Dynamic Competition with Network Externalities: Why History Matters*

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Abstract

We consider dynamic competition with network externalities and vertical differentiation. A platform that dominated the market in the previous period becomes "focal" in the current period, in that agents play the equilibrium in which they adopt the focal platform whenever it exists. In the finite-horizon case, the unique equilibrium is efficient for "patient" platforms; with an infinite time horizon, however, there are multiple equilibria where either platform dominates. If qualities are stochastic, the better average quality platform wins with higher frequency, even when its realized quality is lower. Social welfare may decline as platforms become more forward looking.

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1 Introduction

Platform competition typically involves both network effects and repeated interaction. We often observe that a platform that was dominant in the recent past has the advantage of customers' favorable expectations, meaning that customers expect that this platform will also attract other customers in the current period. We shall refer to such a platform as a *focal* platform. For example, Apple's success with the iPhone 4 resulted in pre-orders for its iPhone 5 exceeding 2 million within a day of its launch in September 2012 — even though there were not yet any applications that could take advantage of the phone's new features. Moreover, analysts predicted that 50 million users would buy the new smartphone within three months of its launch.[1] A similar dynamics was in evidence for the iPhone 6's release, as sales topped 4 million in the first 24 hours.[2] In contrast, neither Blackberry nor Windows phones enjoyed a comparable advantage during this period. Even though the new Blackberry phones — the Q10 and the Z10 — received glowing reviews, the absence of positive expectations made it difficult for the firm to gain substantial market share: application developers were skeptical about the phone's ability to attract users; sales were indeed sluggish, which

[1] Ryan Faughnder and Adam Satariano, "Apple iPhone 5 Pre-Orders Top 2 Million, Doubling Record", Bloomberg (2012). Available at: (http://www.bloomberg.com/news/2012-09- 17/apple-iphone-5-pre-orders-top-2-million-double-prior-record-1-.html).

[2] http://www.cnet.com/news/apple-iphone-6-iphone-6-plus-preorders-top-4m-in-first-24-hours

was due in no small part to the paucity of available apps.[3] We can ascribe the developer' skepticism and the resulting lack of apps to the Blackberry platform's recent history.

Yet even dominant platforms can lose market share, despite winning in the past, when faced with a higher-quality competitor. In the market for smartphones, for instance, Nokia dominated the early stage (along with RIM) with smartphones based on a physical keyboard. Apple then revolutionized the industry by betting on its new operating system, which featured touch-screen technology. A few years later, Samsung managed to gain substantial market share (though not market dominance) by betting on smartphones with large screens. The supplanting of industry leaders was likewise a common theme in the market for video-game consoles. Nintendo, Sony, and Microsoft alternated as the market leader (Hagiu and Halaburda 2009). Thus platforms are sometimes able to overcome the market's unfavorable expectations. In this paper we explore when is it profitable for a platform facing unfavorable position to invest in capturing the market, and when it is profitable for a platform facing a favora ble position to invest in retaining the market, if each firm knows that its current strategy may affect its future position.

If the impact on the future is ignored, then it may not be profitable for the nonfocal platform to overcome unfavorable expectations even if it can offer higher quality than the focal platform. The reason is that network effects provide the focal platform with a short-term competitive advantage: in a one-time interaction, a focal platform can use its position to dominate the market even when competing against a higher-quality platform. Yet we can expect that, in the long run, if platforms are forward looking then a high-quality but presently nonfocal platform can overcome its expectations disadvantage because it can afford short-term losses in order to become focal in the future. At the same time, a low-quality forward looking focal platform also has an incentive to invest in maintaining its dominant market position. So even though the nonfocal platform has a quality advantage, the focal platform has an expectations advantage.

In this paper, we ask whether a low-quality platform currently benefiting from its focal position can continue to dominate the market in a dynamic setting — that is, when platforms account for the future benefits of capturing the market today. For example, would Apple continue to dominate the market for tablets if competitors (Samsung's Tab, Microsoft's Surface, etc.) offered tablets of higher base quality? Is it possible for a video-game console to maintain market leadership when facing higher-quality competitors? More specifically,

[3]On the quality and launch of the Blackberry phones, see Austen (2012) and Bunton (2013).

we are interested in whether the higher-quality platform's likelihood of winning increases with the importance attached by firms to the future. The winning platform's identity affects not only the firms involved but also social welfare, which is higher when the better platform wins.

