At what speed will a tennis ball break a window?

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The kinetic energy of the tennis ball will go into deflecting the window and increasing the internal energy of the tennis ball. The second term is what distinguishes a rock from a tennis ball. Note that I am assuming that the collision happens sufficiently slowly for the forces within the window to spread the load over the entire window. A sufficiently fast projectile (ie: a bullet) will punch a nice clean hole through a window. A slower projectile will shatter the entire window. A still slower projectile will bounce without breaking the window.

Deflection of the window

First, we figure out the breaking point of a window:

Assume a window of height d, deflected by an angle θ by a force in the middle. Consider just the lower half of the window. Its length has stretched from $d/2$ to

$$l = \frac{(d/2)}{\cos(\theta)}$$

and the strain

$$\begin{aligned} s &= \frac{(l - d/2)}{d/2} = \frac{1}{\cos(\theta)} - 1 \\ &\approx \frac{1}{2}\theta^2 \end{aligned}$$

for small θ.

The maximum strain = Tensile strength / Young's Modulus

$$T/Y = \frac{10^8 \mathrm{N/m}^2}{7 \cdot 10^{10} \mathrm{N/m}^2} = 1/700$$

using numbers from an intro physics text.

This gives a maximum angle $\theta_{max} = 0.053$ rad. If $d = 1$ m, then the maximum deflection is 2.5 cm. Impressive, but consistent with reports during hurricanes.

Now we need the work done on the window during this deflection. Let the radius of the tennis ball be r and the window thickness be t. Assume that the force of the deflection is born entirely by a strip of window $2r$ in width. (At worst, this will be too low by a factor of about 3. However, this will compensate for the fact that we are assuming an otherwise perfect window.)

The force of the window on the ball will be

$$\begin{aligned} F_{wb} &= \left(\frac{1}{2}\theta^2 Y\right) t(2r) 2 \sin\theta \\ &\approx 2\theta^3 Ytr \end{aligned}$$

To get the work we need the distance travelled. $x = (d/2)\sin\theta$ is the linear deflection of the window. The work done will be:

$$\begin{aligned} W &= \int F dx = \int_0^{\theta_{max}} 2\theta^3 Ytr(d/2)d\theta \\ &= \frac{1}{4}\theta^4 Ydtr \end{aligned}$$

Plugging in $t = 1$ mm and $r = 4$ cm, we get

$$W = 5.7\text{J}$$

and

$$F_{max} = 830\text{N}$$

Now the difference between a rock and a tennis ball is that the tennis ball can compress.

Compression of the tennis ball

Now I get to make the approximation I have waited my entire professional life to make: **assume the sphere to be a cube!** I can calculate the compression of a cube easily; spheres are hard.

I will assume that the cubic tennis ball compresses by crumpling the sides. This means that a tennis ball of radius r that is compressed a distance c, has the trailing face unaffected, the leading face displaced a distance c, and the side faces have their leading edges crumpled a distance c. This means that the new center of mass is (measured from the trailing face)

$$\begin{aligned} x_{cm} &= \frac{1}{6}\left[0 + (2r - c) + 4\frac{2r-c}{2r}\frac{2r-c}{2} + 4\frac{c}{2r}(2r-c)\right] \\ &= r + \frac{1}{6}\left[-c - \frac{c^2}{r}\right] \end{aligned}$$

where the 1st term (0) refers to the trailing face, the 2nd term to the leading face, the 3rd term to the uncrumpled parts of the side faces and the 4th term to the crumpled parts of the side faces.

Now I will assume that the restoring force is due entirely to air pressure, that the walls of the tennis ball can be neglected. This is not necessarily a good assumption. There are two extremes: completely rigid walls (ie: a rock) and completely flabby walls. The answer will be somewhere in between.

Now the pressure inside the tennis ball will be

$$p = \left(\frac{V}{V_0}\right)^\gamma p_0 = \left(\frac{2r}{2r-c}\right)^\gamma p_0$$

since the process happens so quickly it must be adiabatic.

Now we can get the force exerted by the ball on the window as a function of the compression:

$$F_{wb} = (p - p_{atm})A = (2r)^2 \left[\left(\frac{2r}{2r - c} \right)^\gamma p_0 - p_{atm} \right]$$

Since we know the maximum force that the window can exert, we will use this to solve for the maximum compression of the tennis ball. Assuming that $p_0 = 1$ bar for a dead tennis ball, this gives:

$$c = 2r \left(1 - \left[1 + \frac{F}{4r^2 p_0} \right]^{-1/\gamma} \right)$$

Using $\gamma = 1.4$ for air we get

$$c_{max} = 3.6\text{cm}$$

The initial kinetic energy of the ball will go into deflection of the window and the internal energy of the air in the tennis ball. The change in the internal energy of the ball is

$$\Delta U = \frac{5}{2} N R \Delta T$$

where $\Delta T = T_{max} - T_0$ and

$$T_{max} = T_0 \frac{pV}{p_0 V_0} = T_0 p_0 \left(\frac{V}{V_0} \right)^\gamma \frac{V}{p_0 V_0} = 381\text{K}$$

This gives

$$\begin{aligned} \Delta U &= \frac{5}{2} (8\text{cm})^3 \frac{1\text{mole}}{22.4 \cdot 10^3 \text{cm}^3} 8.3(\text{J/mole K}) \cdot 81\text{K} \\ &= 38\text{J} \end{aligned}$$

Thus most of the energy goes into deforming the ball.

Now we can find upper and lower limits for velocities. Assuming that the ball weighs about 50 g, we get

$$v_{min} = \sqrt{2 * W/0.05\text{kg}} = 15\text{m/s}$$

for the rock and

$$v_{max} = \sqrt{2 * (W + \Delta U)/0.5\text{kg}} = 42\text{m/s}$$

or between 30 and 85 mph. v_{max} is clearly too large but v_{min} seems reasonable. I expect that the answer will be between 20 and 30 m/s.