Resonance magnetoresistance of coupled quantum wells

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An in-plane magnetic field suppresses the quantum coupling between electrons in a double-quantum-well structure. The microscopic theory of this effect is developed and confirmed experimentally. We have shown that the decrease of the resistance resonance peak is sensitive to the mutual orientation of the current and the in-plane magnetic field. The characteristic field required for the suppression of the resonance depends on the elastic-small-angle and electron-electron scattering rates. The study of the characteristic field allows us to verify the temperature and Fermi-energy dependence of the electron-electron scattering rate, providing another experimental tool for its determination.

A physical phenomenon, called resistance resonance (RR), in a double-quantum-well (QW) structure was recently predicted and observed experimentally.[1] The key point of this effect is the following. Let us consider two *tunneling coupled* QW's. The quantity of interest is the *lateral* resistance of the structure (all the electrodes are attached to both QW's). If the tunneling between the QW's is for some reason suppressed, each electron is localized in one of the wells. The resulting lateral resistance is that of two conductors connected in parallel, $R_{\rm off} \sim (\tau_1^{\rm tr}+\tau_2^{\rm tr})^{-1}$, where $\tau_i^{\rm tr}$ is the transport mean-free time in the ith well. In the presence of tunneling, the eigenfunctions form symmetric and antisymmetric subbands, leading to delocalization of electrons between the two wells. The corresponding scattering rate in each of these subbands is $(\tau^{\rm tr})^{-1} = (2\tau_1^{\rm tr})^{-1} + (2\tau_2^{\rm tr})^{-1}$ and the resistance is given by $R_{\rm res} \sim (2\tau^{\rm tr})^{-1}$. One notices that if the mobilities of the two QW's are different ($\tau_1^{\rm tr} \neq \tau_2^{\rm tr}$), then $R_{\rm res} > R_{\rm off}$. The reason is very simple: in the first case of no coupling, the clean well shunts the dirty one, making the resistance small. No such shunting occurs for coupled wells.

The experimental realization of this idea[1–3] was based on the displacement of the energy levels of the QW by the gate voltage. The typical graph of the lateral resistance versus gate voltage is plotted on the inset of Fig. 1. The resonance occurs in the point where the energy levels of the two wells coincide, facilitating the tunneling. In the present letter we propose a different realization of the RR. Namely, the RR (with maximum value at zero magnetic field) is observed as a function of the *in-plane* magnetic field (instead of the gate voltage). The suppression of tunneling between two QW's in the parallel magnetic field was already demonstrated in a different context in Ref. 4 for perpendicular transport and in Ref. 5 for Shubnikov–de-Haas oscillations. The theoretical calculations of magnetization in the coupled QW's in a tilted magnetic field relevant for the case studied in Ref. 5 were published elsewhere.[6] We employ the nice intuitive picture, developed in Refs. 4 and 5, to illustrate

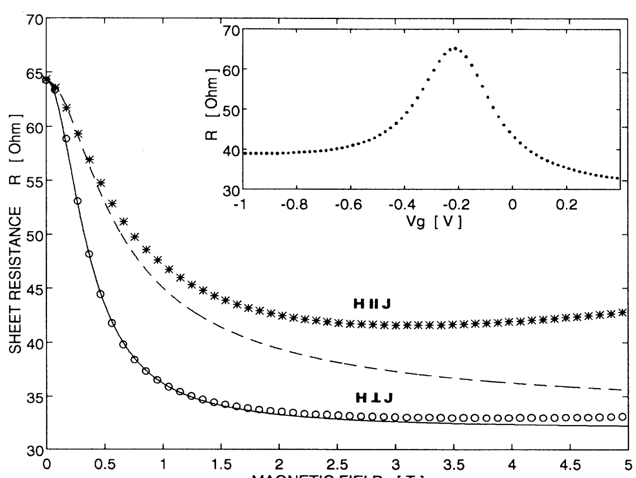

FIG. 1. The resonance resistance (RR) vs magnetic field at 4.2 K: circles and asterisks, experimental data for the perpendicular and parallel orientations; solid and dashed lines, theoretical curves. The inset, line shape of the RR vs top gate voltage, V_g ($V_{bg} = 1.5$ V).