To investigate this research question, we analyze a model of dynamic competition between two platforms. In each period, one of the platforms wins by capturing all of the market. So as to focus on the model's dynamic aspects, we assume that customers are homogeneous.[4] Consumers base their current-period behavior on their observation of past outcomes; thus the platform that won the market in the previous period becomes focal in the current period. It follows that capturing the market in one period gives the platform an advantage in future periods. Hence a nonfocal platform may be willing to sacrifice current profits to gain a better future market position.

We start with the case where each platform's stand-alone quality is constant for all periods and where the time horizon is finite. We show that, when platforms do not care about future profits, the low-quality focal platform maintains its position despite the nonfocal platform's higher quality — provided the quality gap is not too great. But when the future is important for the platforms or the quality gap is sufficiently large, the higher-quality platform wins the market at the start of the game and maintains its leadership. This outcome follows because a high-quality platform can earn higher profits than the low-quality one as the focal platform in the last period. So as compared with the low-quality platform, the high-quality platform has a larger incentive to fight for focality in the game's early stages.

We then consider the infinite-horizon case. We find that, when platforms care moderately about the future, there is a unique equilibrium in which a high-quality platform wins the market. But when the platforms care a lot about the future, the result is an increase both in the focal platform's incentive to maintain its position and in the nonfocal platform's incentive to win the focal position. This gives rise to multiple Markov equilibria. In one Markov equilibrium, the high-quality platform wins the focal position in the first period and then maintains it indefinitely — even if it begins as a nonfocal platform; however, there are equilibria in which the focal platform can maintain its leadership indefinitely even if it is of low quality. Thus an infinite horizon yields a new form of market failure, leading to an outcome where one platform aggressively builds market share under all circumstances (i.e., both on and off the equilibrium path) and succeeds in doing so because the other platform

[4]The consequences of this assumption are addressed in Section 7.

restrains from sacrificing current profit.

In terms of social welfare, these results indicate that when firms' patience increases from low to moderate, the social welfare increases. This outcome reflects the market's movement from the equilibrium in which the low-quality platform wins to the equilibrium in which the high-quality one overcomes its nonfocal position and thereafter maintains its newly acquired focal position indefinitely. However, as firms' patience increases further, the effects on welfare are ambiguous owing to the existence of multiple equilibria.

These results are a unique consequence of network effects. In an alternative model where instead of network effects buyers face switching costs, the market failure that our paper identifies vanishes. Under switching costs, when players sufficiently care about future payoffs, there is a unique equilibrium in which the best platform wins. Intuitively, in the absence of network effects, a buyer's decision to switch from one platform to the other is not affected by the buyer's beliefs concerning the decisions of other buyers. This result highlights the qualitative difference between network effects and switching costs as well as the important role that beliefs play in markets with competing platforms.[5]

When qualities are constant, our model finds that the same platform dominates the market in all periods. Yet there are some cases in which platforms "take turns" at being the dominant platform, as with the Sony, Nintendo, and Microsoft video-game consoles mentioned previously. We therefore study how focality and the importance of the future affect changes in market leadership and market efficiency.

To study this question, we consider the case where the quality of each platform changes stochastically every period — a setup that is consistent with the continuous technology improvements seen in the markets for such products as video-game consoles and smartphones. In this setting it is possible for a low-quality platform to itself become a higher-quality one. Even so, we will assume that one of the platforms is of higher quality in average.

Unlike the case of fixed quality, in this stochastic scenario there is a unique Markov equilibrium. It is possible for either platform to win the market in any period if its quality in that period is sufficiently high. Still, the more platforms care about the future, the more likely it is that the platform with better average quality wins the market, even when its quality realization is lower. In some cases it is also possible for a focal platform to lose its market dominance even if its quality realization is higher than that of the nonfocal competitor. Thus social welfare may decline with increases in platforms' concern for the future.

[5]For an extensive discussion of switching costs, see Farell and Klemperer (2007).

The intuition behind this result is as follows. The platform with a higher quality on average is more likely capable of defending its focal position. Therefore, as future considerations become more important to platforms, this firm has more incentive to compete aggressively in order to capture a focal position even if its current realized quality is low. At the same time, the platform expecting a lower future quality on average has less incentive to win the market even if its current quality is high. This result indicates that the changes in market leadership, following technological improvements, which we observe in several markets for platforms (e.g., video-game consoles, smartphones), may not necessarily result in outcome in which the platform with higher quality wins.

