The choice of flexibility in targeting inflation during normal times and during disinflations

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This paper investigates the relationship between the return path to price stability and the extent of flexibility in targeting inflation under perfect reputation as well as under imperfectly anchored inflation targeting systems characterized by imperfect reputation. The first part of the paper shows that the mapping from the flexibility parameter to the return path to price stability is generally non-unique. It discusses reasons and consequences of this non-uniqueness, and proposes several ways to address the communication and related problems that this fact creates for the conduct of monetary policy. The second part investigates the impact of reputation (defined as the weight given by the public to preannounced interim targets in forming inflationary expectations) on the speed of inflation stabilization. The main result is that higher reputation is associated with faster stabilizations at all levels of the flexibility parameter.

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1. Introduction  

It is widely accepted that, by raising inflation and reducing output, cost shocks create a trade-off between the stabilization of inflation and the stabilization of output for the central bank (CB). Under strict inflation targeting the CB is supposed to use its policy instrument to move the economy back to the inflation target, \( \pi \), immediately. By contrast, under flexible inflation targeting the CB is expected to steer the economy back to target only gradually in order to avoid excessive fluctuations in output and the output gap. The speed with which the CB plans to steer the economy back to the inflation target following a cost shock depends on the relative importance attributed to stabilization of output relative to stabilization of inflation. If it cares relatively much about the stabilization of output, the CB will allow a longer time interval between the realization of the shock and the planned return to the inflation target. If the opposite is true, it will plan to steer the economy back to the inflation target relatively quickly. A standard formulation of this view is embedded in the following loss function:

\[
L_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \delta^t \left[ \alpha x_t^2 + (\pi_t - \pi^*)^2 \right]
\]

where \( x_t \) and \( \pi_t \) are the output gap and inflation respectively, \( \delta \) is a discount factor, \( E_0 \) is the expected value operator conditioned on the information available to the CB in period 0, and \( \alpha \) is the relative importance attributed to the stabilization of output. Rogoff (1985) refers to \( 1/\alpha \) as the level of central bank conservativeness (CBC). Since this parameter

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determines the speed with which the CB plans to steer the economy back to the inflation target following a shock, advocates of flexible inflation targeting also refer to it as the flexibility parameter in targeting inflation. Note that the flexibility parameter and CBC are inversely related.

Even highly transparent central banks like the Bank of England have been remarkably silent about the flexibility parameter, \( \alpha \). While acting as Executive Director and Chief Economist of the Bank of England, John Vickers candidly admitted that this parameter has not been put in the public domain and argued that it is infeasible to do so (Vickers, 1998, p. 370). There are at least two feasibility constraints on publishing a meaningful figure for the flexibility parameter (FP). First, since in most central banks, policy is made by a committee it is normally the case that not all committee members share the same \( \alpha \). Agreeing once and for all on a value of \( \alpha \) to represent the median or average position of the committee is likely to be a more controversial process than agreeing on the setting of the interest rate at each particular point of time. The greater risk of lack of consensus tends to discourage attempts to reach a once and for all agreement.

But even in the case of a unitary policymaker (either because all the authority is vested in one individual or because all committee members share the same views) policymakers may shy away from formulating a precise value for \( \alpha \) due to the formidable task involved in precise measurement of potential output and the output gap that enters the loss function. Even a unitary policymaker may hesitate to make a long term commitment to a relative output gap weight that is attached to a highly unreliable output gap measure. On the other hand, flexible inflation targeters like the Bank of England, the Bank of Israel and other inflation targeters have been at ease to represent their objectives in terms of the path of the inflation gap they are willing to tolerate following a cost shock until the economy goes back to the inflation target. For practical central bankers as well as for communicating with the general public this is a natural and intuitive way of formulating the normative trade-off between inflation and output variability. Obviously, economists with some minimal training in the solution of quadratic dynamic optimization problems may argue that, provided a value of \( \alpha \) implies a unique desired path back to the target, and provided such a path uniquely maps back into a value of \( \alpha \), the two formulations are essentially equivalent. Framing the problem in terms of the tolerable return path or in terms of the FP should lead to identical decisions by the CB and is therefore immaterial. But when those conditions are not satisfied it is necessary to take a position on whether the desired trade-off between inflation variability and output variability should be formulated in terms of the FP or in terms of the inflation gap path that society is willing to tolerate on the way back to the inflation target when inflation is off target.

