DISCUSSION
THE MYSTERY OF THE RAVENS*

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The controversy concerning Professor Hempel's paradox of the ravens is going strong. Yet its purpose is not clear. What Professor Hempel has shown is this. Assume the theory of confirmation by satisfaction or instantiation (e.g. "all ravens are black" is confirmed by "at such and such time and place there was a black raven"). Assume further the principle of equivalence of confirmation (all logically equivalent hypotheses are equally confirmed by a given observation). These two lead to a somewhat surprising result according to which practically any non-disconfirming evidence is confirming evidence (e.g. a red herring, which is a non-ravenous non-black thing, confirms "all non-black things are non-ravenous" and thus its equivalent "all ravens are black").

It is assumed by all writers on the subject that this result calls for some further thinking, but it is seldom clear what kind of thinking, or to what end. It is not clear, to be precise, what is the problem to be solved. Professor Hempel did not try to solve a problem but to resolve a paradox; this, it may be assumed, is in keeping with the analytic approach. Paradoxically, because Professor Hempel followed this approach he fell prey to the confusion which this approach comes to expose; because he did not pose a problem he could use the words "resolve," "paradox," and "confirmation" in senses wavering between the technical and the ordinary. (The previous statement is an application of a general point concerning analytic philosophy made by Popper in the new preface to his Logic of Scientific Discovery.) Let me explain in detail.

I

Being no authority on ordinary English, I looked for a paradigm case before posing an instance to my liking. I am assured that Gilbert and Sullivan excel both in their Englishness and in their ordinariness. So I have chosen my first instance from the Pirates of Penzance, where it is declared to be very much of a paradox that a twenty-four year old fellow should have had but six birthdays. My second choice is the paradoxical fact—or theory—that at present the American economy would improve as the outcome of increased consumption, even though, or particularly if, this is achieved largely by borrowing money. Paradoxically, the national economy will improve if consumers borrow in order to consume luxuries.

What is to be done about a paradox? About many paradoxes we do nothing; some of us record them and use them as witticisms. Some of us want to explain paradoxical phenomena, but this is nothing peculiar to paradoxes: someone wants to explain this, someone that. However, someone may want the paradox explained in order to be relieved of the sense of paradoxicality; some people feel that only puzzling phenomena should be explained, particularly paradoxes. You will find that often what are viewed as puzzling or paradoxical phenomena, are counter-examples to

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deeply seated and widely accepted views. You will also find that those particularly
irked (rather than enchanted) by paradoxes are of two kinds. They may be dogmatists
who look for an auxiliary hypothesis which reconciles their pet theories with the
paradox which is a *seeming* counter-example. Alternatively, they may be neurotics
who cannot stand the feeling of paradoxicalness. In either case the explanation of the
paradox—by the theory of the leap-year or by Keynesian economics, in our instances—
may be of little avail for such people. Treatment may help; explanation surely will
not.

What problems are involved here it is difficult to list. Do we wish to find the source
of the sense of paradoxicalness, or the means of overcoming it, or do we wish to
determine whether it is at all desirable to overcome it, or how to overcome other
difficulties it gives rise to? To take an instance, we may wish to know how borrowing
for the sake of buying luxuries leads to a boom; or we may wish to know why people
deny this claim. We may assume, in reply to the latter question that the public denies
this because it holds a primitive theory of wealth and value, a view that entails that
consumption is loss and debt is poverty. These popular prejudices are a nuisance to
economic planners who may have the problem, how best to get rid of such nuisances.
The solution may be, teach people Keynesian economics—a solution attempted by
President Kennedy in a television talk to the nation. That solution, perhaps because
of the death of Kennedy, did not work fast enough; nor would advertising for easily
available loans. So one might conceive of more ingenious solutions, such as, to create
the illusion in many people that their income-tax is much smaller than it is in fact.
The federal government has tried this solution recently, with a fair measure of success,
but of a kind which cannot be lasting: the illusion was corrected so painfully that it
will not be easily reestablished.

So much for paradoxes in the ordinary sense of this word, and for the diversity
of problems which they may raise. The one problem that is common to all of them
is, whether intuition, which causes our feeling of paradoxicalness, is not thereby
proved fallible? Further, if intuition is fallible, should it not be dismissed? This is
precisely what formalists of all sorts and other kinds of anti-intuitionists often allege.
Intuitionists, however, contend that intuition can only be misled by intellectual
juggling, conjuring, or sleight of hand. Intuition may be misled, they admit, by
creating conditions under which a perfectly intuitive idea looks counter-intuitive,
or else by deducing an idea from premises which are only tacitly assumed so that the
idea looks counter-intuitive. Let us discuss this point in some detail.

