Review of Economic Studies (1999) **66**, 679–691 0034-6527/99/00280679$02.00

Property Rights and Efficiency of Voluntary Bargaining under Asymmetric Information

ZVIKA NEEMAN
Boston University

First version received January 1995, *Final version accepted June* 1998 (*Eds.*)

We show that in public good problems under asymmetric information, the success of voluntary bargaining is closely related to the structure of property rights. We characterize property rights structures and mediated bargaining procedures that either lead to an efficient voluntary resolution to public good problems, or achieve the efficient outcome but slightly coerce the agents into participation. In this respect, we identify "efficient" property rights structures.

1. INTRODUCTION

In this paper we show that in public good problems under asymmetric information, the success of any voluntary bargaining procedure is closely related to the structure of property rights in the economy. For clarity of exposition, we focus our attention on the following example: in a town inhabited by n residents, a firm is considering whether to operate a factory that generates pollution as an inevitable by-product. A higher level of production generates a correspondingly higher level of pollution. The firm's profits are increasing with the level of production, and the welfare of the residents is decreasing with the level of pollution. The firm's profitability and residents' preferences are assumed to be private information. The structure of property rights specifies the level of pollution that the firm is entitled to emit into the environment.[1]

For every such economy, we describe and prove the existence of "efficient" property rights structures. We characterize property rights structures and voluntary mediated bargaining procedures that maximize the sum of residents' utilities and the firm's profit and thus lead to an efficient resolution to the public good problem. Furthermore, we show that these efficient property rights structures are robust to changes in the underlying environment. This suggests that the combination of efficiency-minded legislation that would set the initial structure of property rights appropriately and a mediator who would base his recommendations on an efficient decision mechanism could facilitate the achievement of the efficient outcome. Under some circumstances, however, we show that the mediated voluntary bargaining procedure has to be replaced by compulsory arbitration that coerces agents into agreement. We provide results that show that when the "efficient" property rights structure is adopted, the degree of coercion that is necessary to obtain the efficient outcome is "small". Thus, in contrast to Rob (1989) and Mailath and Postlewaite

1. Our results apply to any public good problem where the costs and benefits of providing the public good are monotonically increasing in the level of provision and where some additional restrictions, to be specified later, are imposed on the form of the utility and profit functions and on the information structure.

(1990) that show that non coercive bargaining over public good provision fails miserably in large economies, we show that public good problems can be *efficiently* solved in any economy through a judicious allocation of property rights and reliance on efficient mediation or arbitration.

Two examples in private goods environments where problems of inefficient voluntary bargaining were solved through a manipulation of property rights were discussed by Samuelson (1985) and Cramton, Gibbons and Klemperer (1987).[2] In a simple model with an "upstream" pollutant firm and a "downstream" firm, Samuelson (1985) suggests that in order to get efficiency, firms should bid for the property rights for pollution. He shows that an auction for the property rights where the firm that submits the higher bid wins the right to pollute and pays the other firm half of the average bid is efficient. Cramton, Gibbons and Klemperer (1987) show that a partnership can be dissolved efficiently if the initial individual shares of its partners are approximately the same size. The intuition behind their result is similar to ours. A partner with a large share of the partnership has strong property rights that need to be weakened to induce him to participate in an efficient, incentive compatible, voluntary bargaining process.

The rest of the paper follows the following plan: in the next section, we demonstrate how the structure of property rights affects the process of voluntary bargaining vis-à-vis individuals' incentives. We present three property rights structures and discuss their efficiency properties. In Section 3, we characterize efficient property rights structures and voluntary bargaining procedures and discuss the issue of coercion. All proofs are relegated to the Appendix.

2. PROPERTY RIGHTS

A property right, in law and economics alike, is the right to exclude everyone else from the use of some scarce resource. Ordinarily, property rights are tradable, they can be bought, sold, or exchanged for other property rights. Since holders of property rights may also refuse to trade them, any *voluntary* bargaining procedure that involves a reallocation of property rights must guarantee property rights holders a level of utility that is at least as high as their utility when they refuse to trade their right. To clarify the role that property rights play in our analysis, we return to the example of the production facility which, if operated, generates pollution. We describe three different property rights structures, and discuss the incentives they induce.

Suppose first that the firm has the privilege to pollute (*i.e.* it holds the right for a clean environment). Because the firm is entitled to produce and pollute, if the town's residents want the firm to reduce the level of pollution they have to compensate it for its corresponding loss of production profits. Any reasonably efficient voluntary scheme to compensate the firm must require those individuals who suffer more from pollution to contribute more towards the compensation of the firm. However, under asymmetric information, the degrees of aversion to pollution are not publicly known, and therefore each individual resident has an incentive to understate his aversion to pollution so that he can pay less and "free-ride" on others' contributions. The analysis in Mailath and Postelwaite (1990) shows that as the number of residents increases, the tendency to understate one's aversion to pollution leads to an inability to compensate the firm. The intuition for this result is that each resident faces the following trade-off: either report the truth and pay accordingly, or lie and pay less, but increase the risk that the residents will not succeed in

2. See also the discussion in Myerson (1989, pp. 198–199).

raising enough money to compensate the firm and a higher level of production and pollution will follow. As the number of residents increases, the risk that a single resident faces by misreporting his true valuation diminishes. On the other hand, reporting his true valuation still costs him the same. Therefore, when the number of residents gets large, the latter effect dominates, and eventually all the residents have an incentive to understate their true valuation.

Consider now a second property rights structure where the town's residents, individually, have the right for a clean environment. This implies that the town's residents can freely insist that no production would take place. If the firm wants to produce and pollute, it has to compensate each and every resident. Under this property rights structure, the town's residents have an incentive to overstate their aversion to pollution so that they can get a higher compensation. The analysis in Rob (1989) shows that when the number of residents increases, the tendency of the town's residents to overstate their aversion to pollution prevents production, because the firm cannot compensate them and still make a profit.

The fact that when the property rights for a clean environment belong to the town's residents they have an incentive to report higher valuations for no-pollution, and when the property rights belong to the firm the residents have an incentive to report lower valuations for no-pollution, suggests that an "intermediate" rights structure may provide the residents with the appropriate countervailing incentives to report the truth. We show that this indeed is the case below.