This paper opens by showing that, generally, the mapping between a given desired return path to the inflation target and the flexibility parameter, \( \alpha \) is non-unique. This non-uniqueness creates a tension between a clean formulation of the CB subjective trade-off between the variabilities of the inflation and the output gaps, on one hand, and transparent communications with the public on the other. Preannouncing a given horizon for price stability is easily comprehensible to the general public and therefore transparent and verifiable. By contrast, although it can be derived from micro-foundations the formulation in terms of the flexibility parameter, \( \alpha \), requires a theoretical apparatus and level of abstraction that most of the public and even some policymakers do not possess. Tversky and Kahneman (1981) have convincingly shown that framing of the same problem in different ways often affects decisions. This problem is compounded by the fact, demonstrated in this paper, that there generally is no one to one relationship between the flexibility parameter and the preannounced horizon to price stability.

In addition, given the FP, the choice of return path is likely to differ between a situation in which the inflation target (IT) system itself is well anchored and between a situation in which it is used as a disinflation device. Given \( \alpha \) the first part of the paper discusses the choice of return path in the case in which the IT system is already well anchored so that deviations between actual and target inflation occur only because of persistent but temporary cost shocks. The second part focuses on the case in which the credibility of the IT system is not fully established yet implying that reputational considerations play a central role. It highlights the crucial role of credibility considerations for the choice of horizon for price stability when the IT system is not fully credible. Smets (2003) and Friedman and Otker-Robe (2009) stress the importance of credibility for the efficient functioning of the IT system but do not discuss its implications for the choice of horizon to price stability.

The paper's organization follows. Sections 2 and 3 investigate the case in which the IT system is already well anchored. Section 2 shows that the mapping between the FP and the desired return path is generally non-unique but that this problem can be bypassed by using a particularization of the return path that is tailored to the stochastic structure of the cost shock. For the general case Section 3 proposes a method for mapping any given desired return path to target path into a unique “equivalent” value of \( \alpha \) that is “nearest” in the least squares sense to the desirable path (denoted \( \alpha^* \)). The section shows that \( \alpha^* \) depends on the parameters of the underlying economic structure including the average magnitude of the cost shock implying that the presence of model uncertainty exacerbates the problem of non-uniqueness of the equivalent FP. Section 4

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1 Cukierman (2009, Section 3) reviews the difficulties this creates for implementation of full transparency in monetary policy. Orphanides (2003) advocates taking the output gap completely out of the loss function because of those measurement problems.


3 Woodford (2003) and Gali (2008) show that the loss function in Eq. (1) is well grounded theoretically as a quadratic approximation of more basic micro-foundations.

4 The works of Cukierman and Livianat (1991) and Cukierman (2000) suggest that, when the IT system is not well anchored yet, reputational considerations play an essential role in the determination of the return path.
analyzes the choice of return path to price stability in an economy with a partially anchored IT system. This analysis is particularly relevant for the case of emerging economies with rates of inflation and inflationary expectations that are initially above the international norm. The section introduces interim inflation targets as a device for speeding up the return to price stability while economizing on the output gap costs of inflation stabilization. The section presents a framework for the analysis of the relation between the reputation of policymakers and the speed of inflation stabilization and uses it to investigate the effect of reputation and of CB conservativeness for the choice of return path to price stability. This is followed by concluding remarks.