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the results of our calculations. Below we present a microscopic description of the lateral magnetoresistance of the coupled QW's, which is verified by the experimental data. The main points, following from our studies, are (i) the in-plane magnetic field destroys the coupling between QW's, leading to the RR; the lateral resistance is essentially *anisotropic*, implying that the shape of the RR depends on the angle between the current and magnetic field; (ii) the *width* (i.e., the characteristic magnetic field, H_c) of the RR is sensitive to the single electron scattering time, providing a new method of measuring the small-angle scattering time on the remote impurities; (iii) the dependence of H_c on temperature and on the Fermi energy suggests that the electron-electron scattering rate (intralayer and interlayer) may be tested as well.

To develop a microscopic model of transport in two QW's, we employ the basis of eigenstates of uncoupled wells. In this basis the Hamiltonian of the system is a 2×2 matrix, the off-diagonal elements of which represent the tunneling coupling between QW's,

$$\hat{H}_{\mathbf{k},\mathbf{p}} = \delta_{\mathbf{kp}} \begin{pmatrix} \frac{(\mathbf{p}-e/c\mathbf{A}_1)^2}{2m^*} & \frac{\Delta}{2} \\ \frac{\Delta}{2} & \frac{(\mathbf{p}-e/c\mathbf{A}_2)^2}{2m^*} \end{pmatrix} + \begin{pmatrix} U_1(\mathbf{p}-\mathbf{k}) & 0 \\ 0 & U_2(\mathbf{p}-\mathbf{k}) \end{pmatrix} . \quad (1)$$

We treat here only the case of coinciding quantized (in the z direction) energy levels of the two wells. In Eq. (1), $-\mathbf{k},\mathbf{p}$ are two-dimensional (2D) momenta of the electrons, Δ is the tunneling gap (we assume tunneling to be momentum conserved), and $\mathbf{A}_i$ is a vector potential of an external field in the ith QW. The second matrix on the right-hand side of Eq. (1) represents an elastic impurity scattering inside each QW. We assume that (1) random potentials $U_i(\mathbf{p}-\mathbf{k})$ have a finite correlation length in a plane of 2D gas, and that (2) there are no correlations between scatterers in different wells. In this case the disorder potential in each well may be described[7] by the single-particle (small-angle) mean-free time τ_i and the two-particle (transport) mean-free time, $\tau_i^{\rm tr} \geq \tau_i$.

In a uniform magnetic field H parallel to the plane of the QW's (say directed along the y direction) the corresponding vector potentials are $\mathbf{A}_i = (Hz_i, 0)$, where z_i are the z coordinates of the effective centers of the QW's. The Fermi surfaces of the two QW's have a form of two circles displaced along the x direction (see Fig. 2) on the relative distance eHb/c, where $b = z_1 - z_2$ is the distance between the centers of the wave functions in the two wells.[4,5] Only the electrons, which occupy the states in the vicinity of the (quasi)crossing points A and B (see Fig. 2), have the same energy (ϵ_F) and momentum in both wells and hence participate in the tunneling. Several important conclusions follow immediately. (i) The resistance approaches its off-resonance value (decreases) as the magnetic field is increased. (ii) The characteristic scale of the magnetic field may be estimated as $v_F k_H \approx \max\{\Delta, \hbar/\tau\}$ (see below). (iii) The lateral resistance of the system in the in-plane magnetic field is *anisotropic*.

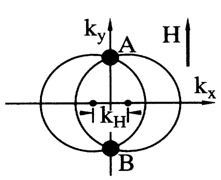

FIG. 2. Fermi surfaces of two QW's in the in-plane magnetic field.

Indeed, the transport in the x direction is dominated by states with large k_x, which are practically decoupled (cf. Fig. 2). As a result, the perpendicular (to the direction of the field) resistance is close to the off-resonance value. On the other hand, the transport in the direction of the field is mostly determined by the states situated near the points A and B of Fig. 2. These states are delocalized, making the parallel resistance closer to the resonance value. In other words, the suppression of the RR occurs in a different way depending on the angle between the magnetic field $\mathbf{H}$ and the current $\mathbf{j}$ used to probe the resistance.