Suppose that the level of production ranges between 0 and 1. The third property rights structure that we consider is where $\gamma \in (0, 1)$ of the property rights belong to the firm, and $1-\gamma$ belong to the town's residents. Thus, the town's residents have the right to insist that the level of production does not exceed γ and the firm has the right to produce at any level up to γ. In the next section we show that this property rights structure may lead to the efficient level of production and pollution through voluntary bargaining among the residents and the firm for appropriately chosen γ's.

The intuition for the result is the following. Efficient voluntary bargaining must succeed in inducing agents to truthfully reveal their private information. This "incentive compatibility" constraint uniquely determines the relationship between agents' valuations and their expected payments. (See Proposition 1 below.) If payments are higher, agents would rather claim that they prefer to pay less and will willingly suffer the associated decrease in the quantity of the public good. Similarly, they will claim that they prefer to pay more and have more of the public good if their payments are too low. If, for some agents, the expected utilities or profit (after the incentive compatible payments are taken into account) happen to be lower than those guaranteed by their property rights, the voluntary bargaining procedure fails. Thus, for the voluntary bargaining to succeed, no agent can have property rights that are too strong. This is achieved by choosing an appropriate intermediate property rights structure.

3. EFFICIENT PROPERTY RIGHTS

3.1. *Assumptions and definitions*

We present the model and discuss the results in terms of the example of the production facility that generates pollution. The economy consists of $n+1$ agents, n individuals or residents and a firm. A decision about the level of production is to be reached. The firm can operate the factory at any level of production $\kappa \in [0, 1]$. Production entails pollution, and a higher level of production generates a higher level of pollution. Individual i's utility is given by the function $u_i(\kappa, v_i)+t$ where $v_i \in [\underline{v}_i, \bar{v}_i]$, $0 \leqq v_i \leqq \bar{v}_i \leqq \infty$, referred to as i's type, determines the benefit individual i derives from less pollution and t is the (net) monetary transfer made to individual i. We assume that $u_i(\kappa, v_i) = \phi_i(\kappa) \cdot v_i$ where ϕ_i is non negative, continuous, decreasing and concave for all $i \in \{1, \ldots, n\}$.[3] We further assume that individuals seek to maximize their expected utilities.

The firm's profit from producing the quantity $\kappa \in [0, 1]$ and receiving a monetary transfer t is given by the function $\pi(\kappa, r)+t$ where $r \in [\underline{r}, \bar{r}]$, $0 \leqq \underline{r} \leqq \bar{r} \leqq \infty$, referred to as the firm's type, determines the firm's profitability and $\pi(\kappa, r) = \psi(\kappa) \cdot r$ where ψ is non negative, continuous, increasing, and concave. The firm's objective is to maximize its expected profit.

Individuals' utility functions and the firms profit function are commonly known. In addition, each individual knows his type v_i, and the firm knows r. None of the agents knows the types of the other agents. We assume that all the agents believe that other agents' types are independent random variables $V_1, \ldots, V_n, R$ with distributions $F_1, \ldots, F_n, G$, respectively. All the distributions are assumed to be differentiable and increasing. The collection $e = \langle \{u_i\}_{i=1}^n, \pi, \{F_i\}_{i=1}^n, G \rangle$ describes a public good economy e.

Because the individuals utilities and the firm's profit are quasi-linear, Pareto efficiency requires the maximization of the sum of individuals' utilities and the firm's profit. For every vector of agents' types $(v, r) = (v_1, \ldots, v_n, r)$, let $\kappa^*(v, r) \in \arg\max_{\kappa \in [0,1]} \{\pi(\kappa, r) + \sum_{i=1}^n u_i(\kappa, v_i)\}$ denote the efficient production rule.[4] It is straightforward to verify that κ^* is non decreasing in r and non increasing in v.

The total welfare in the economy is given by

$$W(v, r) = \pi(\kappa^*(v, r), r) + \textstyle\sum_{i=1}^n u_i(\kappa^*(v, r), v_i).$$

By the revelation principle (see, *e.g.* Myerson (1985)), any outcome that is obtained as a result of a bargaining process among the agents can also be obtained as an equilibrium outcome of an incentive compatible direct revelation mechanism. We shall therefore use the terms "bargaining procedure" and "mechanism" interchangeably. It is also important to note that the suggestiveness of our normative results rely on interpreting the mechanism as representing the recommendation of a mediator, or the decision of an arbitrator or a court.

A direct revelation mechanism is described by $n+2$ functions: a decision rule $\delta: \prod_{i=1}^n [\underline{v}_i, \bar{v}_i] \times [\underline{r}, \bar{r}] \to [0, 1]$ that specifies the level of provision of the public good as a function of the vector of agents' reports of their types to the mediator; and $n+1$ monetary transfer functions $\tau = (\tau_0, \tau_1, \ldots, \tau_n)$ where $\tau_0: \prod_{i=1}^n [\underline{v}_i, \bar{v}_i] \times [\underline{r}, \bar{r}] \to \boldsymbol{R}$ specifies the monetary transfer to the firm and $\tau_i: \prod_{i=1}^n [\underline{v}_i, \bar{v}_i] \times [\underline{r}, \bar{r}] \to \boldsymbol{R}$ specifies the monetary transfer to individual i as a function of the vector of agents' reports.

A direct revelation mechanism $\langle \delta, \tau \rangle$ is incentive compatible if it induces agents to truthfully report their private information. That is,

Definition. A mechanism $\langle \delta, \tau \rangle$ is incentive compatible if for all $i \in \{1, \ldots, n\}$, v_i, $\hat{v}_i \in [\underline{v}_i, \bar{v}_i]$, and $r, \hat{r} \in [\underline{r}, \bar{r}]$

$$E[u_i(\delta(V_{-i}, v_i, R), v_i) + \tau_i(V_{-i}, v_i, R)] \geqq E[u_i(\delta(V_{-i}, \hat{v}_i, R), v_i) + \tau_i(V_{-i}, \hat{v}_i, R)],$$

3. The assumption of multiplicative separability allows us to obtain an explicit characterization of efficient property rights in terms of an interval and simplifies the analysis. It is not necessary. The same applies to the profit function of the firm.

4. In the cases where κ^* is not uniquely defined, choose κ^* to maximize residents' utilities. The results do not depend on which κ^* is chosen.

and

$$E[\pi(\delta(V, r), r) + \tau_0(V, r)] \geqq E[\pi(\delta(V, \hat{r}), r) + \tau_0(V, \hat{r})].$$

The following mechanism properties are of interest to us: budget balance, *ex post* efficiency, and individual rationality.

