# Magnetoresistance Oscillations of Superconducting Al-Film Cylinders Covering InAs Nanowires below the Quantum Critical Point

I. Sternfeld,[1,*] E. Levy,[1] M. Eshkol,[1] A. Tsukernik,[2] M. Karpovski,[1] Hadas Shtrikman,[3] A. Kretinin,[3] and A. Palevski[1]

[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*

[2]*The Center for Nanoscience and Nanotechnology, Tel Aviv University, Tel Aviv 69978, Israel*

[3]*Department of Condensed Matter Physics, The Weizmann Institute of Scienece, Rehovot 76100, Israel*

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When odd multiples of half flux quanta thread a cylindrical superconducting shell with a diameter $d$ shorter than the zero temperature coherence length $\xi(0)$, superconductivity is predicted to be destroyed. We show here that as $d$ is reduced in comparison to $\xi(0)$ the resistance attains the normal state value, which seems to be temperature independent in the vicinity of half flux quanta. The data are in agreement with recent theoretical results.

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Quantum phase transitions (QPT) are an active topic in modern solid-state physics [1–5]. An important and interesting example for such transitions is the superconductor-normal state QPT, where external parameters such as disorder [1,6] or a magnetic field [5,7] force the superconductor quantum ground state to transit into a fundamentally different one. A unique system that can be used to study QPT is the so called destructive regime in a cylindrical superconducting shell with diameter $d$ smaller than the zero temperature superconductor coherence length $\xi(0)$ in a parallel magnetic field [8–13]. When odd multiples of half flux quanta thread such a system the kinetic energy of superconducting current is predicted to exceed the condensation energy so superconductivity would be destroyed even at $T = 0$.

This phenomenon is, essentially, enhanced Little-Parks (LP) oscillations [14] of the critical temperature $T_C$ as a function of the magnetic flux, $\phi$. The period of the LP oscillations is the flux quanta $\phi_0 = hc/2e$, and the maximal reduction of $T_C$, which takes place at odd multiples of $0.5\phi_0$, depends on the ratio $d/\xi(0)$. If $d \leq \xi(0)$ (or $d \leq 1.2\xi(0)$ [10]) the maximal reduction of $T_C$ exceeds $T_C$ in the vicinity of half-integer flux quanta, resulting in a flux tuned QPT. If the external flux threaded only through the nonsuperconducting part of the cylinder (Aharonov-Bohm flux) is raised, the system will periodically shift from the superconducting to the normal state. In practice, the entire cylinder is placed in an external magnetic field, the flux threads both the interior and the exterior superconducting wall of the cylinder and the kinetic energy of the superconducting currents is enhanced. When the magnetic flux is raised, at zero temperature, superconductivity is destroyed at a certain critical flux $\phi_c$, then it may reappear at another critical flux, and so on, but eventually the additional superconducting currents will not enable the reentry of the superconducting state. The lowest critical flux, $\phi_c$ weakly depends on the thickness of the superconducting wall $t$, and was calculated by mean field theory to be $\phi_c/\phi_0 = 0.83R/\xi(0)$ [8,10], where $R$ is the cylinder's radius.

The boundary between the normal and the superconducting phase is at the critical temperature $T_C(n, \phi)$ which solves Eq. (1) [11,12]:

$$\ln\left(\frac{T_C(n,\phi)}{T_C(0,0)}\right) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\alpha(n,\phi)}{2\pi T_C(n,\phi)}\right), \qquad (1)$$

where $\psi$ is the digamma function, $T_C(0, 0)$ is the zero field critical temperature, $n$ is an integer and $\alpha(n, \phi)$ is the pair breaking parameter, given by [12,15]:

$$\alpha(n,\phi) = \frac{\xi(0)^2}{\pi R^2} T_C(0,0)\left[4\left(n - \frac{\phi}{\phi_0}\right)^2 + \frac{t^2}{R^2}\left(\frac{\phi^2}{\phi_0^2} + \frac{n^2}{3}\right)\right]. \qquad (2)$$

Even though there are three parameters in $\alpha(n, \phi)$: $\xi(0)$, $R$ and $t$, $\alpha(n, \phi)$ depends only on the two ratios $R/\xi(0)$ and $t/R$. If $t < R$, $\alpha(0, \phi)$ weakly depends on $t/R$, therefore this line in the phase diagram can be used to obtain the ratio $R/\xi(0)$. On the other hand $\alpha(n, \phi)$ with $n > 0$, strongly depends on $t/R$, resulting in high values of $\alpha(n, \phi)$ even at $\phi/\phi_0 = n$; hence superconductivity is destroyed above a certain flux.