2. Preliminaries and the non-uniqueness of the relation between the flexibility parameter and the desired return path

For concreteness the arguments in the paper are illustrated within the framework of a stylized forward looking New Keynesian framework due to Clarida et al. (1999) (CGG in the sequel). But it is argued in the conclusion that the non-uniqueness of the FP and related qualitative results transcend this particular structure. The behavior of the economy is described by

\[ x_t = -q(t - \bar{E}_t \pi_{t+1}) + \bar{E}_t x_{t+1} + g_t \]  

\[ \pi_t = \lambda x_t + \delta \bar{E}_t \pi_{t+1} + u_t \]

Here \( \bar{E}_t x_{t+1} \) and \( \bar{E}_t \pi_{t+1} \) are the expected values of the output gap and inflation, conditioned on the information available in period \( t \). \( i_t \) is the short term nominal rate of interest, \( g_t \) is a demand shock, \( u_t \) is a cost shock and \( \phi, \lambda \) and \( \delta \) are positive coefficients. The stochastic behavior of the two shocks is stationary but persistent, and is given by

\[ g_t = \mu g_{t-1} + \hat{g}_t, \quad 0 < \mu < 1 \]

\[ u_t = \rho u_{t-1} + \hat{u}_t, \quad 0 < \rho < 1. \]

Here \( \hat{g}_t \) and \( \hat{u}_t \) are the innovations to the cyclical components of demand and of costs respectively, and \( \mu \) and \( \rho \) characterize the persistence of those shocks. The first equation states that the output gap is negatively related to the exante real rate of interest and positively related to the expected future output gap. The latter appears in the output gap equation to reflect the notion that, since individuals smooth consumption, expectations of higher consumption next period (associated with higher expected output) leads them to demand more current consumption, which raises current output. In this framework the CB affects the output gap and inflation through the choice of the short term nominal rate of interest which, given expected inflation, determines the short term real rate. Note that the CB affects inflation through the output gap. As in stylized models of sticky staggered prices pioneered by Calvo (1983), current inflation depends on future expected inflation. In this type of model only a fraction of firms has the opportunity to adjusts its price each period and, due to costs of price adjustment, each firm adjusts its price at discrete intervals. Hence when it is given the chance to adjust its price the firm adjusts it by more the higher is expected future inflation. This interpretation implies that \( \delta \) is a discount factor.

2.1. Non-uniqueness of the mapping between the desired return path to target and the flexibility parameter

The desired return path to the inflation target specifies the speed with which the CB plans to close the gap between actual inflation and the target in each of the periods following the realization of a current (period 0) cost shock. More precisely the desired return path is given by

\[ d_t > 0, \quad t = 0, 1, \ldots T \quad \text{and} \quad d_t = 0, \quad t > T. \]

Here \( d_t \) are non-negative constants that stand for the absolute value of the inflation gap that the CB is willing to tolerate in each of the periods following the shock’s realization and \( T \) is the last period in which it is willing to tolerate a positive deviation from the inflation target. In what follows we will focus our analysis on the optimal monetary policy chosen by the CB under discretion. CGG show that, given a flexibility parameter \( \alpha \), the optimal planned response of discretionary policy following a cost shock satisfies the following set of conditions:

\[ x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi^*), \quad t = 0, 1, 2, \ldots. \]
and that those conditions imply the following planned path for the inflation gap

\[ \pi_t - \pi^* = \frac{\alpha}{\lambda^2 + \alpha(1 - \delta \rho)} u_t. \]  

(7)

Consider now a cost shock, \( u_0 \), that realizes in period 0. Eq. (7) implies that, given the information available in that period, everybody, including the CB, expects that the future path of the inflation gap is given by

\[ \mathbb{E}_0[\pi_t - \pi^*] = \frac{\alpha}{\lambda^2 + \alpha(1 - \delta \rho)} \mathbb{E}_0 u_t = \frac{\alpha}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t u_0, \quad t = 0, 1, 2, \ldots \]  

(8)

Ideally, to attain the desired return path in Eq. (5) it is necessary that

\[ \mathbb{E}_0[\pi_t - \pi^*] = \frac{\alpha}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t u_0 = d_t \quad \text{for all } t \geq 0. \]  

(9)

