First- and second-order phase transitions with random fields at low temperatures

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(Received 11 June 1982)

It is shown, within mean-field theory of the random-field Ising model, that a maximum of the distribution function at zero field does not necessarily imply a second-order transition at low temperature, as was previously suggested. The order of the low-temperature transition is discussed in terms of the maxima of the distribution function.

In a previous work,[1] it was suggested that Ising systems in a random ordering field undergo a first-order (second-order) phase transition at low enough temperatures, provided that the distribution function is symmetric and has a minimum (maximum) at zero field. We present here a correction to this criterion.

The random-field Ising system has the Hamiltonian

$$\mathcal{H}=-J\sum_{\langle ij\rangle} s_i s_j-\sum_i H_i s_i,\quad s_i=\pm 1\ , \tag{1}$$

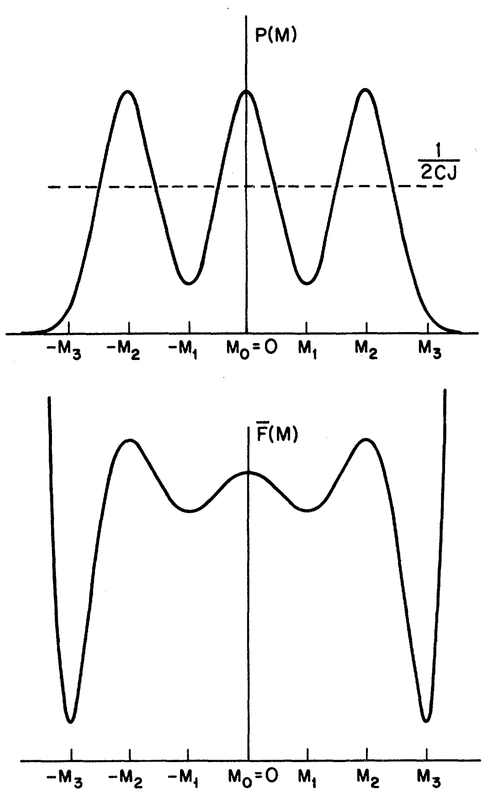

FIG. 1. Solutions of the self-consistent Eq. (6), M_0, $\pm M_1$, $\pm M_2$, and $\pm M_3$ are drawn. $\pm M_1$ and $\pm M_3$ correspond to minima of the free energy. These solutions are represented graphically. They occur when the area under the curve $P(M)$ is equal to the area under the horizontal line $1/(2cJ)$.

where $\{H_i\}$ are uncorrelated local random fields, with a distribution function $P(H_i)$.

Following Aharony,[1] and Schneider and Pytte,[2] within the mean-field theory, the free energy per spin can be written as

$$\bar{F}=\tfrac{1}{2}cJM^2-\frac{1}{\beta}\langle \ln[2\cosh\beta(cJM+H_i)]\rangle_{\rm av}\ , \tag{2}$$

where c is the coordination number and the magnetization M minimizes the free energy, Eq. (2), thus satisfying the self-consistent equation

$$M=\langle \tanh(\beta cJM+\beta H_i)\rangle_{\rm av}\ . \tag{3}$$

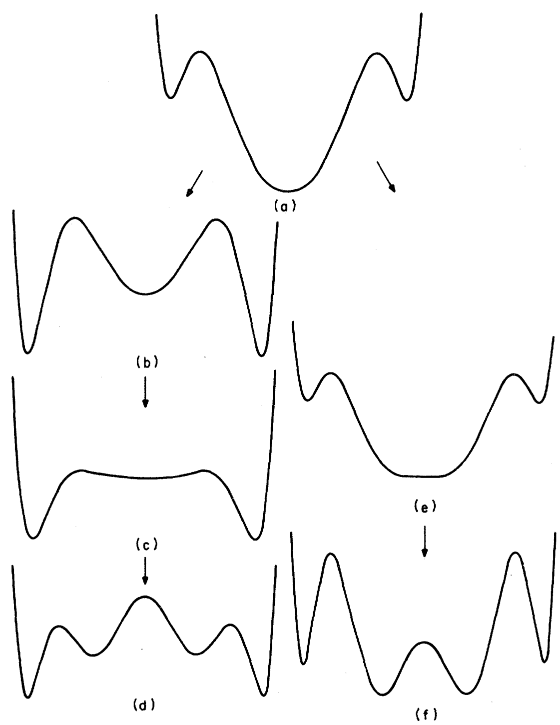

FIG. 2. First-order transition (a)→(b)→(c)→(d) and second-order transition (a)→(e)→(f) when $P''(0)$ is negative.

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For $T \to 0$ limit, we can write the free energy as

$$\bar{F} = \frac{1}{2} cJM^2 - \langle |cJM + H_i| \rangle_{av} \ . \quad (4)$$

For a symmetric distribution $P(-H)$, Eq. (4) reduces to

$$\bar{F} = \frac{1}{2} cJM^2 - 2 \int_0^{cJM} (cJM - H) P(H)\, dH \ , \quad (5)$$

leading to the self-consistent equation

$$M = 2 \int_0^{cJM} P(H)\, dH \ . \quad (6)$$

We investigate the order of the phase transition between the ferromagnetic ($M > 0$) phase and the paramagnetic (spin-glass?) phase ($M = 0$) at $T = 0$. If M is small we can expand the free energy, Eq. (5), in powers of M:

$$\bar{F} = cJ[\tfrac{1}{2} - cJP(0)] M^2 - \tfrac{1}{12} P''(0)(cJM)^4 + O(M^6) \ . \quad (7)$$

For $P''(0) > 0$, the transition cannot be second order, so it has to be first order. For $P''(0) < 0$, the transition can be second order or first order. In the latter case, we need to expand the free energy at least up to order M^8, and the order of the transition will not depend on $P''(0)$ alone.

By considering the most general symmetric distribution, we arrived at the following conclusions:

(a) When $P(H)$ has n maxima, $\bar{F}(M)$ will have at most $n+1$ minima, as seen in Fig. 1.

(b) For n even, $P''(0)$ is positive. In this case, only a first-order transition is allowed at $T = 0$.

(c) For $n > 1$ odd, $P''(0)$ is negative. In this case a second-order transition is possible, if it is not preempted by a first-order transition (Fig. 2). Note that for $n = 1$, the transition is always second order.

For a general symmetric distribution, we can also get phase transitions between two different ferromagnetic phases.

ACKNOWLEDGMENTS

I would like to thank H. Orland, A. N. Berker, A. Aharony, and Y. Shapir for useful comments. This work was supported by the National Science Foundation under Grant No. DMR-79-26405.

[1]A. Aharony, Phys. Rev. B 18, 3318 (1978).

[2]T. Schneider and E. Pytte, Phys. Rev. B 15, 1519 (1977).