Definition. A mechanism $\langle \delta, \tau \rangle$ is *ex post* budget balanced (feasible) if

$$\sum_{i=0}^{n} \tau_i(v, r) \underset{(\leq)}{=} 0 \quad \text{for every } (v, r) \in \prod_{i=1}^{n} [\underline{v}_i, \bar{v}_i] \times [\underline{r}, \bar{r}].$$

Ex post budget balance is desirable because if it is not satisfied, a mechanism is either infeasible when $\sum_{i=0}^{n} \tau_i(v, r) > 0$ or wasteful when $\sum_{i=0}^{n} \tau_i(v, r) < 0$. A weaker notion of budget balance is the following:

Definition. A mechanism $\langle \delta, \tau \rangle$ is *ex ante* budget balanced (feasible) if

$$\sum_{i=0}^{n} E[\tau_i(V, R)] \underset{(\leq)}{=} 0.$$

The notion of efficiency is defined as follows.

Definition. A mechanism $\langle \delta, \tau \rangle$ is (*ex post*) efficient if δ coincides with the efficient production rule κ^*.

We are obviously interested in bargaining procedures that are both efficient and budget balanced. Below, in the proof of Theorem 2, we show that the existence of an *ex ante* feasible and efficient mechanism implies the existence of an *ex post* budget balanced and efficient mechanism.

For each individual i, let $q_i(v_i) = E[\phi_i(\kappa^*(V_{-i}, v_i, R))]$ denote what individual i believes to be the expected level of pollution in the equilibrium where all the agents report their types truthfully and the efficient production rule is used to determine the level of pollution, and let $Q_i(v_i) = \int_{\underline{v}_i}^{v_i} q_i(\theta) d\theta$. Similarly, for the firm, let $q_0(r) = E[\psi(\kappa^*(V, r))]$ denote what the firm believes is its expected profitability in the equilibrium where all the agents report their types truthfully and an efficient production rule is used to determine the level of production and let $Q_0(r) = \int_{\underline{r}}^{r} q_0(\theta) d\theta$. It can be shown that $Q_i(v_i) = E[W(V, R) | V_i = v_i] - E[W(V, R) | V_i = \underline{v}_i]$ for the residents and $Q_0(r) = E[W(V, R) | R = r] - E[W(V, R) | R = \underline{r}]$ for the firm.[5] That is, Q_j, $j \in \{0, 1, \ldots, n\}$, is equal to agent j's expectation of the total welfare in the economy given his valuation, up to a constant.

3.2. *Characterization of efficient property rights*

The following proposition characterizes *ex post* efficient and Bayesian incentive compatible mechanisms.

5. To see this note that by the envelope theorem, both $Q_i(v_i)$ and $E[W(V, R) | V_i = v_i]$ are continuous functions with a derivative equal to $q_i(v_i)$. The same holds for the firm.

Proposition 1. *Suppose that δ is an efficient production rule. The mechanism $\langle\delta, \tau\rangle$ is incentive compatible if and only if*

$$E[\tau_i(V_{-i}, v_i, R)] = Q_i(v_i) - v_i q_i(v_i) + K_i \qquad \forall i \in \{1, \ldots, n\}, \forall v_i \in [\underline{v}_i, \bar{v}_i],$$

and (1)

$$E[\tau_0(V, r)] = Q_0(r) - r q_0(r) + K_0 \qquad \forall r \in [\underline{r}, \bar{r}],$$

where the K_i's are arbitrary constants.

Hence efficient and Bayesian incentive compatible mechanisms give residents an expected utility of $Q_i(v_i) + K_i$ and an expected profit of $Q_0(r) + K_0$ to the firm. To see the intuition for the proposition, recall that $Q_i(v_i) = E[W(V, R)|V_i = v_i] - E(W(V, R)|V_i = \underline{v}_i]$ for the individuals and that a similar relationship holds for the firm. That is, the mechanisms identified above induce truthful reporting by giving agents (expected) transfers that are equal to the difference between agents' expectations of the total welfare and their utility or profit given their valuation, up to a constant. This "aligns" agents' incentives with those of society and induces truth-telling.[6]

We now turn to define the individual rationality constraints. As we have explained in the previous section, when agents bargain voluntarily they may always refuse to trade their property rights if they are dissatisfied with what they get in return. Therefore, voluntary bargaining must satisfy individual rationality constraints that are determined by the structure of property rights in the economy. We assume that the structure of property rights is such that each individual has the right to insist that the firm does not operate its factory at a production level higher than $\gamma \in [0, 1]$, and the firm has the right to operate its factory at any level lower or equal to γ. Thus, every individual is guaranteed to enjoy a utility level which is not lower than his utility when he suffers from a pollution level of γ but retains his full income, and the firm is guaranteed a profit level of at least $\pi(\gamma, r)$.

Definition. A mechanism $\langle\delta, \tau\rangle$ is (interim) individually rational with respect to a property rights structure γ if

$$E[u_i(\delta(V_{-i}, v_i, R), v_i) + \tau_i(V_{-i}, v_i, R)] \geqq u_i(\gamma, v_i) \qquad \forall i \in \{1, \ldots, n\}, \forall v_i \in [\underline{v}_i, \bar{v}_i],$$

and

$$E[\pi(\delta(V, r), r) + \tau_0(V, r)] \geqq \pi(\gamma, r) \qquad \forall r \in [\underline{r}, \bar{r}].$$

Straightforward manipulation of the individual rationality constraints shows that an efficient bargaining procedure $\langle\kappa^*, \tau\rangle$ where τ satisfies (1) is individually rational for the firm with respect to a property rights structure γ if and only if $\gamma \leqq \min_{r \in [\underline{r}, \bar{r}]} \{\psi^{-1}((Q_0(r) + K_0)/r)\}$; it is individually rational for the individuals if and only if $\gamma \geqq \max_{v_i \in [\underline{v}_i, \bar{v}_i]} \{\phi_i^{-1}((Q_i(v_i) + K_i)/v_i)\}$ for all $i \in \{1, \ldots, n\}$. By Proposition 1, we can identify every efficient and incentive compatible mechanism with an $n+1$-tuple vector $(K_0, \ldots, K_n)$. Define the function $\Gamma: \boldsymbol{R}^{n+1} \rightarrow$ as,

$$\Gamma(K_0, \ldots, K_n) = \min_{r \in [\underline{r}, \bar{r}]} \left\{\psi^{-1}\left(\frac{Q_0(r) + K_0}{r}\right)\right\} - \max_{i \in \{1, \ldots, n\}} \left\{\max_{v_i \in [\underline{v}_i, \bar{v}_i]} \left\{\phi_i^{-1}\left(\frac{Q_i(v_i) + K_i}{v_i}\right)\right\}\right\}.$$