In an earlier experiment on the destructive regime Liu *et al.* [9] have reported that the maximum conductivity at $\pm 0.5\phi_0$, is larger than the conductivity of the normal state, even near $T = 0$. The nature of this enhanced conductivity is not yet fully understood. While some studies attribute it to superconducting fluctuations [15–17], others claim that the origin of this finite conductivity is the formation of sections with local superconducting order in the cylinder [11].

Lopatin *et al.* [16] calculated the fluctuation correction to the normal state conductivity in the vicinity of a superconducting-normal metal QPT for a low-dimensional system in a parallel magnetic field. While their calculations qualitatively agreed with the mentioned enhanced conductivity, some of their predictions, like a nonmonotonic resistivity as a function of magnetic field, were not observed experimentally. Vafek *et al.* [8] argued that even at odd multiples of $0.5\phi_0$, fluctuations of $\ell$, the mean free path, and $R$ lead to the construction of rare regions with local superconducting order. They calculated the cylinder's resistance by modeling the system to Josephson coupled regions and obtained a qualitative agreement with the experimental data. These calculations, however, are limited to high temperatures. Recently, Dao and Chibotaru [11] argued that the experimental data presented in [9,17] are mainly due to the inhomogeneities of the cylinder that lead to local $T_C(n, \phi)$ along the cylinder, rather than superconducting fluctuations. By simulating a cylinder in which $\xi(0)$ varies along the cylinder they quantitatively produced the features presented in previous experiments.

In this Letter we present the results of an experimental study of the destructive regime of a cylindrical superconducting shell in vicinity and below the QPT by making the parameter $d/\xi(0)$ smaller than ever achieved before. When $d \sim \xi(0)$ the mentioned enhanced conductivity is observed. However, as $d/\xi(0)$ is further reduced, the resistance is raised to the normal state value. It is remarkable that this value seems to be temperature independent (saturates). The data analysis is consistent with the model invoking the inhomogeneity of the cylinder.

The cylindrical superconducting shells are obtained by sputtering Al onto an InAs nanocylinders. The sample fabrication consists of three major steps: (i) Au-assisted vapor-liquid-solid (VLS) growth of InAs nanowires using molecular beam epitaxy (MBE), (ii) deposition of a superconducting coating film, and (iii) nanofabrication of electrical leads. The details of each step are described elsewhere [18]. A SEM image of a typical sample, after all fabrication stages af presented in Fig. 1.

The InAs wires are not conducting since they are not doped and we do not apply any gate voltage. This was checked experimentally in samples were the Al metal was absent or discontinuous (for thin Al layers) between the Ohmic contacts. Furthermore, since the oxide layer surrounding the InAs nanowire is not removed, there is a barrier for the electrons between the InAs and the Al. Therefore, the superconducting proximity effect can be ruled out [19]. The nanocylinder is manually aligned and bonded to a socket sample holder, so that the nanowire is parallel to the socket's edge and consequently to the applied magnetic field. The resistance was measured using a low noise analog lock-in amplifier (EG&G PR-124A) in a dilution refrigerator with a base temperature of 60 mK.

The inset of Fig. 2 shows the transition of sample "e7" into the superconducting phase, as the normal state resistance ($R_N = 14.7\ \Omega$) drops to zero. The width of the transition can be explained by thermal fluctuations above $T_C$ and by phase slips below $T_C$ [20]. Thickness fluctuations due to the granular structure of the Al surrounding the InAs can also broaden the transition. It is usually assumed that