The detailed diagrammatic calculation based on the Kubo formula[8] leads to the following dependence of the resonance resistance on the in-plane magnetic field:

$$R^{-1}(H) - R^{-1}_{\rm off} = [R^{-1}(0) - R^{-1}_{\rm off}] f(H/H_c), \quad (2)$$

where

$$f(x) = \frac{2(\sqrt{1+x^2}-1)}{x^2} \times \begin{cases} 1, & \mathbf{H} \,||\, \mathbf{j}, \\ (1+x^2)^{-1/2}, & \mathbf{H} \perp \mathbf{j}, \end{cases} \quad (3)$$

and the characteristic field is given by

$$H_c = \frac{\hbar c}{e} \frac{1}{v_F \tau b} \sqrt{1 + \left(\frac{\Delta}{\hbar}\right)^2 \frac{\tau_1^{\rm tr} + \tau_2^{\rm tr}}{2} \tau}, \quad (4)$$

with $2\tau^{-1} \equiv \tau_1^{-1} + \tau_2^{-1}$. The above relations are valid until $H \approx H_F$, where $H_F \equiv 2\pi\hbar c/(e\lambda_F b)$; λ_F is a Fermi wavelength. Note also that $H_c \ll H_F$, when $\epsilon_F\tau/\hbar \gg 1$. In agreement with our expectation, the RR is suppressed faster in the perpendicular configuration, $R^{-1}(H) - R^{-1}_{\rm off} \propto H^{-2}$, whereas in the parallel configuration $R^{-1}(H) - R^{-1}_{\rm off} \propto H^{-1}$, for $H_c \ll H < H_F$.

The double QW structure was grown on a N^+ GaAs substrate by molecular-beam epitaxy and consists of two GaAs wells of 139 Å width separated by a 40 Å $Al_{0.3}Ga_{0.7}As$ barrier. The tunneling gap for this structure is estimated as $\Delta = 0.55$ meV. The electrons were provided by remote δ-doped donor layers set back by 250-Å and 450-Å spacer layers from the top and the bottom well correspondingly. In order to obtain the difference in the mobilities, an enhanced amount of impurities was introduced at the upper edge of the top well (Si, 10^{10} cm^{-2}). The schematic cross section of the device may be found in Ref. 1. Measurements were done on 10 μm wide and 200 μm long channels with Au/Ge/Ni Ohmic contacts. Top and bottom gates were patterned using the standard photolithography fabrication method. The top Schottky gate covered 150 μm of the channel. The data were taken using a lock-in four terminal technique at $f=5.5$ Hz. The voltage probes connected to the gated segment of the channel were separated by 100 μm. The complementary measurements of the resistance and Hall coefficient lead to the following parameters of the structure (as grown, i.e., $V_g = V_{bg} = 0$ and $T = 4.2$ K): $\mu_1 = 47\,000$ cm^2/V sec, $\mu_2 = 390\,000$ cm^2/V sec, $n_1 = 4.7 \times 10^{11}$ cm^{-2}, $n_2 = 2.5 \times 10^{11}$ cm^{-2}. The values of these parameters for each temperature and gate voltage were determined independently and used in the fits (see below).

The variation of the top gate voltage V_g (for a fixed bottom gate voltage V_{bg}) allows one to sweep the potential profile of the QW's through the resonant configuration. The resistance vs top gate voltage ($V_{bg} = 1.5$ V; $T = 4.2$ K) is plotted in the inset of Fig. 1. The resistance resonance is clearly observed at $V_g \approx -0.2$ V. The value of the resistance in resonance is $R_{\rm res} = 65$ Ω, whereas the off-resonance value is estimated as $R_{\rm off} = 32$ Ω (cf. Fig. 1, inset). Next we fix the gate voltages, corresponding to the exact resonance position, and measure the resistance as a function of the in-plane magnetic field. Figure 1 shows the behavior of the RR for the two orientations of the magnetic field with respect to the direction of the current ($\mathbf{H} \,||\, \mathbf{j}$ and $\mathbf{H} \perp \mathbf{j}$). The experimental data clearly demonstrate the suppression of the RR by the magnetic field, as well as the expected anisotropy. In the perpendicular orientation the resistance decreases faster than in the parallel one. The theoretical curves, using Eqs. (2) and (3) with $H_c = 0.44$ T (this is the only fitting parameter) are shown on the same plot. For the perpendicular configuration, we obtained a perfect fit for the magnetic fields up to $H \approx 3$ T (note that $H_F \approx 4.2$ T for $\epsilon_F = 15$ meV).

