated over the electron energy distribution, here approximated as a sum over the cold and hot electrons with temperatures of 5 and 50 eV and mean densities of .06 and .003 cm^{-3}, respectively, [*Zhang et al.*, 1991]; the corresponding impact ionization rate coefficients are 9.4×10^{-10} and $2.5 \times 10^{-8}\,\mathrm{cm}^3\mathrm{s}^{-1}$ [*Lotz*, 1967]. T is the ion temperature, T_{pk} is the pickup energy and $\dot{Q}$ represents various forms of energy exchange as follows:

1. Collisions with neutral hydrogen atoms:

$$\dot{Q} = v_i n_a(\sigma_{ce}(T - T_{pk}) + \sigma_c T)Y \qquad (5)$$

where v_i is the mean relative speed between ions and neutral atoms, given for a corotating Maxwellian ion distribution by

$$v_i = \left(\Omega L + \left(\frac{GM}{L}\right)^{\frac{1}{2}}\right)\left(1 + \frac{T}{T_{pk}}\right)\mathrm{erf}\left(\left(\frac{T_{pk}}{T}\right)^{\frac{1}{2}}\right) \qquad (6)$$

with $\Omega = 1.07 \times 10^{-4}\,\mathrm{s}^{-1}$ the planet's rate of rotation, G the gravitational constant, M the mass of the planet; σ_{ce} is the symmetric charge exchange cross section and σ_c is the ion$-$neutral inelastic collision cross section. For proton$-$hydrogen atom collisions we take σ_{ce} from *Newman et al.* [1982] with the energy dependence given by

$$\sigma_{ce} = 5.8 \times 10^{-15}\left(1 - 0.05\log_{10}\frac{E}{1\,\mathrm{eV}}\right)^2\ \mathrm{cm}^2$$

and $\sigma_c = 5 \times 10^{-15}\mathrm{cm}^2$ [*Book*, 1977]. The temperature of the neutral atoms has been assumed to be negligible compared to both T and T_{pk}.

2. Collisions between charged particles:

$$\dot{Q} = \nu^{\alpha/\beta}(T_\beta - T_\alpha)Y_\beta \qquad (7)$$

with a similar expression for energy exchange between N^+ and neutral atomic nitrogen. The rate coefficient is given by *Book* [1977]

$$\nu^{\alpha/\beta} = \frac{1.8 \times 10^{-19}(A_\alpha A_\beta)^{\frac{1}{2}} n_\beta \log\Lambda}{m_p^{\frac{1}{2}}(A_\alpha T_\beta + A_\beta T_\alpha)^{\frac{3}{2}}} \qquad (8)$$

(A is the atomic mass number).

Because of the very low density of charged particles in the Triton torus region, the process described by equation 7 is most unlikely there, and we shall not consider it further.

Multiplying the particle content equation 2 by $TL^{\frac{8}{3}}$ and subtracting the result from the energy equation 3, we obtain, after some manipulation, an equation with the diffusion operator acting on temperature alone;

$$YL^2\frac{d}{dL}\left\{\frac{D}{L^2}\frac{d}{dL}\left(TL^{\frac{8}{3}}\right)\right\}+2D\frac{dY}{dL}\frac{d}{dL}(TL^{\frac{8}{3}}) = YL^{\frac{8}{3}}n_a\left\{\left(v_i\sigma_{ce}+\frac{\phi+\alpha_e n_e}{n_i}\right)(T-T_{pk})+v_i\sigma_c T\right\} \quad (9)$$

where n_i is the ion number density and the contribution of 7 has been ignored, as stated above. Equation 9 clearly displays the two distinct effects that collisions with a neutral cloud have on the temperature of plasma ions:

1. Ionization processes and charge exchange both tend to bring the temperature towards the local pickup *i.e.* the corotation energy. They have, therefore, little effect if the temperature is already near that value, as is the case beyond 8 R_N.
2. Inelastic ion–neutral collisions, on the other hand, always act to reduce the plasma temperature towards the extremely low temperature of the neutral cloud, regardless of the value of T/T_{pk}. They have not been included in previous models of the plasma temperature at Neptune[*Richardson et al.*, 1991; *Richardson*, 1993]. Neither has the fact that they provide a sink for neutral atoms, since they produce them with velocities greater than the local escape velocity from Neptune, been included in previous models of the neutral cloud[*Cheng and Decker*, 1993; *Smyth and Marconi*, 1993].