Philosophical intuitionism is the doctrine (associated with the name of Descartes)
according to which by the use of our mental eye we can see with assuredness the truth
of certain ideas; it is analogous to philosophical sensationalism, the doctrine (associated
with the name of Bacon) according to which by the use of our physical eye we can
see with assurance the truth of certain factual information. Taken literally both of these
doctrines as formulated above may be viewed as having been refuted by paradoxes
and sense illusions respectively. But the doctrines can easily be saved by the claim
that under certain conditions we can avoid such errors, that certain conditions
guarantee the absence of paradoxes or sense-illusions. One may now try to formulate
a theory concerning these conditions and submit that theory for critical examination.
One may also try to formulate a theory concerning the causes of the paradoxes or
sense-illusions and how these can be avoided. Here one may go into detail and suggest
how to avoid such errors from the start and how to rectify such errors after they were
committed. For instance, one may suggest that though a slow and cautious progress
may prevent the error in question, the elimination of the error requires more than caution. It may be claimed that to eliminate a paradox one must see how it was created, by what default or sleight of hand.

One may claim that Professor Hempel's study is an instance of this kind of elimination of paradox: by laying bare all the elements which go into causing us the feeling of paradoxicalness he has expelled that feeling. If so, then the very existence of a literature on the paradox of the ravens is empirical evidence for his failure to expel the feeling of paradoxicalness, and so an empirical refutation of the intuitionist thesis under examination. One may, however, insist on the intuitionist thesis and reject the thesis that Hempel has succeeded in laying bare all the ingredients which have led to the sense of paradoxicalness.

This may be true. Moreover, however many unsuccessful attempts were made to expel the sense of paradoxicalness by trying to lay bare the ingredients that gave rise to it, one may still blame the attempts rather than the intuitionist thesis. One may stick to that thesis and propose it as a criterion for success: we may say that one has found all the ingredients which went into the making of a paradox, if and only if, one has thereby succeeded to eliminate the sense of paradoxicalness in question.

But such a policy is somewhat dangerous, since the intuitionist view may be false while the policy leads us to uphold it in spite of any number of failures. Indeed, Michael Polanyi has proposed in his *Personal Knowledge* just this characteristic of a policy as a mark of its being pseudo-scientific. Any procedure is pseudo-scientific, he suggests, when and only when failures of its application are automatically viewed as the results of shortcomings of those who apply it, never as the possible results of the shortcomings of the procedure itself. Polanyi's characterization is very much in the spirit of Popper's view of science.

This seems to be an obvious argument against the policy of viewing every failed attempt to dispel the sense of paradoxicalness as a failure to reveal all the ingredients which make the paradox rather than as evidence that intuitionism is false. Still, there is one good reason for the persistence of the above policy, and it is the unsatisfactoriness of the view opposed to intuitionism—namely formalism. As Hadamard has shown empirically in his *The Psychology of Invention in the Mathematical Field*, we need our intuitions badly, if not to judge truths, at least for heuristic purposes. But whenever two opposite views are unsatisfactory the obvious policy is to try and invent a new one, as William James and John Dewey have so amply stressed. Indeed, a new theory of intuition has developed recently, though it was adumbrated already in Russell's 'Mysticism and Logic'. It is as follows. We need our intuition not for the purpose of judging truths but for heuristic purposes. Therefore, we have to improve our intuitions so as to come to a stage where what once was felt to be paradoxical is felt paradoxical no longer. Professor Lakatos is a notable proponent of this view amongst our contemporaries, especially in his already classical, though quite recent, papers "Proofs and Refutations" in the *British Journal for the Philosophy of Science* (1964).

The suggestion that intuition may be in error but is capable of improvement, is rather common sense; and the view that it is indeed useful to improve it is not striking either. Yet some philosophers have often claimed that some intuition is the final authority. Even Russell, who endorses the view that intuition can and should always be improved, accepts one intuition as final, namely the *a priori* intuition we have of the principle of induction. As Hume had argued before and as Russell has argued in his "The Limits of Empiricism" and other works, the principle of induction cannot itself be based on experience without appeal to a (higher level) principle of
induction. And so he has advocated the principle on the authority of an \textit{a priori} intuition. The only other alternative is to claim that the principle of induction belongs to logic (Carnap, \textit{Logical Foundations of Probability}). Hempel, it seems, oscillates between the intuitionist and the logical view of the foundation of the principle of induction. If he is intuitionist, then his use of "paradox" in the ordinary sense of the word is adequate; if he holds that the principle of induction belongs to logic, he may well have to use the word in the logician's technical sense which I shall now present.