Obviously this cannot be achieved with a unique, stable over time, value of \( \alpha \). Hence, a unique value of the FP that corresponds to any arbitrarily given desired return path does not exist in general. On the other hand, a given value of the FP can be mapped into a specific return path but this mapping will also depend on the parameters of the economy and on the realization of the cost shock. This can be seen by using Eq. (9) to specify the return path parameters as

\[ d_t(u_0) \equiv \frac{\alpha u_0}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t \quad \text{for all } t \geq 0. \]  

(10)

As expected Eq. (10) possesses the desirable property that the coefficients of the return path are larger the larger is the FP, \( \alpha \). However, a bothersome aspect of this mapping is that the return path coefficients depend on the particular realization of the cost shock. This is likely to be a serious drawback for internal decisions and communications, as well as for external communications. This problem can be handled by requiring that the return path coefficient should reflect the FP for the average of the cost shocks rather than for each individual realization. This leads to the following respecification of the return path coefficients:

\[ d_t \equiv \frac{\alpha \mathbb{E}[u]}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t \quad \text{for all } t \geq 0. \]  

(11)

Eq. (11) shows that in addition to its dependence on \( \alpha \) and \( \mathbb{E}[u] \) the return path parameters depend on the persistence, \( \rho \), of the cost shock, on the discount factor, \( \delta \), and on the impact, \( \lambda \), of the output gap on inflation implying that the FP is not the only determinant of the return path. The Bank of England and several other central banks have stated that they are committed to move inflation back to the target within two years implying that the weights, \( d_t \), are identically zero beyond a two years horizon. Strictly speaking this is in violation of the return path weights implied, through Eq. (11), by the stochastic structure of the cost shock in Eq. (4). A possible reason for such discrepancies is that policymakers neglect sufficiently small deviations from the target or judge that the stochastic structure of their economies is such that cost shocks die out within a two years horizon. Be that as it may, the preceding discussion implies that the two years horizon restriction is totally unrelated to the FP, \( \alpha \). The following proposition summarizes the impacts of the structure of the economy on the return path parameters:

**Proposition 1.** Given a FP, \( \alpha \), the return path coefficients are larger (and the return to the IT therefore slower):

(i) the larger the variability of the cost shock as characterized by \( \mathbb{E}[u] \),

(ii) the weaker the impact of the output gap on inflation (the lower is \( \lambda \))

(iii) the higher is the discount factor, \( \delta \),

(iv) the higher is the persistence, \( \rho \), of the cost shock.

**Proof.** The proof follows immediately by inspection of Eq. (11).\(^\circ\)

The intuition underlying the proposition follows. When the variability of the cost shock is larger the CB spreads the consequent larger costs of being away from target over both the inflation and the output gaps implying that the return path to the inflation target is longer. The weaker the impact of the output gap on inflation the higher the cost in terms of output gap variability of returning to the IT implying that the optimal return path should be longer. The higher is the public’s discount factor, \( \delta \), the stronger the impact of (assumed model consistent) expectations on current inflation implying that the optimal return path is more extended. Finally, the higher the persistence of the cost shock the more extended over time are its inflationary consequences. Optimal spreading of those higher costs over the two gaps implies that the return path to the target should be longer.

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\(^{10}\) CGG implicitly assumes that the inflation target is zero. Eqs. (6) and (7) in the text follow from Eqs. (3.3) and (3.5) in CGG by reinterpreting their inflation term as a deviation from target. Note that when \( \pi^* = 0 \) those equations reduce respectively to Eqs. (3.3) and (3.5) in CGG.

\(^{11}\) This can be seen by noting that the partial derivative of \( d_t \) in Eq. (11) is given by \( \frac{\partial d_t}{\partial x} = \frac{\alpha \mathbb{E}[u]}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t \) which is positive for all \( t \)’s.