The Conjectured Hydrogen Cloud

At this point, we shall narrow our discussion to the proton temperature profile and the hydrogen cloud which we take to be its heat sink. We do this for several reasons:

- The escape rate of an atom as heavy as nitrogen from Triton will be smaller than that of hydrogen by more than an order of magnitude[*Krasnopolsky et al.*, 1993] and thus a major nitrogen neutral cloud should not be expected to exist in the magnetosphere.
- The observed N^+ may be explained as the result of the direct outflow of ionosphere nitrogen plasma[*Strobel et al.*, 1990; *Ip*, 1990; *Yung and Lyons*, 1990; *Majeed et al.*, 1990]. Such plumes were observed during the *Voyager* 1 flyby of Titan[*Hartle et al.*, 1982; *Eviatar et al.*, 1982] and it is reasonable to expect them at Triton.
- The data are considerably poorer than those of protons. The higher energy channels of the PLS instrument have a somewhat higher noise level. As a result, the temperature minimum seen clearly in the proton data and to some extent in the nitrogen outbound measurements is less clearly defined.

The discrepancies between the magnetosphere data and the aeronomic models of Triton in the literature as well as the possibility of such plumes is a topic worthy of a separate study and will not be considered in this paper.

Near Triton, we assume a dense cloud of neutral hydrogen, such as was observed by Voyager 1 near Titan. We postulate that the magnetosphere of Neptune is dominated by a cloud of atomic hydrogen of Triton origin from 14 R_N in as far as 8 R_N, and we attribute the nonadiabatic behavior of plasma temperature to cooling by collisions with this cloud.

To test this hypothesis quantitatively, we first estimate the density of neutral hydrogen that is required to produce, by means of the cooling described by equation 9, a broad minimum of temperature near $L = 10 R_N$ apparent in figures 1 and 2. To satisfy the conditions for a minimum,

$$\frac{dT}{dL} = 0 \quad \text{and} \quad \frac{dT}{dL^2} > 0,$$

equation 9 requires

$$\frac{n_a R_N^p}{kL^{p-2}}\left\{\left(v_i\sigma_{ce}+\frac{\phi+\alpha_e n_e}{n_i}\right)\left(1-\frac{T_{pk}}{T}\right)+v_i\sigma_c\right\} > \frac{8}{3}\left(p-\frac{1}{3}\right)+\frac{16}{3}\frac{d\log Y}{d\log L} \quad (10)$$

where a diffusion coefficient given by equation 1 has been assumed. With

$$v_i \;=\; \Omega L + \sqrt{GM/L} = 31\,\mathrm{km\,s^{-1}}$$

at 10 R_N, *i.e.* without the thermal–speed enhancement of equation 6 and with the typical value $n_p \approx 3\times10^{-2}\mathrm{cm^{-3}}$ [*Richardson et al.*, 1991], the process rates within the left hand side of inequality 10 are as given in Table 1. We neglect the second term on the right hand side because of the slow variation of Y with L (see figure 1 of *Richardson* [1993]), and we take $p = 3$.

We assume first that $T = T_{pk}$ so that only inelastic scattering contributes to the cooling. With the enhancement of v_i by T/T_{pk} as given in equation 6 taken into account, we obtain at $L = 10 R_N$

$$\frac{n_a}{k} > 2.71\times10^9\,\mathrm{cm^{-3}\,s\,R_N^{-2}}$$

Table 1. Process Rates

Process	Rate	Source
charge exchange	$1.8 \times 10^{-8}\mathrm{cm}^3\mathrm{s}^{-1}$	*Newman et al.* [1982]
photoionization	$9.8 \times 10^{-11}\, s^{-1}$	*Giguere and Huebner* [1978]
total electron impact ionization	$0.8 \times 10^{-8}\mathrm{cm}^3\mathrm{s}^{-1}$	*Lotz* [1967]; *Zhang et al.* [1991]
inelastic scattering	$1.6 \times 10^{-8}\mathrm{cm}^3\mathrm{s}^{-1}$.	*Book* [1977]

or, with $k = 2.5 \times 10^{-8}\mathrm{R}_\mathrm{N}^2\,\mathrm{s}^{-1}$[*Richardson et al.*, 1991],

$$n_a > 68\, cm^{-3}$$

as the lowest hydrogen density needed to explain the temperature minimum. If $T > T_{pk}$, the required density is reduced both because ionization and charge exchange contribute to cooling and because v_i is further increased; with $T = 2T_{pk}$,

$$n_a > 46\, cm^{-3}.$$

While no Lyman-α emissions from the magnetosphere of Neptune and no enhancements near the orbit of Triton were detected by the *Voyager* UVS experiment against the background of the interstellar medium [*Broadfoot et al.*, 1989], the general weakness of the solar source at this distance coupled with the short extent of the available line of sight may have made the concealment of a fairly large hydrogen density possible. A 2σ upper limit for the Neptune system emission between 10 and 15$\mathrm{R_N}$ is 35 Rayleighs, and a 3σ upper limit is about 50 Rayleighs [*D.T. Hall, private communication*]. For a resonant scattering rate at Neptune of $g = 1.68 \times 10^{-6}\,\mathrm{s}^{-1}$, a viewing angle of $\theta \approx 45°$ [*B.R. Sandel, private communication*] and an equatorial line of sight of length $l = 20\,\mathrm{R_N}$, the intensity will be given by

$$I = g n_H l \cos\theta \tag{11}$$

and $n_a = 68\,\mathrm{cm}^{-3}$ yields a predicted Lyman $-\,\alpha$ intensity of about 6 Rayleighs, well below the upper limit implied by *Voyager* observations.