\section{II}

Though the word "paradox" belongs to ordinary English, the expression "to resolve a paradox" does not. Intelligent non-philosophers usually interpret this expression to mean "remove the sense of paradoxicalness," but confess not to have encountered it before (this is an empirical finding of mine). When asked how a paradox is resolved people scarcely know how to answer this question, even if they are philosophers (this is also an empirical finding of mine). This is not to say that the expression "to resolve a paradox" is meaningless or that there is no established procedure of sorts by which to resolve a paradox: indeed, logicians understand all this with no great difficulty as long as they do not confuse "paradox" in their sense with "paradox" in the ordinary sense.

In the technical sense a paradox is an antinomy, a proven contradiction. When we prove that the normal class is normal and also that the normal class is non-normal, we have a paradox—Russell's paradox—on our hands; and we must resolve it. "Resolve" is here used, as elsewhere in mathematics, to mean, "solve the problem posed by." The paradigm case is "to resolve an equation"; it means, to solve the problem, what is the value of the unknown in that given equation? Since "resolve" is often used by the ignorant as a synonym for "solve" it must be stressed that this is a misuse of that word; an equation cannot properly be solved since it is not a problem, nor can a problem be properly resolved. Since to resolve something (whether an equation or a conflict or anything else) is to solve the problem posed by that thing, there is ample room for ambiguity here. Since a paradox in the ordinary sense of this word may pose many problems, or no problem at all, the application of the verb "resolve" in such a context would be intolerably ambiguous. The problem raised by a paradox in the technical sense of the word, that is to say by an antinomy, is one very clear-cut and very urgent problem: It is, how can we eliminate it at the minimal cost? The verb "resolve" may be used here with no fear of ambiguity.

The problem raised by a logical paradox has nothing to do with intuition: it is a matter of formal logic. The resolution of the paradox, indeed, may be highly counter-intuitive. In particular, the attempts to eliminate the paradoxes of set-theory, such as Cantor's or Russell's paradox, may lead to the highly counter-intuitive result that we cannot speak of a class-complement in the abstract: when we speak of non-ravens it is no more automatically clear what we mean. Thus, non-ravens may be all birds other than ravens, or all animals other than ravens, or all numbers other than ravens, depending on what is known as the universe of discourse (that is to say, the context).

Though logicians are at present pretty much agreed about logical paradoxes in general, this is no reason to accept their view. In particular, one may endorse the view proposed by Wittgenstein (in the \textit{Tractatus}). Wittgenstein denied that there exist any philosophical problems, and he claims that what may give us a sense of a problem
or of puzzlement or of paradox (in the ordinary sense) is a confusion which has to be dispelled, and with it the seeming problem would dissolve. As instances of such pseudo-problems Wittgenstein presented both traditional philosophical material and new logical material, including Russell’s paradox. Thus, according to Wittgenstein we should not resolve a (logical) paradox but dissolve it. It is of some importance here, if confusion is to be avoided, to note that, properly and rigorously speaking, we can resolve a logical paradox only if we think it poses a genuine problem; if we share Wittgenstein’s view according to which the logical paradoxes are not genuine antinomies but merely paradoxical—i.e., seemingly counter-intuitive results rooted in confusions—then we cannot resolve them though we can dissolve them. As a genuine antinomy a paradox is to be resolved, and as a mere puzzle or a pseudo-problem it is to be dissolved.

Does Hempel try do dissolve the paradox of confirmation or does he try to resolve it? Does he think that it poses a seeming problem which results from the fact that our intuition is misled? Or does he think that there is an antinomy here to be eliminated and does he try to solve the problem how to eliminate it at minimum cost? It is hard to say. In some places (“Inductive Inconsistencies”) he may be thinking of paradoxicalness in the logician’s sense, in some he may be thinking of paradoxicalness in the ordinary sense. We must first find out what he means by “confirmation.”

Let me first briefly show that confirmation in its ordinary sense is different from confirmation in its technical sense, the sense so well captured by Whewell and by Popper. In the ordinary sense of the word one may be legitimately confirmed in one’s belief by reassurances of friends and acquaintances, or at the very least by assurances from experts. If experts are confirmed in their beliefs by other experts, then confirmation is very different from what scientists in the modern world call confirmation or empirical confirmation. Also, the conditions for empirical confirmation in the ordinary use of the term are much more lax than the conditions for empirical confirmation in the technical sense. Scientists are capable of and ready to doubt doctrines amply confirmed by ordinary criteria, and these facts are reflected in usages. To take one instance, we are confirmed in our belief in our doctor since many of his patients recover, whereas—according to the Consumers Report—expert students of medical practices in this country are very skeptical of all private practitioners, and of many who practice in clinics, because their practices do not compare favorably in any way to those which take place in hospitals, especially hospitals attached to medical schools. Thus, common people are more credulous than expert examiners; their conditions for confirmation are more lax.