The situation is markedly different for the parallel configuration. The fit to the data is obtained only in the narrow range of fields up to H_c; at high magnetic fields the resistance does not approach the value $R_{\rm off} = 32$ Ω. Moreover, a positive magnetoresistance contribution is well resolved. Large positive magnetoresistance ($\propto H^2$) in the parallel configuration was also observed for the one QW (the second well was totally depleted by a large negative voltage on the top gate). We tend to attribute this positive contribution to some normal component of the magnetic field to the plane of QW's, which is due to a nonperfect flatness of our structure in one direction. In the following, we thus restrict ourselves mostly on the perpendicular ($\mathbf{H} \perp \mathbf{j}$) orientation.

We now employ Eq. (4) and the extracted value of the characteristic field, $H_c = 0.44$ T, to establish the small-angle scattering time τ [note that all other parameters entering Eq. (4) are known, see above]. As a result, one has $\hbar/\tau$=1.7 meV at T=4.2 K, which implies the ratio between the transport and the small-angle scattering times to be equal to ≈ 3.2. Measurements of this ratio for different values of the Fermi energy (see below) result in a slow decrease from 3.2 at $\epsilon_F = 15$ meV to 2.5 at $\epsilon_F = 7$ meV. These data are in good agreement with those measured using Shubnikov–de-Haas oscillations in the 2D gas with a similar mobility.[9] It becomes evident now why one should complicate the theory to account for the long-range nature of scatterers. The simpler theory with short-range scatterers only ($\tau_i^{\rm tr} = \tau_i$) fails to explain quantitatively the observed width of the RR. Thus we conclude that the suppression of the RR in the magnetic field gives rise to a new and relatively simple way of measuring the small-angle scattering time.

We now repeat the same procedure for the perpendicular orientation at different temperatures in the range between 4.2–40 K using H_c as the only fitting parameter and then extracting τ. The experimental data and a set of theoretical curves are presented in Fig. 3. The width of the curves increases with temperature indicating the increase of H_c. The same type of data was obtained for a different set of voltages applied to the top and the bottom gates corresponding to the resonant conditions at different Fermi energies. The values of the Fermi energy are in the range 7–15 meV. These data were also analyzed in the same fashion and the values of $\tau(T, \epsilon_F)$ were determined. We stress again that all relevant parameters, besides τ, were established independently for each value of T and ϵ_F.

In Fig. 4 we plot $\epsilon_F[\hbar/\tau(T) - \hbar/\tau(0)]$ versus temperature on a logarithmical scale for three different values of the Fermi energy. The values of the small-angle scattering rate $\hbar/\tau(0)$ are as follows: 1.74 meV, 1.86 meV, and 3.36 meV for Fermi energies ϵ_F 15 meV, 11 meV, and 7 meV, respectively.

At small enough temperature ($k_B T \ll \epsilon_F$) all experimental points collapse to the same line. The slope of this line implies the quadratic temperature dependence of the displayed quantity. In this way the following relation is established:

$$\frac{\hbar}{\tau(T)} - \frac{\hbar}{\tau(0)} \propto \frac{(k_B T)^2}{\epsilon_F}. \tag{5}$$

Equation (5) suggests that the single-particle scattering rate $\tau^{-1}(T)$ consists of two parts: the small-angle scat-

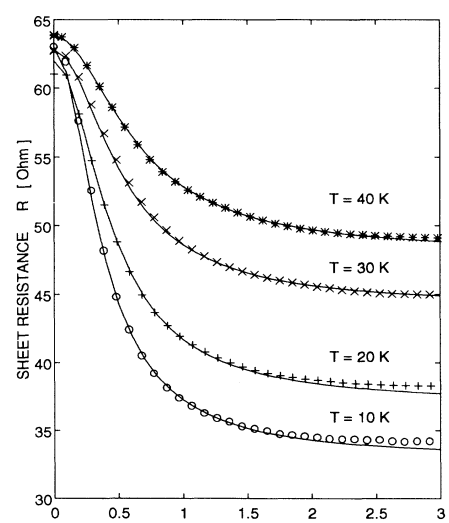

FIG. 3. The resistance vs magnetic field in the perpendicular configuration at different temperatures: circles, experiment; solid lines, theory ($V_{gb} = 1.5$ V; $V_G = -0.2$ V, corresponding to $\epsilon_F = 15$ meV).

