Current inhomogeneities in an electric wire

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August 24, 2002

In an electric wire, the current is the source of a magnetic field that exerts a force on the moving electrons and drives them to the surface. This results in an inhomogeneous electron density and drives a compensating electrostatic force. I propose a dimensional approach to determine the order of this inhomogeneity and find that it can be ignored.

We call ρ the free electron density, $\Delta\rho$ the order of the inhomogeneity in this density, j the mean current density, v the mean velocity of free electrons and a the radius of the wire. The electrostatic and magnetic field are given by the Coulomb and Ampere laws:

$$E \sim \frac{1}{\varepsilon_0} a \Delta\rho \tag{1}$$

$$B \sim \mu_0 a j \tag{2}$$

The force they induce per charge unit is:

$$\vec{F} = \vec{E} + \vec{v} \times \vec{B} \tag{3}$$

The terms of the right handside are along the radial direction and opposite, at the equilibrium, they are equal so that $E \sim vB$. In other words:

$$\frac{\Delta\rho}{\rho} \sim \varepsilon_0 \mu_0 j v \tag{4}$$

Using $j = \rho v$ and $\varepsilon_0 \mu_0 c^2 = 1$, we derive:

$$\frac{\Delta\rho}{\rho} \sim \left(\frac{v}{c}\right)^2 \tag{5}$$

To be able to conclude, we need the order of v/c in an electric wire. v can be connected to more intuitive quantities:

$$v = \frac{I}{\pi a^2 \rho} \tag{6}$$

The free electron density equals the atomic density in most cases, in other words:

$$\rho = \frac{\mathcal{N} e d}{\mathcal{M}} \tag{7}$$

where $\mathcal{N}$ is the Avogadro constant, e the elementary charge, $\mathcal{M}$ the molar mass of the wire and d its density. Finally:

$$v = \frac{I\mathcal{M}}{\pi a^2 \mathcal{N} e d} \tag{8}$$

With typical values of $d = 5$ and $M = 0.05\,\text{kg/mol}$, we derive

$$\frac{v}{1\,\text{cm/s}} \sim 3 \left(\frac{I}{1\,\text{A}}\right) \left(\frac{a}{1\,\text{mm}}\right)^{-2} \tag{9}$$

$$\frac{\Delta\rho}{\rho} \sim 10^{-20} \left(\frac{I}{1\,\text{A}}\right)^{2} \left(\frac{a}{1\,\text{mm}}\right)^{-4} \tag{10}$$

Conclusion: j is constant!