Now when Hempel resolves the paradox of confirmation he sometimes argues as if once we see where our intuition went wrong it will be rectified all by itself. Our intuition, he says, goes wrong when we feel it paradoxical that a red herring confirms the hypothesis that all ravens are black because we know too much; we may see a red thing and, wishing to test the hypothesis, we may wish to know if it is not a red raven. In this case, obviously, says Hempel, the red herring is a confirming instance; but if we are told in advance that the red thing is a herring which, as we already know, is very different from any raven, we feel the difference to be the reason for the felt irrelevance of the evidence.

I agree with this point. Suppose I told you that all black things are ravens. You will want to know what about my shoe, which happens to look black; and if it turns out to be a raven, or not-black after all, you will be impressed. That is to say, you will be ready to consider the non-blackness of my shoe, or its being made of a raven,
as confirmation. (If you don’t like this example, take one from modern physics: all black surfaces are carbonic.)

Now Hempel uses the example of testing a hypothesis as support for his theory of confirmation as instantiation. He thereby confuses the technical (test) and the ordinary (instance) senses of confirmation; he does so because of his analytic or linguistic approach which leads him to search for a linguistic or analytic criterion of confirmation. If in one case—the test case—the statement “here is a red herring” confirms the hypothesis “all herrings are red” or the hypothesis “all ravens are black,” then it does so in all cases, since the criterion depends on the logical relation between propositions and not on the cases. And the logical relation between these propositions is that of instantiation. Thus the analytic approach imposes on us the acceptance of the lax and ordinary sense of confirmation and the rejection of the technical scientific sense.

To conclude with a clear and concise presentation of Hempel’s view as I understand it. There is a contradiction between the scientific theory of confirmation as the result of examination and the analytic theory of confirmation as a result of finding an instance to the hypothesis. Not noticing this we are baffled. Once we notice this we shall be baffled no more. All this may be viewed as Hempel’s dissolution of a seeming problem, his elimination of an inconsistency which results from confusing two incompatible theories; or it may be viewed as his elimination of a sense of counter-intuitiveness. For my part, I agree that the causes of the uneasiness or the sense of paradoxicalness result from confusing the idea of confirmation as applied in science and the analytic theory of confirmation. But I neither endorse his preference for the analytic theory, nor do I think that this theory can be rendered intuitive. The counter-intuitiveness of the analytic theory, or rather of the corollary to it that a red herring confirms “all ravens are black,” may pose a problem for the dogmatist or for the neurotic. Others may find cause here for relinquishing—at least temporarily—the analytic approach. If they would, they might reject the task of dissolving a paradox (as meaningless) and pose instead clear-cut problems to be solved.

Nelson Goodman’s view (Fact, Fiction, and Forecast), is somewhat similar to Hempel’s, equally convincing as far as it goes, and breaks down when stretched further than permissible due to the desire to have a formal criterion of confirmantion. Rather than speaking of tests, Goodman speaks of a special case of testing, crucial experiments. A red herring does not serve as a crucial experiment, say, between “all ravens are black” and “all ravens are non-black,” so we feel it is no confirmation of the one vis-a-vis the other. Had Goodman said that confirmations only occur as results of crucial experiments, he would be offering rather too stringent a condition for confirmation; but this cannot be, because this condition is not formal. The search for a formal criterion, then, is what stands behind much of the present discussion.

III

Is the analytic requirement for a formal criterion, rather than a material criterion, so very intuitive that intuitionists must endorse it? The statement that intuition supports the requirement for a formal criterion of confirmation is not in accord with facts (this is another empirical finding of mine). Yet Hempel and Goodman, who are intuitionists of sorts, endorse it. They endorse it, I suppose, because it leads to the satisfaction or instantiation theory of confirmation (a black raven confirms “all ravens are black”), which is intuitive, even though this latter theory leads to the counter-
intuitive results. Intuitionists, therefore, wish to show that the intuitive theory does not lead to counter-intuitive result. To show this they (both Hempel and Goodman) describe material circumstances under which the paradoxical results do not seem paradoxical. These circumstances only show the theory to be intuitively comfortably applicable in some circumstances. Were the intuitionist thesis true, and were the requirement for a formal criterion intuitive, the paradox would disappear. It does not. Hence at least one of these theses is mistaken. Indeed, one is empirically refuted and the other has no leg to stand on.