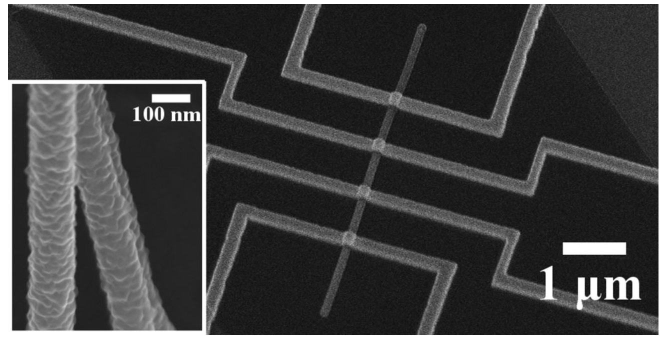


FIG. 1. A SEM image of a typical nanowire connected to Ohmic contacts. The dark area surrounding the nanowire is the hardened PMMA. The inset shows typical superconducting coated nanowires with $d = 100$ nm.

the zero field transition is not broadened due to nonmagnetic defects [21]. However, if $t$ is small enough, the transition can also be broadened due to variations of the density of states or the electron phonon interaction [11].

Figure 3 shows the residual resistance of sample "b4" as a function of magnetic field at 70 mK (this sample was measured in a 2-probe configuration and a residual resistance of 100 $\Omega$ was subtracted from the resistance). The basic features of this graph are similar to those presented in Ref. [9], i.e., at zero field, the sample is in the superconducting state, there are narrow peaks of the resistance at fields corresponding to $\pm 0.5\phi_0$ and $\pm 1.5\phi_0$, with finite resistances lower than $R_N$. At $\pm\phi_0$ the resistance is again close to its minimal value and at $\pm 2\phi_0$, $\pm 3\phi_0$ the resistance is reduced from $R_N$ to a finite value. We evaluate $\xi(0) = \sqrt{\pi\hbar v_F \ell / 24 k_B T_C}$ to be 72 nm, where $\hbar$ is Planck constant, $v_F = 2.03 \times 10^6$ m/sec [22] is the Fermi velocity, $k_B$ the Boltzmann constant, $T_C = 1.24$ K (taken at the middle of the transition) and the mean free path $\ell = 3.2$ nm, is estimated from $R_N$ using a free electron gas model.

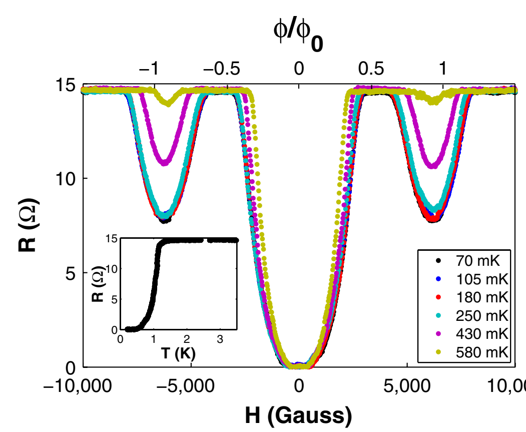


FIG. 2 (color online). Resistance as a function of magnetic field of sample e7 at several temperatures. The diameter of the cylinder is estimated to be 66 nm, and the zero temperature coherence length 110 nm. The inset shows the transition of sample e7 into the superconducting phase.

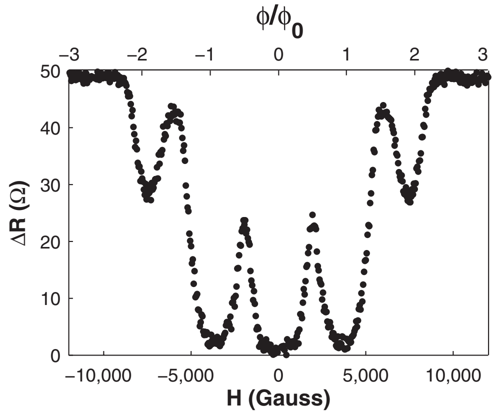


FIG. 3. The residual resistance as a function of magnetic field (bottom axis) and flux (top axis) of sample b4. The diameter of the cylinder is estimated to be 83 nm, and the zero temperature coherence length 79 nm.