6. It is straightforward to show that mechanisms that satisfy (1) are subjectively discretionary (d'Aspremont and Gerard Varet (1979)) or "Groves in expectations" (Makowski and Mezzetti (1994)) and hence coincide with the class of mechanisms identified by d'Aspremont and Gerard Varet (1979) and Arrow (1979) for the class of economies studied here.

Thus, a property rights structure γ may give rise to voluntary efficient bargaining only if there exists a mechanism, or a non negative vector $(K_0, \ldots, K_n)$, such that $\Gamma(K_0, \ldots, K_n) \geqq 0$. To check whether such a mechanism, that in addition is also feasible, exists, denote the value of the following maximization problem by S. $S = \max_{K_0, \ldots, K_n \geq 0} \Gamma(K_0, \ldots, K_n)$ subject to the (*ex ante*) feasibility constraint $\sum_{i=0}^{n} K_i \leqq E[Rq_0(R) - Q_0(R)] + \sum_{i=1}^{n} E[V_i q_i(V_i) - Q_i(V_i)]$. The solution to this maximization problem identifies a family of *voluntary* and efficient bargaining procedures (namely, the family of mechanisms $\langle \kappa^*, \tau \rangle$ where τ satisfies (1) and the K_i's are given by the solution) that satisfy the feasibily constraint.

Theorem 2. *Consider a public good economy* $e = \langle \{u_i\}_{i=1}^n, \pi, \{F_i\}_{i=1}^n, G \rangle$.

(a) *If* $S \geqq 0$ *then there exists a mechanism that is efficient, ex post budget balanced, Bayesian incentive compatible, and interim individually rational with respect to any property rights structure*

$$\max_{i \in \{1, \ldots, n\}} \left\{ \max_{v_i \in [\underline{v}_i, \bar{v}_i]} \left\{ \phi_i^{-1} \left(\frac{Q_i(v_i) + K_i}{v_i} \right) \right\} \right\} \leqq \gamma \leqq \min_{r \in [\underline{r}, \bar{r}]} \left\{ \psi^{-1} \left(\frac{Q_0(r) + K_0}{r} \right) \right\} \qquad (2)$$

where the vector $(K_0, \ldots, K_n)$ *is determined by the mechanism.*

(b) *Conversely, if* $S < 0$ *or* γ *does not satisfy* (2) *for any non negative vector* $(K_0, \ldots, K_n)$, *then there does not exist an efficient, ex ante feasible, Bayesian incentive compatible, and interim individually rational mechanism for the economy e.*

A possible criticism of this result is that the efficient property rights and voluntary bargaining mechanism depend on the parameters of the problem such as the agents' utility and profit functions. In addition, having modelled the bargaining process as a Bayesian game, the efficient property rights and mechanism depend on the prior that describes agents' beliefs about each other's valuations. While a mediator may be more flexible in adjusting his recommendation to specific bargaining environments, it is more difficult to believe that legislators would be capable of establishing different property rights structures for different environments. However, the fact that when efficient property rights structures exist they can be described by an interval of γ's implies that efficient property rights structures are robust to (small) changes in the underlying environment, that is, to changes in both agents' utility and profit functions and beliefs. Thus, it is sufficient that legislators specify property rights structures for a finite number of "broad classes" of bargaining environments.

Theorem 2 also has some positive implications. Specifically, performing comparative statics on the interval of efficient property rights identified in (2) reveals that they respond positively to agents' utilities (*i.e.* an increase in individuals aversion to pollution implies that their (efficient) right for clean air should be strengthened). This suggests that one may interpret, say, the changes in environmental regulation in the last 30 years in the U.S. as an efficient adjustment of property rights in response to the increasing awareness of the dangers of industrial pollution.

Example. Consider an economy with fifteen residents. Residents' utilities from no-pollution are given by the function $(1-\kappa)^{\alpha} \cdot v$ and the firm's net profit is given by the function $\kappa^{\alpha} \cdot r$. The agents' valuations are independently drawn from a chi-square distribution where residents' distributions have one degree of freedom, and the firm's distribution has fifteen degrees of freedom. In the case where $\alpha = 0{\cdot}7$ for example, the vector $(K_0, \ldots, K_{15})$ that maximizes Γ subject to the budget balance constraint is given by $(5{\cdot}627, 0{\cdot}019, \ldots, 0{\cdot}019)$ and for every $\gamma \in [0{\cdot}531, 0{\cdot}538]$ the mechanism specified in the proof of part (a) of the previous theorem is efficient, *ex post* budget balanced, incentive compatible, and interim individually rational with respect to γ. The size of the efficient property rights intervals depends on the curvature of the agents' utilities and profit. In this example, it is increasing with α. When $\alpha = 0{\cdot}6$, for example, $S = -0{\cdot}0005$ and no efficient property rights structure exist. The intuition for this result is that more concavity (*i.e.* lower α) implies that κ^* is less responsive to agents' valuations. The agents therefore must get a higher "premium" to maintain incentive compatibility. Because all agents must receive this premium, it may sometimes happen that the mechanism cannot break even, and some agents receive a lower utility/profit than what their property rights give them and consequently refuse to participate in the bargaining.[7]

The previous theorem shows that the condition $S \geqq 0$ characterizes those economies where efficient property rights and mechanisms exist. Obtaining a more straightforward characterization of this condition is desirable. While we cannot obtain such a characterization in general, we are able to do so for the case where agents' utilities and profits are linear and one agent's valuation is public information. In particular, in the case where residents utilities are identical and are given by $\kappa \cdot r$, and residents' beliefs about each other are identical, Proposition 4 in the appendix shows that the condition $S \geqq 0$ is equivalent to the following simpler condition,

$$\frac{1}{n} E\{\max\{\textstyle\sum_{i=1}^{n} V_i, r\}\} - \frac{r}{n} \geqq E[W(V,R)] - E[W(V,R) \mid V_i = r/n]. \tag{3}$$

