How many drops are there in a cubic centimeter of fog if the visibility is 100m and the fog disappears within an hour?

P. L. Kapitza

Solution:

Note: The present solution follows, in general lines, the solution of Acad. Kapitza.

1. Visibility distance:

The visual contrast ε of an object placed at a distance *x* from an observer follows the Lambert law

$$\varepsilon = \varepsilon_0 e^{-\alpha x}, \qquad (1)$$

where α is the attenuation coefficient and ε_0 – the visual contrast of an object placed in front of the eyes:

$$\varepsilon_0 = \frac{B_b - B_o}{B_b} \qquad (2)$$

Eq. (2) is the Weber-Fechner law, ε_0 being a physiological quantity and *B* a physical one. B_o is the brightness of the object and B_b is the brightness of the background (the sky at the horizon). The maximum visual contrast is obtained for objects placed on the ground, where $B_o=0$, for which $\varepsilon_0=1$.

From (1) follows that the visibility distance is

$$x = \frac{1}{\alpha}\ln\frac{\varepsilon_0}{\varepsilon} \qquad (3)$$

This distance attains its maximum value when ε_0 is maximal ($\varepsilon_0=1$) and ε takes its minimal value (ε_{min} =3% for most of the people). Under these conditions the maximum visibility distance is

$$L = x_{\max} = \frac{3.5}{\alpha} \qquad (4)$$

2. Attenuation coefficient

In the case of fog, the water drops concentration is small! From this two major statements emerge: a) light attenuation occurs due to scattering and not to absorption; b) Water drops scatter the light independently. So, if *n* is the concentration of water drops in the fog and *S* the attenuation cross section of light on a single drop, then

$$\alpha = nS \tag{5}$$

If the light wavelength is much smaller than the drop radius *R*, then $S = \pi R^2$. If they are of the same order of magnitude, due to diffraction, this effective surface of scattering doubles. Assuming the later case, from (4) and (5) it follows that

$$n = \frac{3{,}5}{2\pi R^2 L} \tag{6}$$

3. <u>water drop radius</u>

Assuming that the fog lasts the time *t* needed by water drops to reach the ground and that they fall with constant speed (their weight is counterbalanced by Stokes viscosity force) $v=H/t$, then $mg=6\pi\eta Rv$, where all these quantities have their usual meaning, or

$$R = \sqrt{\frac{9\eta H}{2\rho g t}} \tag{7}$$

Finally, from (6) and (7) we get

$$n = \frac{7}{18\pi}\frac{\rho g t}{L\eta H} \tag{8}$$

So, taking $\rho=1000$ kg/m^3, $g=9.81$ m/s^2, $t=3600$ s, $L=100$ m, $\eta=1.05 \cdot 10^{-3}$ kg/m$\cdot$s, and $H=6$ m (this is the usual ground fog layer height according to WECA – weather glossary: www.weca.org/nws-terms.html), it follows that $n = 7$ drops/cm^3 and $R = 28$ µm.