Meteorites - Problem 06/03

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Abstract

What is the cumulative effect of all the meteorites that fell on the Earth during the last billion years on the length of a day? Did it become longer or shorter? By how much?

1 Meteorites

We denote the average mass of a meteorite by m and we assume that the number of meteorites that fall -on average- every year is $1/\tau$ (where is the elapsed time between 2 successive hits). Thus, the mass of the Earth increases at an -almost- constant rate $\lambda = dM/dt = m/\tau$. From that assumption, it is evident that the Earth's mass as a function of time is $M(t) = M_0 + \lambda t$, where M_0 is the mass at $t = 0$.

The moment of inertia of the Earth is $I = (2/5)MR^2$, since the Earth can be approximated by a sphere. Then, the Earth's angular momentum changes at time t before it is hit by a meteor of mass dM, is:

$$\mathbf{L}(t) = \frac{2}{5}MR^2\omega + (dM)vR\sin\phi, \tag{1.1}$$

where v is the velocity of the meteorite and ϕ is the latitude of the impact point. The angular momentum at time $t + dt$ after the impact is:

$$\mathbf{L}(t+dt) = \frac{2}{5}(M+dM)(\omega+d\omega) = \frac{2}{5}MR^2\omega + \frac{2}{5}(dM)R^2\omega + \frac{2}{5}MR^2 d\omega. \tag{1.2}$$

Thus, the rate of change of the angular momentum, which equals the torque, is:

$$\dot{\mathbf{L}} = \mathbf{N} = \frac{2}{5}\dot{M}R^2\omega + \frac{2}{5}MR^2\dot{\omega} - \dot{M}vR\sin\phi. \tag{1.3}$$

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Taking the average over all the impact points, and assuming the absence of external torque, we have:

$$\dot{M}R^2\omega = -MR^2\dot{\omega}, \tag{1.4}$$

which can be integrated to yield:

$$\omega = \omega_0\left(1 - \lambda t/M_0\right). \tag{1.5}$$

Solving for the relative increase in the period, we find:

$$\frac{\Delta T}{T_0} = \frac{1}{\frac{M_0}{\lambda t} - 1}, \tag{1.6}$$

where M_0 is the mass of the Earth at $t = 0$, which is equal to $M_0 = M_\oplus - \lambda t$ ($M_\oplus$ is the mass of Earth now). Thus, we have,

$$\frac{\Delta T}{T_0} = \frac{1}{\frac{M_\oplus}{\lambda t} - 2} \approx \frac{m}{M_\oplus}\frac{t}{\tau}. \tag{1.7}$$

Substituting typical values for large meteorites $m \approx 9\cdot 10^1$ kgr, $M_\oplus \approx 6\cdot 10^{24}$ kgr, $t \approx 10^9$ years and $\tau \approx 10^{-2}$ years, we find $\Delta T/T_0 \approx 10^{-12}$. This corresponds to an increase in the period of rotation of order 10^{-5} sec.