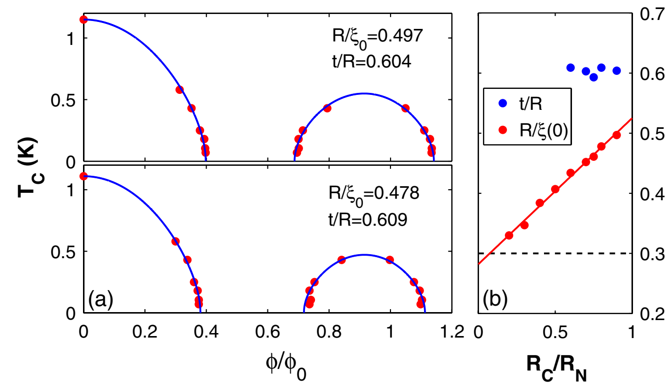


FIG. 4 (color online). (a) Flux-temperature phase diagram of the cylindrical superconducting shell e7. Top: $R_C/R_N = 0.9$. Bottom: $R_C/R_N = 0.8$. The solid lines are fits to the theory presented in Eq. (1). (b) The ratios $R/\xi(0)$ (red points) and $t/R$ (blue points) for different critical resistances. Each point is the result of a fit of the kind presented in Fig. 4(a). The red line is a linear fit to the red points and the dashed black line is the ratio $R/\xi(0) = 0.3$ as estimated in the text.

A different method to estimate $\xi(T)$ is by using the expression for the parallel critical field of a thin planar film, $H_C^{\|} = \frac{\sqrt{3}\phi_0}{\pi t \xi(T)}$, as was done by Liu *et al.* [9]. However, the definition of the critical field in this system is not clear, since the kinetic energy is not a monotonic function of the field. Furthermore, contrary to a cylindrical superconducting shell, where there are macroscopic supercurrents, in the derivation of $H_C^{\|}$, one assumes that the total current is zero [23]. Nonetheless, this estimation gives $\xi(T) = 79$ nm at $T = 70$ mK for sample b4, which agrees well with the former evaluation. From the value of the field at the peak corresponding to $0.5\phi_0$ in Fig. 3, we find that the diameter is $d = 83$ nm, which is consistent with the SEM observations. According to Ginzburg-Landau theory the criterion for the destructive regime ($d < \xi(0)$) is almost met, whereas according to Ref. [10] the criterion ($d \leq 1.2\xi(0)$) is met.

The resistance of sample e7 as a function of magnetic field at several temperatures is presented in Fig. 2. In contrast to sample b4 shown in Fig. 3, the resistance attains the normal value in a flux range near $\pm 0.5\phi_0$, and close to $\pm\phi_0$ the resistance is reduced from $R_N$ to a finite value, resembling the behavior of sample b4 near $\pm 2\phi_0$. To the best of our knowledge this is the first example of a cylindrical superconducting shell in which the resistance in the vicinity of $\pm 0.5\phi_0$ attains the normal value, and is not reduced due to superconducting fluctuations or regions with superconducting order. The reason why superconducting fluctuations do not alter the resistance is that as the ratio $R/\xi(0)$ is reduced $\phi_C$ is reduced as well, so the points ($\pm 0.5\phi_0$, $T$) on the phase diagram [see Fig. 4(a)] are shifted away from the phase transition. Similarly, when $R/\xi(0)$ is reduced all the cross sections in the cylinder meet the destructive regime criteria; thus, from a certain value of $R/\xi(0)$ the entire cylinder will become normal at $\pm 0.5\phi_0$.

For this sample we evaluate $\xi(0) = \sqrt{\pi \hbar v_F \ell / 24 k_B T_C}$ to be 110 nm ($T_C = 1.06$ K and $\ell = 6.5$ nm), while from $H_C^{\|}$ we get $\xi(T) = 95$ nm at 70 mK. From the value of the field at the minimal resistance (approximately at $\phi_0$) we find that the cylinders diameter is $d = 66$ nm, so $R/\xi(0) = 0.3$ and the destructive regime criterion is surely met.

In order to construct the flux-temperature phase diagram of the cylindrical superconducting shell and compare our results with Eqs. (1) and (2) we define a critical resistance $R_C$ separating the normal and superconducting states. The intersection of $R_C$ with the graphs in Fig. 2 yields the points of the phase diagram shown in Fig. 4(a) for $R_C/R_N = 0.9$ and 0.8. The ratios $R/\xi(0)$ and $t/R$, the fitting parameters for Eq. (2), are 0.497 and 0.604 for $R_C/R_N = 0.9$, respectively, and 0.478 and 0.609 for $R_C/R_N = 0.8$, respectively.