The intuition for condition (3) is the following. The L.H.S. of inequality (3) is the difference in expected welfare per individual that is generated by adopting the efficient decision rule instead of letting the firm pollute. The R.H.S. is the difference between the expected welfare and the expected welfare conditional on individual i having a valuation that is equal to r/n. Note that because the L.H.S. of (3) is non negative, constraint (3) becomes less binding the smaller ("on average") V_i is in comparison to r/n. When V_i is "large on average" in relation to r/n, production and pollution is inefficient, and the residents, realizing that an efficient decision is likely to mandate no pollution, have stronger incentives to free ride. Thus, whether efficient property rights and voluntary bargaining procedures exist or not depends on the relationship between the welfare per person generated by adopting the efficient decision (L.H.S. of (3)) and the severity of the free-rider problem as measured by the R.H.S. of (3).

Theorem 2 characterizes economies where efficient property rights and voluntary bargaining procedures exist. In other economies, efficient bargaining procedures still exist but they potentially violate some agents' property rights. That is, some agents may need to be coerced into participating in the efficient bargaining process. yet, we show that when the property rights structure is appropriately chosen (so as to maximize S) the degree of coercion that is needed is "small." Thus, whereas until now we focused on voluntary bargaining, henceforth we assume that any agent may force the others into bargaining with him by exercising a right to compulsory arbitration where the arbitrator uses an efficient decision rule that satisfies (1).

7. The example suggests that less concavity has a positive effect on the sizes of the efficient property rights intervals and more importantly on their existence in more general settings as well. We have been unable to prove this intuition correct.

We define the degree of coercion that is imposed by such an arbitrator on an agent as the difference between the R.H.S. and the L.H.S. of the agent's individual rationality constraint.

Definition. Given a property rights structure $\gamma \in [0, 1]$, the degree of coercion that is imposed on individual $i \in \{1, \ldots, n\}$ with type v_i by the bargaining procedure $\langle \delta, \tau \rangle$ is

$$\max\{u_i(\gamma, v_i) - E[u_i(\delta(V_{-i}, v_i, R), v_i) + \tau_i(V_{-i}, v_i, R)], 0\},$$

and the degree of coercion that is imposed on the firm with type r is

$$\max\{\pi(\gamma, r) - E[\pi(\delta(V, r), r) + \tau_0(V, r)], 0\}.$$

The total degree of coercion that is imposed on all the agents in the economy by the bargaining procedure $\langle \delta, \tau \rangle$ is given by the sum of the degrees of coercion that are imposed on the individuals and the firm.

We make two observations. First, in spite of the fact that the agents' type spaces need not be bounded, it can be shown that for every $\gamma \in (0, 1)$ the maximal degree of cercion that has to be imposed on each single agent in order to achieve efficiency when the bargaining procedure $\langle \delta, \tau \rangle$ is such that δ is an efficient decision rule and τ satisfies (1) is bounded.

More interestingly, we show that under more restrictive assumptions, the total degree of coercion that needs to be imposed on all the agents *together* converges to zero exponentially fast as the economy gets large. Assume henceforth that the valuation of the firm $r > 0$ is commonly known and that $\psi(\kappa) = \kappa$ for all $\kappa \in [0, 1]$. Residents' utility functions are identical and are given by $\phi(\kappa) \cdot v_i$ where $\phi(\kappa) = 1 - \kappa$. The residents believe that other residents' types are given by i.i.d. random variables $V_1, \ldots, V_n$ that are distributed according to a differentiable and increasing distribution function F such that $\underline{v}_i = 0$ for all $i \in \{1, \ldots, n\}$. As before, each resident still knows his own valuation. Because all individuals are identical, $q_i(v_i) = \Pr(v_i + \sum_{j \neq i} V_j \geq r)$ is independent of i, but depends on the number of residents n. To emphasize this fact and minimize notation, we denote it by q^n. Such economies can be described by the vector $\langle n, F, r \rangle$.

It can be shown that under these more restricted assumptions, the residents need not be coerced into participating in the bargaining process and the degree of coercion to which the firm needs to be subjected is small enough for the firm to still enjoy positive profits. Moreover, the next proposition shows that the maximal degree of coercion the firm has to be subjected to in order to guarantee efficiency and hence the total level of coercion that is imposed on all the agents in the economy becomes negligible in large economies.

Proposition 3. *Consider a sequence of public good economies* $\{\langle n, F, r_n \rangle_n\}$ *such that* $\lim_{n \to \infty} r_n / n = r$, $E[V_i] \neq r$, *and* $r_n \geq \bar{v}_1$ *for large enough n. If the property rights structures in these economies are* $\gamma_n = 1 - q^n(0)$, *respectively, then, when the number of residents is sufficiently large, the total degree of coercion required to achieve the efficient outcome is bounded from above by* $2rne^{-nC}$ *where C is a positive constant.*

The necessity of sometimes using coercion to achieve efficiency highlights the fact that if property rights are not defined at all (*i.e.* agents have zero on the R.H.S. of their individual rationality constraints), and disputes are settled *via* compulsory arbitration, it is much easier to achieve the efficient outcome. Any mechanism that utilizes an efficient decision rule and mandates a scheme of transfers that satisfies (1) will generate the efficient outcome. Barzel (1989) and North (1990) have observed that, sometimes, property rights are not fully delineated and argued that this is because the transaction costs involved in measuring, monitoring, and enforcing them may be higher than the benefits of doing so. Here, we present a different rational. Namely, the transaction costs that are associated with fully delineating property rights are due to the fact that doing so encumbers the process of voluntary bargaining. This insight is also the (implicit) motivation behind the recent interest in law and economics literature in the relative performance of liability rules and property rules.[8] (See, *e.g.* Kaplow and Shavell, 1996 and Ayres and Balkin, 1996). In fact, our results imply that the superiority of liability rules over the more rigid property rules in situations that involve externalities is due to the superior performance of compulsory procedures for settling disputes over the establishment of rights followed by voluntary bargaining.

APPENDIX

Proof of Proposition 1.