\(^{12}\) The expectation of the absolute value of the cost shock is used since, the expected value of the cost shock itself is zero by Eq. (4).
3. The equivalent flexibility parameter

We saw in the previous section that many central banks are more comfortable making public announcements about the return path to the IT than about their FP, $\alpha$. The author of this paper actually witnessed cases in which members of a monetary policy committees were either unable or unwilling to specify or even think in terms of $\alpha$ but willingly subscribed to a two years or a one year horizon for return to the IT. This is probably due to the fact that, since it does not require a background analytic apparatus it is easier to form an intuition and to communicate it to the public in terms of the return path than in terms of the flexibility parameter. On the other hand, the formulation in terms of the flexibility parameter is preferable from a conceptual point of view since it focuses directly on policymakers’ subjective trade-offs between the output and the inflation gaps that enter their linear quadratic loss function. Most of the literature on endogenous monetary policy utilizes such linear quadratic loss functions. Furthermore, Woodford (2003) and Gali (2008) have shown that these loss functions can be viewed as approximations of the more fundamental costs of inflation due to relative price distortions caused by the interaction between inflation and temporary price stickiness. It would, therefore, be desirable to map any given return path into a FP. But, as explained in the previous section such mapping does not generally exist. A second best is to find a single value of $\alpha$ that minimizes the square deviations of the expected path in Eq. (10) from the desired return to target path. This leads to the following problem. Find the value of $\alpha$ (denoted by $\alpha^*$) that solves the following problem:

$$
\min_{\alpha} \frac{1}{2} \sum_{t=0}^{\infty} \left\{ \frac{\alpha E[u]}{\lambda^2 + \alpha(1 - \delta \rho)} \rho^t - d_t \right\}^2.
$$

(12)

The (implicit) solution for $\alpha^*$ is determined by the condition

$$
\frac{\alpha^* E[u]}{\lambda^2 + \alpha^*(1 - \delta \rho)} \frac{1}{1 - \rho^2} - \sum_{t=0}^{T} \rho^t d_t = 0.
$$

(13)

The explicit solution is given by

$$
\alpha^* = \frac{(1 - \rho^2)\lambda^2 \sum_{t=0}^{T} \rho^t d_t}{E[u] - (1 - \rho^2)(1 - \delta \rho) \sum_{t=0}^{T} \rho^t d_t}.
$$

(14)

This is the flexibility parameter that, within a loss function based formulation, is nearest to the desired return path to the inflation target. I shall refer to it as the “equivalent flexibility parameter”. The solution in Eq. (14) makes sense only if the denominator of this expression is positive.\footnote{This is obtained by a rearrangement of the first order condition. It is easily checked that the second order condition for a minimum is satisfied.} The term $(1 - \rho^2)(1 - \delta \rho)$ is smaller than 1 implying that the average variability of the shock $u$ has to be sufficiently large in comparison to the parameters of the desired return path.

3.1. Determinants of the equivalent flexibility parameter

Not surprisingly, Eq. (14) implies that a main determinant of the equivalent flexibility parameter, $\alpha^*$, is the path flexibility as characterized by the vector $d = [d_0, d_1, \ldots]$. But Eq. (14) also implies that $\alpha^*$ is affected by economic parameters like the structural parameters $\lambda$ and $\delta$ and by the variability of the cost shock as characterized by $\mathbb{E}[u]$. It follows by inspection of Eq. (14) that, not surprisingly, the larger the return path flexibility as characterized by each of the $d_t$ coefficients, the larger the equivalent flexibility parameter. The following proposition summarizes the impacts of the structural parameters on the magnitude of the equivalent flexibility parameter:

**Proposition 2.** The equivalent flexibility parameter, $\alpha^*$, is larger,

(i) the larger the impact, $\lambda$, of the output gap on inflation;
(ii) the lower the impact, $\delta$, of inflationary expectations on inflation;
(iii) the lower the variability of the cost shock as characterized by $\mathbb{E}[u]$.