Related Literature

Our paper's main conclusion is that even long-term considerations may not lead to an outcome in which the best platform wins. For example, there is disagreement in the economics literature as to whether the presence of network effects leads to long-term market inefficiency. David (1985) argues that the QWERTY keyboard's prevalence is an example of long-term inefficiency due to network effects; that claim is based on evidence that the Dvorak keyboard enables faster typing and requires less training. Liebowitz and Margolis (1990) criticize David's argument by claiming, on the basis of a case study, that the success of QWERTY is due not to network effects but rather to its superior quality vis-à-vis Dvorak. In an experiment, Hossain and Morgan (2009) find that the more efficient platform always wins over time, which would seem to support the claim of Liebowitz and Margolis. Our paper contributes to this debate by demonstrating that, when platforms strategically set prices to compete for users, both efficient and inefficient equilibria are possible in the long term.

Most theoretical analyses of platform competition focus on static games. Caillaud and Jullien (2001, 2003) introduce the notion of favorable beliefs — with respect to networks in the context of two-sided markets — as a tool that can be used to characterize the full equilibrium set for competition between undifferentiated platforms. This concept was used in subsequent research on two-sided markets (see Hagiu 2006; Jullien 2011; Halaburda and Yehezkel 2013) as a way of modelling market leadership when one platform benefits from favourable expectations; it has also been used in the literature on telecommunications to model consumer inertia (Lopez and Rey 2016). Though all of these papers acknowledge the dynamic nature of platform competition, their aim is to approximate market characteristics using static models. Halaburda and Yehezkel (forthcoming) extend this concept to partial beliefs advantage and explore how platform's pricing strategies affect their future profits; th e article employs a simple multi-period setup in which the extent of beliefs advantage depends on the market's history.

There has been some work addressing dynamic price competition between networks, but so far as we know our paper is the first to address the dynamics of consumer expectations. One strand of the literature assumes that consumers' decisions are based on current market shares (e.g., Doganoglu 2003; Mitchell and Skrzypacz 2006; Markovich 2008); the other strand assumes that consumers are forward looking (e.g., Fudenberg and Tirole 2000; Laussel and Resende 2007; Cabral 2011). In all these papers, the competition is dynamic because switching costs render participation decisions irreversible to some extent. Our paper abstracts from switching costs in order to focus on the dynamics of consumer expectations. Consequently, consumers in our model need not form beliefs about the market's future. The real-life examples that we cite (i.e., the markets for smartphones and video games) may include both network externalities and switching costs (stemming, for example, from the adjustments required after adoption of a new operating system). However, we share with those other models the dynamic-game feature of firms competing aggressively to build their market share in the current period so as to gain a long-term advantage.[6]

In a contemporaneous work, Crémer and Biglaiser (2016) propose an alternative equilibrium concept capturing the reluctance of consumers to migrate from one platform to another. Their concept, as well as ours, induces consumer inertia. They show that consumer heterogeneity may lead to inefficient market fragmentation with free entry of equally efficient platforms. We focus on consumers' beliefs and identify conditions for an inefficient platform to dominate the market under duopoly competition.

Argenziano and Gilboa (2012) consider a repeated coordination game where players use the game's history to form beliefs regarding the behavior of other players. Our paper adopts that approach in the context of platform competition, as we study how platforms should compete given such belief formation by consumers. But we add the feature that each platform can alter beliefs by capturing the market and thereby shifting the coordination of consumers in its favor.

The rest of the paper is organized as follows. After describing the model in Section 2, in

[6]For an analysis of such strategies in games involving dynamic competition, see Besanko, Doraszelski, and Kryukov (2014).

Section 3 we consider the benchmark case of a dynamic game with finite horizon. Section 4 characterizes Markov equilibria under an infinite time horizon, and Section 6 considers the case where platform qualities change stochastically over time. We conclude in Section 7 by summarizing our results and touching on some related considerations.