Constraints from Plasma Transport

In the previous section, we found that the existence of a temperature minimum requires the ratio n_a/k to be larger than a lower limit defined by equation 10. On the other hand, a more general upper limit on n_a/k is imposed by plasma transport, which must be fast enough to remove ions at the rate at which they are produced. Obviously, our proposed mechanism of cooling by a neutral cloud can work only if the lower limit implied by cooling is smaller than the upper limit imposed by transport.

We consider a simple model with a constant ionization source from a distance L_c outwards to at least the distance L_s at which Y reaches a maximum (about at 10$\mathrm{R_N}$, see figure 1 of *Richardson* [1993]). We solve the flux−tube−content equation 2 with the usual boundary condition

$$Y = 0 \quad \text{at} \quad L = L_1 = 1\mathrm{R_N}$$

(see the Appendix for details) and obtain a constraint on n_a/k which, put into a form analogous to equation 10, reads

$$\frac{n_a R_N}{kL}\left(\frac{\phi + \alpha_e n_e}{n_i}\right) \le \frac{1}{\zeta} \tag{12}$$

where

$$\zeta = \left(1 - \frac{L_c}{L_s}\right)\left\{\log\frac{L_c}{L_1} - \frac{1}{3}\left(1 - \frac{L_c}{L_s}\right) + \cdots\right\} \tag{13}$$

and

$$\zeta = 0.4 \quad \text{for} \quad L_c = 8\,\mathrm{R_N} \quad L_s = 10\,\mathrm{R_N} \quad \text{and} \quad L = L_1 = 1\mathrm{R_N}.$$

A necessary condition that both inequalities 10 and 12 hold is obtained by eliminating n_a/k between them:

$$\left(\frac{\chi_{ce}}{\chi_i} + 1\right)\left(1 - \frac{T_{pk}}{T}\right) + \frac{\chi_c}{\chi_i} > \left(\frac{8}{3}\right)^2 \zeta \tag{14}$$

where $\chi_{ce} \equiv v_i\sigma_{ce}, \chi_i \equiv (\phi + \alpha_e n_e)/n_i$ and $\chi_c \equiv v_i\sigma_c$. This is independent of both n_a and k and depends only on charge exchange and inelastic collision rates divided by the ionization rate and on T_{pk}/T. With the rate coefficients listed above, v_i given by equation 6 and $\zeta = 0.4$, 14 becomes

$$F\left(\frac{T_{pk}}{T}\right) \equiv \frac{\mathrm{erf}\left(\left(\frac{T_{pk}}{T}\right)^{\frac{1}{2}}\right)\left(4.25\frac{T}{T_{pk}} + 2 - 2.25\frac{T_{pk}}{T}\right)}{1.8 + \frac{T_{pk}}{T}} - 1 > 0 \tag{15}$$

In the upper panel of figure 3 we plot $F\left(\frac{T_{pk}}{T}\right)$ and note that the condition 15 is satisfied if $T > 0.906T_{pk}$; in the lower panel, we plot it as a function of L with our outbound proton temperature data and note that $F > 0$ throughout the region in which we have assumed the neutral cloud to exist and to be cooling the plasma.

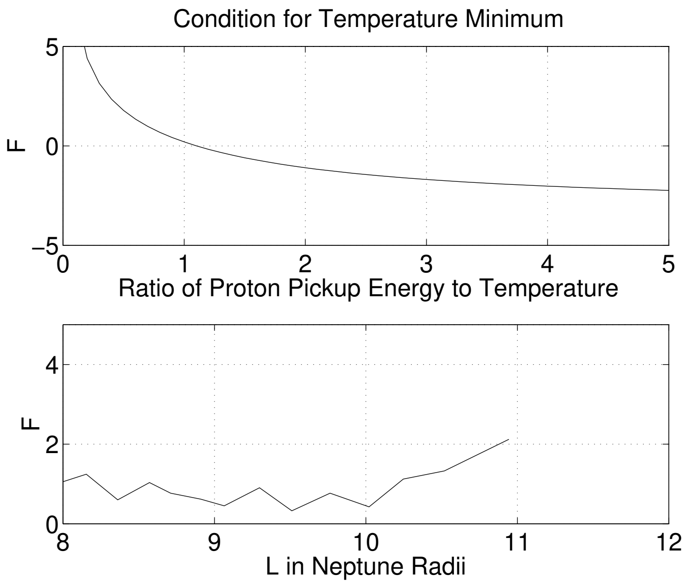

Figure 3. Upper panel: The function $F\left(\frac{T_{pk}}{T}\right)$ given in equation 15; lower panel: the same function for the outbound proton temperature data.