**Proof.** Immediate from differentiation of Eq. (14).\footnote{Otherwise $\alpha^*$ is negative.}

The intuition underlying the proposition follows. When the impact of the output gap on inflation is larger the CB can achieve a given return path with more emphasis on stabilization of the output gap. Consequently $\alpha^*$ and $\lambda$ are positively related. The larger is the impact of inflationary expectations on inflation the more costly it is to let the expected output gap deviate from its desired level since it has a stronger undesirable impact on the inflation gap. Consequently, the desired return path is achieved with a lower equivalent flexibility parameter. Finally, the larger the variability of the cost shock, the larger the currently expected future path of this shock. Hence, to achieve a given return path to the inflation target the CB needs to reduce the weight on stabilization of output, implying that the equivalent flexibility parameter is lower.
3.2. The equivalent flexibility parameter and model uncertainty

Since the equivalent flexibility parameter (EFP) depends on the structural parameters of the economy uncertainty about those parameters imparts uncertainty onto the EFP. This has both positive and normative implications. On the positive side it provides one explanation for the notorious reluctance of even relatively transparent central banks to announce their relative weight on output stabilization. On the normative side ardent supporters of transparency like Svensson (2002, 2003) advocate publication of the relative weight on output gap stabilization. Proposition 2 suggests that uncertainty about the parameters of the economy complicate the task of revealing this weight for central bankers that frame their objectives in terms of return time to the inflation target. Those difficulties may take several forms. First, since different committee members may have different view about the economic structure, they will come up with different equivalent flexibility parameters. Second, Proposition 2 suggests that central bankers that publicly announce a value of $\alpha$ will have to revise those announcements as they learn about changes in structural parameters of the economy. This may confuse the public and hurt the credibility of the CB.

4. Choice of stabilization path to price stability for a given level of imperfect reputation

During the last two decades inflation targets have been used by several countries to stabilize inflation from levels persistently above the current international standard of price stability down to this standard. The main problem in such cases is that, in view of past inflationary experiences, the public is not sure about the extent to which policymakers will be able to deliver their preannounced inflation target. Prior to full anchoring of expectations through IT the mere announcement of an inflation target does not generate immediate credibility. On the other hand the announcement of a target usually has some effect on inflationary expectations. During periods characterized by excessive inflation imperfect credibility of inflation targets is frequently due to the fact that, although it intends to achieve the target, the CB, as well as fiscal authorities, are subject to political and other constraints. Since they are aware of this tension individuals give some, but not full, credence to preannounced targets. How large is the effect of announced targets on expectations depends on the past record of policymakers to which I refer as "reputation". This reputation is valuable because it enables policymakers to influence inflationary expectations and through them actual inflation and real variables up-front by just announcing an intermediate inflation target.

A major impediment to quickly establish the international price stability norm is that, in the presence of persistently high inflationary expectations, a cold turkey stabilization requires a large recession. One way to reduce this cost is to stabilize gradually. This can be gainfully supplemented by preannouncement of relatively modest interim targets and by delivering them. Even if reputation is imperfect the announcement of temporary interim targets improves CB objectives through the following mechanism: by partially reducing inflationary expectations and therefore inflation (see Eq. (3)) it reduces the recession required to deliver a given decrease in the rate of inflation. Such a strategy is preferable to the absence of interim targets since it partly conveys the future stabilization plans of the CB to the public and this affects expectations even before the required restrictive policies are implemented. Obviously, in order to maintain its reputation the CB has to deliver rates of inflation that are consistent with the announced interim targets. It should be intuitively clear from this discussion that the reputation of the CB will turn out to be one important determinant of the path to price stability.

The main problem during disinflation is bringing down inflationary expectations to the international standard at minimal possible costs in terms of recessions. The loss function remains Eq. (1). For simplicity and focus I will abstract from economic shocks by setting all the shocks in Eqs. (2) and (3) at their zero expected values. To focus on the impact of reputation on the speed of stabilization with minimal algebraic distractions the New-Keynesian (NK) Phillips curve in Eq. (3) is replaced by

$$\pi_t = \lambda x_t + E_{t-1} \pi_t.$$  \hspace{1cm} (15)