The proof employs a standard technique in mechanism design literature. It follows Myerson (1981) and Myerson and Satterthwaite (1983). The fact that the q_i's, $i\in\{0,\dots,n\}$, are non decreasing functions implies that (1) implies incentive compatibility. We prove the other direction. Incentive compatibility implies that,

$$-v_i(q_i(\hat v_i)-q_i(v_i))\leq E[\tau_i(V_{-i},\hat v_i,R)]-E[\tau_i(V_{-i},v_i,R)]\leq -\hat v_i(q_i(\hat v_i)-q_i(v_i)),$$

for all $i\in\{1,\dots,n\}$ and $v_i,\hat v_i\in[\underline{v}_i,\bar v_i]$. When $\hat v_i \gtrless v_i$,

$$-\frac{v_i(q_i(\hat v_i)-q_i(v_i))}{\hat v_i-v_i}\underset{(\geq)}{\leq}\frac{E([\tau_i(V_{-i},\hat v_i,R)]-E[\tau_i(V_{-i},v_i,R)]}{\hat v_i-v_i}\underset{(\geq)}{\leq}-\frac{\hat v(q_i(\hat v_i)-q_i(v_i))}{\hat v_i-v_i}.$$

Letting $\hat v_i\to v_i$, this implies that the derivative of $E[\tau_i(V_{-i},v_i,R)]$ with respect to v_i equals $-v_iq_i'(v_i)$ for all $i\in\{1,\dots,n\}$ and $v_i\in[\underline{v}_i,\bar v_i]$ whenever q_i is differentiable, which because of differentiability of $F_1,\dots,F_n,G$ is everywhere on $[\underline{v}_i,\bar v_i]$. Integrating both sides, we obtain, $E[\tau_i(V_{-i},v_i,R)]=Q_i(v_i)-v_iq_i(v_i)+K_i$ for all $i\in\{1,\dots,n\}$ and $v_i\in[v_i,\bar v_i]$ where K_i is an arbitrary constant. A similar proof establishes this fact for the firm. ||

Proof of Theorem 2.

Proposition 1 in conjunction with the revelation principle (see, for example, Myerson (1985)) imply that there is no loss of generality in restricting our attention to mechanisms $\langle\kappa^*,\tau\rangle$ where τ satisfies (1). Consider a mechanism $\langle\kappa^*,\tau\rangle$ where τ is given by (1) and is *ex ante* budget balance, that is $\sum_{i=0}^n K_i=E[Rq_0(R)-Q_0(R)]+\sum_{i=1}^n E[V_iq_i(V_i)-Q_i(V_i)]$. In the truth-telling equilibrium, the firm's interim expected profit is $Q_0(r)+K_0$. Interim individual rationality is satisfied for the firm if and only if $Q_0(r)+K_0\geq\psi(\gamma)r$ for all $r\in[\underline{r},\bar r]$, if and only if

$$\gamma\leq\min_{r\in[\underline{r},\bar r]}\left\{\psi^{-1}\left(\frac{Q_0(r)+K_0}{r}\right)\right\}.$$

Similarly, in the truth-telling equilibrium, individual i with valuation v_i enjoys an interim utility of $Q_i(v_i)+K_i$. $\langle\kappa^*,\tau\rangle$ is interim individually rational for the residents if and only if $Q_i(v_i)+K_i\geq\phi_i(\gamma)v_i$ for all $i\in\{1,\dots,n\}$ and $v_i\in[\underline{r}_i,\bar v_i]$, if and only if

$$\gamma\geq\max_{v_i\in[v_i,\bar v_i]}\{\phi_i^{-1}((Q_i(v_i)+K_i)/v_i)\}$$

Thus, $\langle\kappa^*,\tau\rangle$ is *ex ante* budget balanced and interim individually rational with respect to γ if and only if there exists a vector $(K_0,\dots,K_n)$ such that $\sum_{i=0}^n K_i=E[Rq_0(R)-Q_0(R)]+\sum_{i=1}^n E[V_iq_i(V_i)-Q_i(V_i)]$, and

$$\max_{i\in\{1,\dots,n\}}\left\{\max_{v_i\in[\underline{v}_i,\bar v_i]}\left\{\phi_i^{-1}\left(\frac{Q_i(v_i)+K_i}{v_i}\right)\right\}\right\}\leq\gamma\leq\min_{r\in[\underline{r},\bar r]}\left\{\psi^{-1}\left(\frac{Q_0(r)+K_0}{r}\right)\right\},$$

8. Property rules prohibit nonconsensual takings; liability rules permit nonconsensual takings in return for payment of damages. (Ayres and Balkin (1996), p. 704).

if and only if $S \geq 0$. Denote the solution to the problem of maximizing S subject to the budget balance constraint by $(K_0^*, \ldots, K_n^*)$. Note that because Γ is increasing in $(K_0, \ldots, K_n)$, $\sum_{i=0}^n K_i^* = E[Rq_0(R) - Q_0(R)] + \sum_{i=1}^n E[V_i q_i(V_i) - Q_i(V_i)]$. To complete the proof consider the mechanism $\langle \kappa^*, \tau^* \rangle$ where

$$\tau_i^*(v, r) = Q_i(v_i) - v_i q_i(v_i) + \frac{1}{n}(rq_0(r) - Q_0(r)) - \frac{1}{n} E[Rq_0(R) - Q_0(R)] + K_i^* \qquad \forall i \in \{1, \ldots, n\},$$

$$\tau_0^*(v, r) = Q_0(r) - rq_0(r) + \textstyle\sum_{i=1}^n (v_i q_i(v_i) - Q_i(v_i)) - \sum_{i=1}^n E[V_i q_i(V_i) - Q_i(V_i)] + K_0^*.$$

It is straightforward to verify that the constants induced by $\langle \kappa^*, \tau^* \rangle$ are equal to $(K_0^*, \ldots, K_n^*)$, and that τ^* satisfies (1) and *ex post* budget balance. ||

Proof of Proposition 3.