The mean field result for $T_C$ presented in Eq. (1), assumes a sharp phase transition, so the definition of $R_C$ should not affect the phase diagrams and the ratios obtained from the fits. Different phase diagrams are obtained for different definitions of $R_C$ because the superconducting fluctuations and inhomogeneities, perhaps do not alter the resistance at $\pm 0.5\phi_0$, but they do affect the transition itself.

While we are not aware of a theoretical calculation for the resistance that takes fluctuations into account for such a system, Dau and Chibotaru [11] presented a model to determine the resistance of the cylinder due to inhomogeneities. In this model each cross section along the cylinder has a different $T_C(n, \phi)$ determined by the ratios $R/\xi(0)$ and $t/R$ at that point. When the flux is changed different cross sections along the cylinder shift from the superconducting to the normal state or vice versa; thus, the resistance is altered; i.e., each resistance $0 < R < R_N$ in Fig. 2 corresponds to ratios $R/\xi(0)$ and $t/R$. In order to quantify the inhomogeneity of the cylinder we plot in Fig. 4(b) the ratios $R/\xi(0)$ and $t/R$ as a function of $R_C/R_N$. Each ratio in this graph is obtained from a fit similar to the fits presented in Fig. 4(a). As mentioned earlier, $R/\xi(0)$ can be obtained from $\alpha(0, \phi)$; hence, it can be estimated for a wide range of critical resistances. On the other hand, in order to estimate $t/R$ one needs at least one higher order in the pair breaking parameter, so this ratio can only be determined for critical resistances bigger than the minimal resistance near $\phi_0$ in Fig. 2.

It is quite clear that while $t/R$ hardly varies along the cylinder, $R/\xi(0)$ does change significantly and can be fitted to a linear curve. The ratio $R/\xi(0) = 0.3$, estimated from the mean free path or parallel critical field [dashed line in Fig. 4(b)], is smaller than all the ratios obtained from the fits to Eq. (1) [red points in Fig. 4(b)]. According to the linear fit $R/\xi(0) = 0.3$ corresponds to $R_C/R_N = 0.074$, close to the end of the transition. It seems that, according to this model, the calculations of $\xi(0)$ as explained earlier gives a somewhat overestimated value or perhaps an upper limit rather than the average value of $\xi(0)$. A similar conclusion was obtained by Ref. [11] where the calculations were compared with the data of Ref. [9].

As mentioned, the resistance at the destructive regime is $R_N$, and it does not vary for a rather wide range of temperatures. Similarly, Liu *et al.* [9] witnessed a plateau in the resistance vs temperature measurement. According to the conventional theory, a metallic state is not possible at zero temperature [24] for low-dimensional electronic systems. However, some studies argue [3,4] that when superconductivity is destroyed in the superconductor-insulator QPT, an anomalous finite resistance metallic phase emerges rather than the predicted insulating state. It is of course possible that the sample is not cooled. One possible reason for this is Joule heating. However, the measurement current is 10 nA, which dissipates only $10^{-14}$ W to the sample which is orders of magnitude lower than the cooling power through the sample's leads. Another possible reason is that the electronic temperature is higher than the temperature of the reservoir. Yet, when we perform the measurement with 100 nA current, we do not see any evidence for increasing electronic temperature. It was argued [25] that the absence of the current dependence of the resistance does not necessarily mean the lack of electronic heating, and that the only criterion which should be met is $eV/k_BT < 1$, where $e$ is the electron charge and $V$ is the total voltage across the sample. In our case $eV/k_BT < 0.1$ so such heating can be ruled out. Furthermore, our experience with this measurement setup is that electrons in both metallic and semiconductor low-dimensional systems were cooled much below 200 mK [26–28]. A thorough examination of the temperature dependence of the resistance is needed in order to clarify this point.

In conclusion, we have observed the normal metal-superconductor QPT in a cylindrical superconducting shell with $\xi(0) \gg R$. In contrast to previously reported experiments, the resistance of the nonsuperconducting phase at $0.5\phi_0$ attains values of the normal state. The value of this resistance seems to be temperature independent in contrast to the accepted predictions that a low-dimensional metallic state is not possible at low temperatures. We find that mean field theory fits the data reasonably well and that our data implies that inhomogeneity of the cylinder could play a role in the determination of the cylinder resistance.

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*itayst@post.tau.ac.il


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