As in CGG it reflects the idea that current expectations have a positive impact on future, yet unrealized inflation. But the lead structure is somewhat different. In particular period’s $t-1$ inflation forecast for period $t$ has a positive impact on actual (yet unknown in period $t-1$) inflation in period $t$ and the coefficient $\delta$ in Eq. (3) is set at one. The analysis starts from an inflationary equilibrium in which actual and expected inflation are equal and are above the long term price stability target, $\pi^*$ implying that the initial inflation gap

$$d_0 = \pi_0 - \pi^*$$

16 The discussion in this section is an adaptation of the analysis in Cukierman and Liviatan (1991) to a variant of the New-Keynesian model.
17 In many countries this standard is 2 percent.
18 The Bank of Israel followed such a strategy during the stabilization of the 1990s (details appear in Section 4 of Cukierman and Meltzer, 2015).
19 By contrast in CGG’s specification inflation in period $t$ is affected by the inflation expected in period $t$ for period $t+1$ and the coefficient $\delta$ is taken to be 0.99 when data is quarterly.
is positive. Due to imperfect credibility expected inflation in each period is given by

$$E_{t-1} \pi_t = \beta \pi^*_t + (1 - \beta) \pi_{t-1}, \quad 0 < \beta < 1$$

(16)

where $\pi^*_t$ is the short term interim target for period $t$ announced in period $t-1$ and $\pi_{t-1}$ is the actual rate of inflation during period $t-1$. The reputation of the CB for delivering the target is characterized by $\beta$. When $\beta=1$ reputation is perfect and the IT system in use is fully anchored. The analysis in the two preceding sections corresponds to this particular case. When $\beta=0$ reputation is non-existent and the announcement of interim targets has no impact on expectations. In the more common intermediate cases reputation is neither perfect, nor non-existent and is higher if existing reputation at this

Thus, the preannouncement of an interim target along with some non-zero reputation leads to a NKPC in which the impact of the current output gap on next period's inflation is stronger. Minimization of the losses in Eq. (1) subject to the modified NKPC in Eq. (15) leads to the following string of Euler equations:

$$\chi_t = -\frac{1}{1 - \beta} (\pi_t - \pi^*), \quad t = 1, 2,...$$

(20)

It is instructive to compare this condition to its counterpart in the absence of reputational considerations in Eq. (6). The main difference is that, in the presence of interim targets and some reputation, inflationary expectations are reduced up-front. This leads to a stronger impact of the output gap on inflation and, other things the same, induces the CB to create lower inflation gaps at the cost of higher output gaps. Substituting Eq. (20) into Eq. (19) we obtain

$$\pi_t = \pi_{t-1} - \frac{1}{\alpha} \left( \frac{\lambda}{1 - \beta} \right)^2 (\pi_t - \pi^*), \quad t = 1, 2,...$$

(21)

Subtracting $\pi^*$ from both sides of Eq. (21) and rearranging

$$d_{t+1} = \theta d_t, \quad t = 0, 1, 2,...$$

(22)

where the initial inflation gap, $d_0$, is exogenous and

$$0 < \theta = \frac{1}{1 + \frac{\lambda^2}{\alpha(1 - \beta)^2}} < 1.$$

(23)

Using the recursion in Eq. (22) to express period’s t inflation gap in terms of the initial inflation gap, $d_0$, and using the definition of $\theta$ yields

$$d_t = \frac{1}{\left(1 + \frac{\lambda^2}{\alpha(1 - \beta)^2}\right)^t} d_0, \quad t = 1, 2,...$$

(24)

Since $\theta$ is bounded between zero and one inflation converges monotonically toward the long term IT, $\pi^*$. The following proposition summarizes the impact of various parameters, including in particular the level of reputation, $\beta$, on the speed of convergence to the IT:
Proposition 3. The speed of convergence to the target is higher:

(i) the higher is the reputation, $\beta$, of the CB,
(ii) the higher the impact of the output gap, $\lambda$, on inflation,
(iii) the lower is the flexibility parameter, $\alpha$.