Suppose that the arbitrator relies on the mechanism $\langle \kappa^*, \tau^D \rangle$ where $\tau_i^D(v, r) = W(v, r) - u_i(\kappa^*(v, r), v_i) - E[W(V, r) | V_i = 0]$ for every $i \in \{1, \ldots, n\}$, and $\tau_0^D(v, r) = -\sum_{i=1}^n \tau_i^D(v, r)$. It can be verified that $\langle \kappa^*, \tau^D \rangle$ satisfies (1), *ex post* budget balance, and that it induces truth-telling as a dominant strategy for the residents. The fact that $\gamma_n = 1 - q^n(0)$ together with $E[W(V, r) | V_i = v_i] - E[W(V, r) | V_i = 0] = Q^n(v_i) \geq q^n(0) v_i$ implies that this mechanism is individually rational for the residents. The degree of coercion that has to be imposed on the firm to guarantee efficiency is given by $\gamma_n r_n - E[\max\{\sum_{i=1}^n V_i, r_n\}] + nE[Q^n(V_i)]$. We show that the last expression converges to zero at a rate ne^{-nC} where $C > 0$. (the proof is adapted from the theory of large deviations, see, *e.g.* Durrett (1991), p. 59.)

Distinguish between two cases: (a) $E[V_i] > r$, and (b) $E[V_i] < r$. We prove the assertion for case (a) first. Note that because $\gamma_n = 1 - q^n(0)$ and $Q^n(v) \leq v$ for all $v \geq 0$, $\gamma_n r_n - E[\max\{\sum_{i=1}^n V_i, r_n\}] + nE[Q^n(V_i)] \leq r_n(1 - q^n(0))$. We show that $1 - q^n(0) \leq e^{-nC}$ for some $C > 0$. For all $\theta \geq 0$, let $\varphi(\theta) \equiv E[e^{-\theta V_1}]$ denote the moment generating function of $-V_1$. Applying Chebayshev's inequality (see Durrett (1991), p. 15) to $\varphi(\theta)$ we obtain

$$e^{-\theta r_n} \Pr(\textstyle\sum_{i=1}^{n-1} V_i \leq r_n) \leq \displaystyle\int_0^{r_n} \exp(-\theta \textstyle\sum_{i=1}^{n-1} V_i)\, dF_{n-1} \leq E[\exp(-\theta \sum_{i=1}^{n-1} V_i)] = \varphi(\theta)^{n-1},$$

where F_{n-1} is the cumulative distribution of $\sum_{i=1}^{n-1} V_i$. Upon rearranging this can be expressed as,

$$\Pr(\textstyle\sum_{i=1}^{n-1} V_i \leq r_n) \leq \exp\left[(n-1)\left(\dfrac{r_n}{n-1} r\theta + \log \varphi(\theta)\right)\right].$$

We now show that the last exponent is negative for an appropriate choice of θ (and large enough n).

Lemma 1. *If $E[V_1] > r$, there exists a $\theta > 0$ and a large enough N such that for all $n \geq N$,*

$$\frac{r_r}{n-1}\theta + \log \varphi(\theta) < 0.$$

Proof. Denote $f(\theta) = r_n \theta/(n-1) + \log \varphi(\theta)$. When $\theta = 0$, $f(\theta) = 0$. We show that the right-hand derivative of f at 0, $\lim_{h \to 0, h \geq 0} [(f(h) - f(0))/h]$ denoted $D^+ f(0)$ exists and is negative for large enough n. $d/d\theta[(r_n \theta/(n-1)] = r_n/(n-1)$ exists and is arbitrarily close to r for large enough n. Hence, it suffices to show that $\exists d^+ \log \varphi(0) < -r$. Now, $\varphi(0) = 1$. Therefore it suffices to show that $\exists d^+ \varphi(0) < -r$. However, $\varphi(\theta) = \int_0^\infty e^{-\theta v} dF(v)$. It follows from standard arguments (see Durrett (1959, p. 59) for details) that for $\theta > 0$, $\exists \varphi'(\theta) = -\int_{\underline{v}}^{\bar{v}} ve^{-\theta v} dF(v)$. Hence,

$$\exists D^+ \varphi(0) = \lim_{\substack{\theta \to 0 \\ \theta \geq 0}} \varphi'(\theta) = -\int_{\underline{v}}^{\bar{v}} v dF(v) = -E[V_1] < -r. \quad ||$$

Since $q - 1^n(0) = \Pr(\sum_{i=1}^{n-1} V_i \leq r_n)$, the lemma establishes the existence of a positive constant C and an integer N such that $1 - q^n(0) \leq e^{-nC}$ for all $n \geq N$. Hence, $r_n(q^n(\bar{v}) - q^n(0)) \leq r_n e^{-nC} \leq 2rne^{-nC}$ for large enough n.

The proof for case (b) is similar. $\gamma_n r_n - E[\max\{\sum_{i=1}^n V_i, r_n\}] + nE[Q^n(V_i)] \leq r_n - r_n + nE[Q^n(V_1)] \leq nE[V_1]q^n(\bar{v}) < r_n q^n(\bar{v})$. We prove that $q^n(\bar{v}) \leq e^{-nC}$ for some $C > 0$. For all $\theta \geq 0$, let $\psi(\theta) \equiv E[e^{\theta V_1}]$ denote the moment generating function of V_1. By applying Chebyshev's inequality to $\psi(\theta)$ we obtain

$$\exp(\theta(r_n-\bar{v}))\Pr(\textstyle\sum_{i=1}^{n-1}V_i\geqq r_n-\bar{v})\leqq E[\exp(\theta\textstyle\sum_{i=1}^{n-1}V_i)]=\psi(\theta)^{n-1},$$

or

$$\Pr(\textstyle\sum_{i=1}^{n-1}V_i\geqq r_n-\bar{v})\leqq\exp\left[(n-1)\left(\log\psi(\theta)-\frac{r_n-\bar{v}}{n-1}\theta\right)\right]$$

As before, we have,

Lemma 2. *If $E[V_1]<r$ there exists a $\theta>0$ and a large enough N such that for all $n\geqq N$, $\log\psi(\theta)-[(R(n)-\bar{v})/(n-1)]\theta<0$.*

Finally, since $q^n(\bar{v})=\Pr(\sum_{i=1}^{n-1}V_i\geqq r_n-\bar{v})$, the lemma establishes the existence of a positive constant C and an integer N such that $q^n(\bar{v})e^{-nC}$ for all $n\geqq N$. Hence, $r_n(q^n(\bar{v})-q(0))\leqq r_ne^{-nC}\leqq 2rne^{-nC}$ for large enough n in this case as well. ||