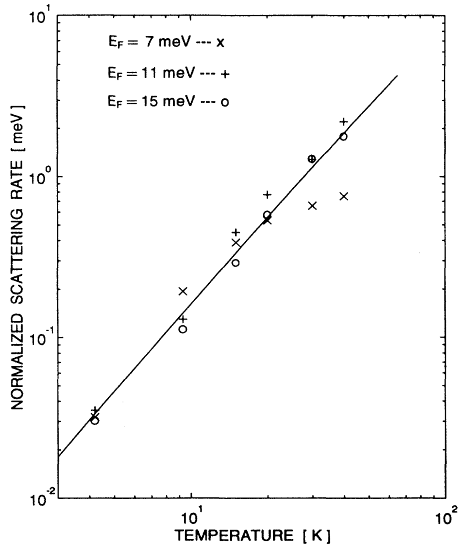

FIG. 4. Normalized scattering rate: $\frac{\epsilon_F}{11\text{ meV}}(\frac{\hbar}{\tau(T)} - \frac{\hbar}{\tau(0)})$ vs temperature for different Fermi energies. The solid line is $\frac{\epsilon_F}{11\text{ meV}}\frac{\hbar}{\tau_{ee}}$, where $\hbar/\tau_{ee}$ is given by Eq. (6) with $\xi = 0.5$.

tering rate on the remote impurities, $\tau^{-1}(0)$, and the electron-electron (e-e) scattering rate, τ_{ee}^{-1}. This is in contrast to the transport (two particle) scattering rate, $1/\tau^{tr}$, which is practically not affected (in the clean limit, see below) by the e-e scattering (due to the momentum conserved nature of the latest). To verify this idea quantitatively we use the result[10] for the e-e scattering rate in a clean 2D gas [the criterion is $\hbar/\tau(0) \ll k_BT \ll \epsilon_F$, which is fulfilled in our case]:

$$\frac{\hbar}{\tau_{ee}} = (1+\xi)\frac{1}{\pi}\frac{(k_BT)^2}{\epsilon_F}\left(1 + \ln 2 + \ln\frac{\lambda_F}{\lambda_{TF}} - \ln\frac{k_BT}{\epsilon_F}\right), \tag{6}$$

where $\lambda_{TF} = 276$ Å is the Thomas-Fermi screening length in the GaAs. We have introduced in Eq. (6) an additional factor $(1+\xi)$, which intends to simulate intralayer and interlayer contributions to the e-e scattering. In the original theory[10] only one 2D gas was considered, thus $\xi \equiv 0$. In the case of two closely spaced QW's, one expects that $0 < \xi < 1$, depending on the ratio between the screening length λ_{TF} and the mean distance between the wells b. This is indeed the case; the best fit (the solid line in Fig. 4) to our data is achieved with $\xi = 0.5$ in Eq. (6). We conclude that in our structure the interlayer e-e scattering rate is 0.5 of the corresponding intralayer value. This seems to be reasonable since the distance between the wells is of the order of the screening length. Equation (6) is valid only in the limit $k_BT \ll \epsilon_F$. Therefore, the deviations of the experimental points from the theory at high temperatures (especially for the smallest ϵ_F) are not surprising. Our results may be considered as another confirmation of the theory (apart from $\xi = 0.5$) (Ref. 10) in the range of relatively large temperatures. In a low temperature regime the theory[10] was excellently confirmed in the interference experiment.[11] A theory of e-e interactions in two tunneling coupled QW's would be desirable.

The central point is that the resistance of the coupled QW's is sensitive not only to the transport scattering time but also to the single-particle scattering time. This enables us to determine the small-angle scattering time, $\tau(0)$ [from the low temperature measurements, $k_BT < \hbar/\tau(0)$] as well as the e-e scattering time, $\tau_{e\text{-}e}$ [from the measurements at $k_BT > \hbar/\tau(0)$]. The comparison with the theory leads to the reasonable estimation of the ratio between intrawell and interwell e-e scattering rates. Thus we believe that the RR in two coupled QW's with different mobilities provides a powerful and relatively simple method of measuring of the small-angle and e-e scattering rates. Some unresolved questions raised in the present letter require further theoretical and experimental investigation of this phenomenon.

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