Shape Functions for Quadrilateral Elements

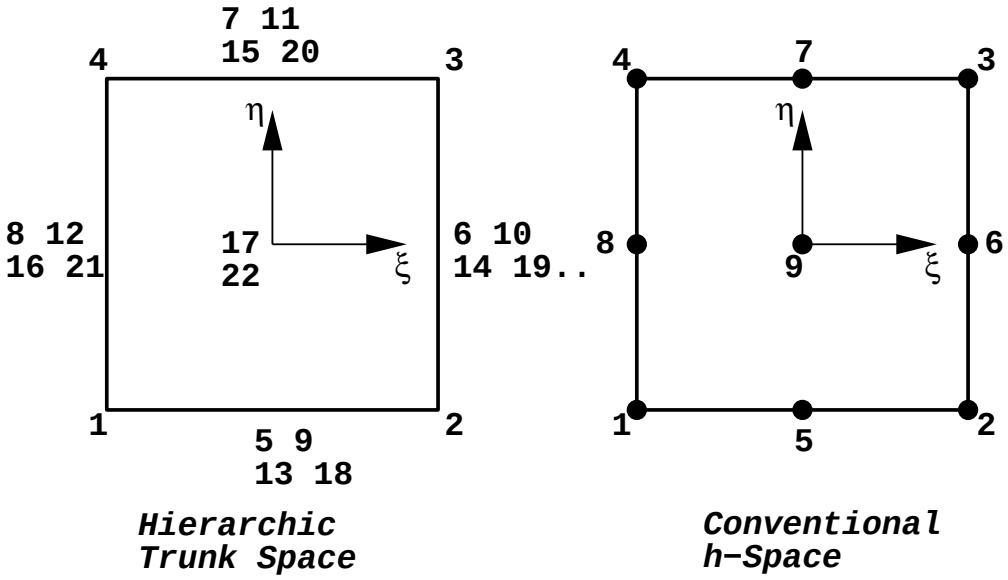

Figure 1: Standard Element and notation of shape functions.

Conventional Parabolic (second-order) h-Space

Serendipity (8-nodes)

Vertex

$$\Phi_1(\xi,\eta) = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$
$$\Phi_2(\xi,\eta) = -\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$
$$\Phi_3(\xi,\eta) = -\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$
$$\Phi_4(\xi,\eta) = -\frac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$

Edge

$$\Phi_5(\xi,\eta) = \frac{1}{2}\left(1-\xi^2\right)(1-\eta)$$
$$\Phi_6(\xi,\eta) = \frac{1}{2}(1+\xi)\left(1-\eta^2\right)$$
$$\Phi_7(\xi,\eta) = \frac{1}{2}\left(1-\xi^2\right)(1+\eta)$$
$$\Phi_8(\xi,\eta) = \frac{1}{2}(1-\xi)\left(1-\eta^2\right)$$

Product (9-nodes)

Vertex

$$\Phi_1(\xi,\eta) = \frac{1}{4}\xi\eta(\xi-1)(\eta-1)$$
$$\Phi_2(\xi,\eta) = \frac{1}{4}\xi\eta(\xi+1)(\eta-1)$$
$$\Phi_3(\xi,\eta) = \frac{1}{4}\xi\eta(\xi+1)(\eta+1)$$
$$\Phi_4(\xi,\eta) = \frac{1}{4}\xi\eta(\xi-1)(\eta+1)$$

Edge

$$\Phi_5(\xi,\eta) = \frac{1}{2}\left(1-\xi^2\right)\eta(\eta-1)$$
$$\Phi_6(\xi,\eta) = \frac{1}{2}\left(1-\eta^2\right)\xi(1+\xi)$$
$$\Phi_7(\xi,\eta) = \frac{1}{2}\left(1-\xi^2\right)\eta(1+\eta)$$
$$\Phi_8(\xi,\eta) = \frac{1}{2}(1-\eta)\xi(\xi-1)$$

Face

$$\Phi_9(\xi,\eta) = \left(1-\xi^2\right)\left(1-\eta^2\right)$$

Hierarchical (p-version) Trunk Space

Vertex

$$\Phi_1(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta)$$
$$\Phi_2(\xi,\eta) = \frac{1}{4}(1+\xi)(1-\eta)$$
$$\Phi_3(\xi,\eta) = \frac{1}{4}(1+\xi)(1+\eta)$$
$$\Phi_4(\xi,\eta) = \frac{1}{4}(1-\xi)(1+\eta)$$

Edge

$$\Phi_5(\xi,\eta) = -\sqrt{\frac{3}{32}}\left(1-\xi^2\right)(1-\eta)$$
$$\Phi_6(\xi,\eta) = -\sqrt{\frac{3}{32}}(1+\xi)\left(1-\eta^2\right)$$
$$\Phi_7(\xi,\eta) = -\sqrt{\frac{3}{32}}\left(1-\xi^2\right)(1+\eta)$$
$$\Phi_8(\xi,\eta) = -\sqrt{\frac{3}{32}}(1-\xi)\left(1-\eta^2\right)$$

Edge (cont.)

$$\Phi_9(\xi,\eta) = -\sqrt{\frac{5}{32}}\,\xi\left(1-\xi^2\right)(1-\eta)$$
$$\Phi_{10}(\xi,\eta) = -\sqrt{\frac{5}{32}}\,\eta(1+\xi)\left(1-\eta^2\right)$$
$$\Phi_{11}(\xi,\eta) = -\sqrt{\frac{5}{32}}\,\xi\left(1-\xi^2\right)(1+\eta)$$
$$\Phi_{12}(\xi,\eta) = -\sqrt{\frac{5}{32}}\,\eta(1-\xi)\left(1-\eta^2\right)$$
$$\Phi_{13}(\xi,\eta) = \sqrt{\frac{7}{512}}\left(1-\xi^2\right)\left(1-5\xi^2\right)(1-\eta)$$
$$\Phi_{14}(\xi,\eta) = \sqrt{\frac{7}{512}}(1+\xi)\left(1-\eta^2\right)\left(1-5\eta^2\right)$$
$$\Phi_{15}(\xi,\eta) = \sqrt{\frac{7}{512}}\left(1-\xi^2\right)\left(1-5\xi^2\right)(1+\eta)$$
$$\Phi_{16}(\xi,\eta) = \sqrt{\frac{7}{512}}(1-\xi)\left(1-\eta^2\right)\left(1-5\eta^2\right)$$

Face

$$\Phi_{17}(\xi,\eta) = \frac{3}{8}\left(1-\xi^2\right)\left(1-\eta^2\right)$$
$$\vdots$$