Proposition 4. *Consider a public good economy $e=\langle\{u_i\}_{i=1}^n,\pi,\{F_i\}_{i=1}^n,r\rangle$ where $u_i(\kappa,v_i)=(1-\kappa)\cdot v_i$, the firm's type r is publicly known, $\pi(\kappa,r)=\kappa\cdot r$, $F_i=F$ for all $i\in\{1,\ldots,n\}$ where F is continuous and increasing, $\underline{v}=0$ and $0<r<n\bar{v}$. Then, $S\geqq 0$ if and only if*

$$(1/n)E[\max\{\textstyle\sum_{i=1}^n V_i,r\}]-r/n\geqq E[W(V,R)]-E[W(V,R)\mid V_i=r/n].$$

Proof. Under the assumptions $\kappa^*(v,r)=1$ when $\sum_{i=1}^n v_i\geqq r$ and $\kappa^*(v,r)=0$ when $\sum_{i=1}^n v_i<r$. q_i is thus simplified to $q_i(v_i)=\Pr(v_i+\sum_{j\neq i}V_j\geqq r)$ and is independent of i. We henceforth omit the index i whenever possible.

$$q_0(r)=\Pr(\textstyle\sum_{i=1}^n V_i<r)\quad\text{and}\quad\min_{r\in[\underline{r},\bar{r}]}\left\{\psi^{-1}\left(\frac{Q_0(r)+K_0}{r}\right)\right\}=\frac{Q_0(r)+K_0}{r}.$$

We rely on the following fact. Its proof is straightforward and is omitted.

Fact. For $i\in\{1,\ldots,n\}$ if $K_i=v_iq(v_i)-Q(v_i)$ for some $v_i\in[\underline{v},\bar{v}]$ then

$$\max_{\theta\in[\underline{v},\bar{v}]}\left\{\phi^{-1}\left(\frac{Q(\theta)+K_i}{\theta}\right)\right\}=\phi^{-1}(q(v_i)).$$

If $K_i>\bar{v}q(\bar{v})-Q(\bar{v})$ then

$$\max_{\theta\in[\underline{v},\bar{v}]}\left\{\phi^{-1}\left(\frac{Q(\theta)+K_i}{\theta}\right)\right\}$$

is obtained at $\bar{v}$.

Because Γ is monotonically increasing in $(K_0,\ldots,K_n)$, no loss of generality is involved in assuming that $K_0=rq_0(r)-Q_0(r)+\sum_{i=1}^n E[V_iq_i(V_i)-Q_i(V_i)]-\sum_{i=1}^n K_i$. It follows that

$$\frac{Q_0(r)+K_0}{r}=\frac{1}{r}[rq_0(r)+\textstyle\sum_{i=1}^n E[V_iq_i(V_i)-Q_i(V_i)]-\sum_{i=1}^n K_i]$$

$$=\frac{1}{r}[E[\max\{\textstyle\sum_{i=1}^n V_i,r\}]-\sum_{i=1}^n E[Q_i(V_i)]-\sum_{i=1}^n K_i].$$

Therefore, $S\geqq 0$ if and only if

$$\frac{1}{r}E[\max\{\textstyle\sum_{i=1}^n V_i,r\}]-\sum_{i=1}^n E[Q_i(V_i)]-\sum_{i=1}^n K_i]-\max_{i\in\{1,\ldots,n\}}\left\{\max_{v_i\in[\underline{v},\bar{v}]}\left\{1-\frac{Q(v_i)+K_i}{v_i}\right\}\right\}\geqq 0.\qquad(4)$$

Because

$$\min_{i\in\{1,\ldots,n}\left\{\max_{v_i\in[\underline{v}_i,\bar{v}_i}\left\{1-\frac{Q_i(v_i)+K_i}{v_i}\right\}\right\}$$

requires setting $K_1 = \cdots = K_n$, the last inequality is equivalent to

$$\min_{v_i \in [\underline{v}, r]} \{q(v_i)[nv_i - r] - nQ(v_i)\} \leq E[\max\{\textstyle\sum_{i=1}^n V_i - r, 0\}] - nE[Q(V_1)].$$

Note that we have restricted our attention to K_i's such that $K_i = v_i q(v_i) - Q(v_i)$ for some $v_i \in [\underline{v}, \bar{v}]$. This restriction is without loss of generality. Smaller K_i's violate the non negativity constraint and larger K_i's need not be considered because if (4) is satisfied with $K_1 = \cdots = K_n = K' > \bar{v}q(\bar{v}) - Q(\bar{v})$ it is also satisfied with $K_1 = \cdots = K_n = \bar{v}q(\bar{v}) - Q(\bar{v})$ because by the fact above if $K_i > \bar{v}q(\bar{v}) - Q(\bar{v})$, $\max_{\theta \in [\underline{v}, \bar{v}]} \{\phi^{-1}([(Q(\theta) + K_i)/\theta])\}$ is obtained at $\bar{v}$ and because $n\bar{v} \geq r$, the L.H.S. of condition (4) with $K_1 = \cdots = K_n = \bar{v}q(\bar{v}) - Q(\bar{v})$ is larger or equal to the L.H.S. of (4) with $K_1 = \cdots = K_n = K' > \bar{v}q(\bar{v}) - Q(\bar{v})$.

Finally, $(d/dv_i)[q(v_i)[nv_i - r] - nQ(v_i)] = q'(v_i)(nv_i - r)$ and since $q'(v_i) > 0$ for all $v_i \in [\underline{v}, \bar{v}]$ the minimum is obtained where the derivative equals zero, namely at r/n, and equals $nQ(r/n)$. Upon rearranging, we obtain that

$$(1/n)E[\max\{\textstyle\sum_{i=1}^n V_i, r\}] - r/n \geq E[Q_i(V_i)] - Q_i(r/n).$$

To finish the proof, recall that $Q_i(v_i) = E[W(V, R) | V_i = v_i] - E[W(V, R) | V_i = \underline{v}_i]$. ||

Acknowledgements. I am grateful to Roger Myerson for suggesting the intuition that lead to the development of the ideas presented in this paper. I would also like to thank Enriqueta Aragonès, V. V. Chari, George Mailath, Steve Matthews, Dilip Mookherjee, Torben Tranæs, Amit Pazgal, and especially Asher Wolinsky and Itzhak Gilboa for their comments and suggestions. Two anonymous referees and a managing editor provided useful comments.

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