There was one further suggestion, which has been adumbrated in a paper by H. G. Alexander (and already by Goodman, in a way), which has been repeated by others, and which ascribes the counter-intuitiveness of Hempel’s result (a red herring confirms “all ravens are black”) to the smallness of the confirmation provided by a red herring as compared with the one provided by a black raven. The relative paucity of ravens and of black things makes a black raven so much more of a rare phenomenon than a non-raven non-black; and this makes the former so much more of a confirmation than the latter, that the latter is negligible by comparison and we tend to forget it. The counter-intuitiveness of Hempel’s result, in other words, is the same as we have when we notice that a drop of water is something, yet compared with the Niagara Falls it is nothing. Let us examine this contention by a simple substitution. Ravens and blacks are rare indeed; let us replace them by common things and see what happens. Instead of black we take opaque, whose complement is transparent; intuitively, most things seem to be opaque. Instead of ravens let us take inorganic things. We shall have “all inorganic things are opaque” as confirmed by the opaque inorganic thing my wrist-watch, and disconfirmed by the transparent inorganic thing the glass-cover of my wrist-watch; also, it is confirmed by the opaque organic thing myself, as well as by a transparent organic thing like a jelly-fish. People feel that it is counter intuitive that a transparent jelly-fish confirms “all inorganic things are opaque” (this is an empirical finding). So, the argument from the rarity of black ravens must be rejected.

Moreover, it may look from Hempel’s study—and some have so claimed—that the cause of the defect is in his presenting the ravens hypothesis as a conditional, or in our having forgotten the way to contraposit a conditional. If this were so, biconditionals would be unproblematic. Take the theory “all and only amorphous (i.e., non-crystalline) carbonic surfaces are black” which is declared by physicists to be true. Everything is black if and only if its surface is sooty. What is black is sooty, and what is sooty is black. Ask your intuitions these questions. Will black soot confirm this? I think the answer is yes. Will a seemingly black but really gray blackboard? I think it is yes again. Will a very seemingly black, but still gray, crystalline carbon (graphite)? I think the answer is, emphatically, yes. Will a red herring? Hardly. But a red herring is a non-black non-soot. The biconditional is paradoxical. But take another instance of a biconditional: “All and only Americans are wise” which is the same as “All and only foreigners are unwise.” Here the counter-intuitiveness, or the sense of paradox, totally disappears, no matter who is commoner, the American or the foreigner, the wise or the unwise.

The reason for this variance between the two biconditionals is in the implicit choice of a proper universe of discourse. When Hempel takes the hypothesis “(x)Raven(x) ⊃ Black(x)” he does not dwell on “x”. If x ranges over atoms, think how highly confirmed the hypothesis is. But let us take “x” to be spacio-temporal regions and reformulate “all ravens are black” to say “no spacio-temporal region contains
any unblack-raven,” and upon careful experimentation you will find it difficult to
decide what confirmation is counterintuitive and what is not. The logical schema is
now “(x) something (x),” or “¬(∃x) something (x)” etc. This enables us to include
biconditionals, and even equations. Take Boyle’s Law of gases: “No spatio-temporal
region contains a non-ideal gas,” where gases are ideal if and only if they obey the
equation \( PV = RT \). The law may be a conjunction of instances, in series: spatio-
temporal region \( a \) contains no non-ideal gas, spatio-temporal region \( b \) contains no
non-ideal gas, etc. Similarly, consider “no spatio-temporal region contains a perpetual-
motion machine,” “no spatio-temporal region contains elementary electric charges
different from that of the electron,” etc., etc. Remembering that almost all spatio-
temporal regions are empty, it follows that they do not contain non-ideal gases, or
perpetual-motion machines, or flying horses, and so there is an \( a \) priori high degree
of confirmation—indeed near 1—to Boyle’s law and to other laws as so stated and
understood. Similarly, there is an \( a \) priori high degree of confirmation to the contrary
of each of these laws. This, I think, is a powerful generalization of Hempel’s result
and I would love to claim the authorship of it, but the fact is, it is in Popper’s \textit{Logik
der Forschung}, which preceded even Hempel’s and Goodman’s works by some years.
It is a pity that the discussion has centered round black ravens and not round the more
powerful examples above. As Popper has shown, as long as our intuition hankers
after the vulgar doctrine of confirmation by satisfaction or instantiation it will not
escape the sense of paradoxicalness.