Proof. The proof follows immediately from inspection of Eq. (24).\textsuperscript{20}

In comparison to previous sections the main new element here concerns the impact of reputation on the speed of stabilization. The proposition states that the higher reputation the faster the speed of convergence to the target. Intuitively, the higher reputation, the higher the credence given by the public to preannounced interim targets. This reduces the output cost of a given inflation reduction and incentivizes the CB to announce and deliver more ambitious interim targets. This speeds up, in turn, the process of convergence to $\pi^e$. Eq. (24) implies that when $\beta = 1$ so that reputation is perfect return to the IT, $\pi^e$, is immediate. The intuition underlying the impacts of the other parameters on the speed of inflation stabilization is similar to the intuition underlying the results in and around Proposition 1: the higher the impact of the output gap on inflation the more advantageous it is to stabilize quickly. Similarly, when $\alpha$ is lower the CB is more conservative, and naturally chooses a faster return path to price stability since it is relatively more averse to inflation than to output gaps.

5. Concluding remarks

This paper investigates the relationship between the return path to price stability and central bank conservativeness (also referred to as inflation targeting flexibility) during normal times, when inflationary expectations are well anchored and deviations from target are due only to temporary shocks, as well as during periods of inflation stabilization characterized (at least initially) by persistently high inflation and imperfect anchoring of inflationary expectations. The first two sections following the introduction consider the first case and the last one extends the analysis to the case of imperfect anchoring. The paper shows that the mapping from the flexibility parameter, $\alpha$, to the return path to price stability is generally non-unique since it also depends on the structural parameters of the economy.\textsuperscript{20} This non-uniqueness can be overcome by tailoring the return path to the stochastic structure and other parameters of the economy at the risk of having to change it later due to changes in the structure of the economy and parameter uncertainty. For the same reason a unique inverse mapping from a preannounced horizon to price stability on one hand and the flexibility parameter on the other does not generally exist.

Given a publicly announced return path to price stability the paper proposes a procedure for calculating an “equivalent flexibility parameter” that is nearest in the least squares sense to the flexibility parameter that would have generated such return path and investigates the impact of structural parameters on this “equivalent flexibility parameter”. On the positive side those results provide an explanation for the reluctance of inflation targeting central banks to publish this weight and to, instead, communicate with the public by publishing a maximal horizon to price stability without precise specification of the return path. On the normative side it suggests that a possible way to alleviate the potentially adverse impact of parameter uncertainty on credibility is to announce a target range (as currently done by many inflation targeters) rather than a point target. For concreteness the main arguments are illustrated within the framework of a standard New Keynesian model. But those arguments obviously apply to any other economic structure as long as policymakers are not fully certain about the structural parameters of the economy.

The second part of the paper investigates the impact of reputation (defined as the weight given by the public to preannounced interim targets in forming inflationary expectations) and of its interaction with structural parameters of the economy on the speed of inflation stabilization. The main result is that higher reputation is associated with faster stabilizations. In a broad sense this part of the paper extends the discussion in Smets (2003) to the case of imperfect reputation and provides a conceptual framework for understanding the recent utilization of inflation targeting by some emerging markets. Policy oriented descriptions of such stabilizations appear in Freedman and Otker-Robe (2009).

For analytical simplicity the analysis in the paper is conducted under a fixed, exogenously given, level of reputation. However, when the CB persists in delivering rates of inflation that are consistent with preannounced interim targets reputation rises eventually. Evidence presented in Section 5 of Cukierman and Melnick (2015) shows that, following the gradual stabilization of Israeli inflation during the nineties and the early twenty first century, the anchoring of inflationary expectations to the target as characterized by reputation increased. By Proposition 3, this should induce an even quicker return to price stability in comparison to the case of fixed reputation. An interesting extension of the analysis presented here is to examine the consequences of gradual learning within the context of a New-Keynesian representation of the economy.\textsuperscript{21}

This is left for future work.

\textsuperscript{20} Smets (2003) makes a similar observation in the context of policy choices for the European Central Bank.

\textsuperscript{21} Walsh (2000) models learning by introducing two types of policymakers within a signaling framework with overlapping wage contracts. Cukierman (2000) utilizes Bayesian analysis to discuss changing levels of reputation within a reduced form economy under inflation targeting.
References

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