Discussion

We have found that the apparently anomalous temperature profiles in the Triton torus region can be understood to be the result of interaction of the plasma with a cloud of neutral hydrogen and nitrogen derived from the atmosphere of Triton. The densities required to nullify the adiabatic heating associated with conservation of the first magnetic moment are well below the Lyman $-\ \alpha$ detection level and are not in contradiction to the torus density predictions of *Cheng and Decker* [1993] and *Smyth and Marconi* [1993]. It is, therefore, unnecessary to invoke gross violation of corotation near Triton as called for by *Richardson* [1993]. Our model differs from theirs and other previous models in that it includes inelastic ion$-$neutral collisions.

Appendix: Flux tube content with ionization source

Consider the diffusion equation 2 with an ionization source:

$$L^2 \frac{d}{dL}\left\{\frac{kL^{p-2}}{R_N^p}\frac{dY}{dL}\right\} + S = 0 \qquad \text{(A1)}$$

in which we take the source term S, defined by equation 4 to be constant, $S = S_0$ for

$$L_c \quad < \quad L \quad < \quad L_s$$

and $S = 0$ for

$$L_1 \quad < \quad L \quad < \quad L_c.$$

We assume the boundary conditions:

$$\begin{aligned} Y &= 0 \quad \text{at} \quad L = L_1 \\ \frac{dY}{dL} &= 0 \quad \text{at} \quad L = L_s \end{aligned}$$

and take $p = 3$. The solution of equation A1 is then easily obtained:

$$\frac{kY}{R_N^3 S_0} = \begin{cases} \frac{1}{L_c} - \frac{1}{L} + \frac{\log\left(\frac{L_c}{L_1}\right)}{L_c} - \frac{\log\left(\frac{L}{L_1}\right)}{L_s} & \text{for} \quad L_c < L < L_s \\ \left(\frac{1}{L_c} - \frac{1}{L_s}\right)\log\left(\frac{L}{L_1}\right) & \text{for} \quad L_1 < L < L_c \end{cases} \qquad \text{(A2)}$$

With the relation $Y = n_i V$ where n_i is the ion density and V is the flux tube volume, equation 4 can be written

$$S = n_a\left(\frac{\phi + \alpha_e n_e}{n_i}\right)Y. \qquad \text{(A3)}$$

We have assumed S rather than n_a to be constant beyond L_c and zero within L_c, in order to gain a considerable simplification of the mathematics. The variation of Y and hence of n_a in the range $L_c < L < L_s$ is not, however, large. We may then approximate n_a by its mean value, which we obtain by integration of A3:

$$\begin{aligned} \int_{L_c}^{L_s}\frac{dL}{L^2}S &= \int_{L_c}^{L_s}\frac{dL}{L^2}n_a\left(\frac{\phi+\alpha_e n_e}{n_i}\right)Y \\ &\approx n_a\left(\frac{\phi+\alpha_e n_e}{n_i}\right)\int_{L_c}^{L_s}\frac{Y\,dL}{L^2} \end{aligned} \qquad \text{(A4)}$$

(the factor $1/L^2$) is included in order to have the integral represent the ion flux, *cf.* the first integral of A1. With $S = S_0$ and Y given by A2, the result is, after division by $S_0(1/L_c - 1/L_s)$ on both sides,

$$1 = \frac{n_a R_N^3}{kL_c}\left(\frac{\phi+\alpha_e n_e}{n_i}\right)\zeta \qquad \text{(A5)}$$

where

$$\zeta = \left(1 - \frac{L_c}{L_s}\right)\log\frac{L_c}{L_1} + \frac{1}{2}\left(1 - \frac{3L_c}{L_s}\right) + \left(\frac{L_c}{L_s}\right)^2 \frac{\log\frac{L_s}{L_c}}{1 - \frac{L_c}{L_s}}.$$

With $L_c = 8\,\mathrm{R_N}$ and $L_s = 10\,\mathrm{R_N}$, expansion of ζ in powers of $(1 - \frac{L_c}{L_s})$ is adequate and gives equation 13 in the text. We are interested in n_a/kL for $L > L_c$, which turns equation A5 into the inequality 12 in the text.

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