2 The Model

Consider a homogeneous consumer population of size 1 and two competing platforms, $i = A, B$, with the same cost (normalized to 0).[7] There are T periods, $t = 1, 2, \ldots, T$, where T may be finite or infinite. Each platform i offers to the customers a stand-alone value, $q_i > 0$, which we refer to as *quality*.[8] Additionally, consumers, benefit from network effects. A consumer's utility from adopting platform i is $q_i + \beta n_i - p_i$; here n_i is a measure of the other consumers who have adopted i, β denotes the strength of network effects, and p_i is the price of platform i.[9]

Every period, each platform i sets a price $p_i(t)$, and then consumers decide which platform to adopt for the current period. In what follows, a negative price is interpreted as one below cost.[10] The two platforms operate for T periods and discount future profits by δ, where $0 \leq \delta < 1$. There are no switching costs, so consumers' current payoff is not directly affected by past or future periods. Path dependency will arise solely from belief formation.

Competition in an environment with network effects often results in multiple equilibria, and it is also the case here. Consider the allocation of consumers that emerges for given prices. If $q_i - p_i(t) > q_j - p_j(t) + \beta$, then all consumers adopt platform i. Yet if

$$|q_A - q_B + p_B(t) - p_A(t)| < \beta \tag{1}$$

then there are two possible allocations: either all consumers adopt A or all adopt B. This multiplicity makes it difficult to discuss dynamic competition in environments with network effects, and several solutions have been proposed to address the issue. We rely on the notion of pessimistic beliefs and a focal platform, as developed in Caillaud and Jullien (2003), Hagiu

[7] In Section 7 we discuss how our results are affected when agents are instead heterogeneous.

[8] We consider the case where the q_i are fixed over time (Sections 3 and 4) and also the case where qualities change from one period to the next (Section 6).

[9] Because consumers are homogeneous, they will all adopt the same platform in an equilibrium.

[10] To allow for the possibility of negative prices, we must assume that agents who collect the resulting subsidy do indeed adopt the platform to the benefit of other users.

(2006), and Jullien (2011). Thus we say that platform i is *focal* if, under condition (1), a consumer adopts platform i. We assume that, during any period, there is a focal platform.

In a dynamic model with $t = 1, \dots, T$, the identity of the focal platform in $t > 1$ may be related to the market's history. In this paper we explore how allowing for such historical dependency affects the market's future outcomes. To simplify matters, we focus on one-period dynamics.

During each period t, the market outcome is expressed by a pair (w_t, f_t), where $w_t \in \{A, B\}$ is the identity of the active platform — i.e., the platform that wins the market in t[11] — and $f_t \in \{A, B\}$ is the identity of the focal platform in t. Note that it is possible for the nonfocal platform to win the market. Based on their observation of past outcomes, consumers form conjectures about the platform most likely to win in the current period. These conjectures are assumed to converge to a single focal platform. In $t = 0$, one of the platforms is arbitrarily set as the focal platform, which we call platform A. The identity of the focal platform f_t is always common knowledge and is the only payoff-relevant variable. The dynamics of platform focality is then expressed by transition probabilities: $\Pr(f_t = i \mid w_{t-1}, f_{t-1})$. We consider a deterministic rule whereby the last period's market winner becomes focal; thus, $\Pr(f_t = w_{t-1} \mid w_{t-1}, f_{t-1}) = 1$.

As a benchmark case for our analysis, consider a static one-period game in this environment. Network externalities may create market inefficiencies in the equilibrium of a static game. Although A is the focal platform, it can be of higher or lower quality than platform B. When $q_A - q_B + \beta > 0$, in equilibrium the two platforms set their respective prices as $p_A = q_A - q_B + \beta$ and $p_B = 0$, and all customers adopt A. If $q_A - q_B + \beta < 0$ then A would derive negative profits from such a pricing strategy, so that strategy is not an equilibrium strategy. In this case, the platforms set $p_A = 0$ and $p_B = q_B - q_A - \beta > 0$ in equilibrium, and all customers adopt platform B.

Thus, if $q_A < q_B$ yet $q_A > q_B - \beta$, then platform A wins despite offering lower quality. It wins because it happens to be focal. This effect is called *excess inertia*, and it leads to inefficient outcomes in equilibrium.

When there are multiple periods, a nonfocal platform may find it worthwhile to win the market by setting negative price in an earlier period. Doing so would yield the platform a negative current profit, but if the focal position is thereby captured then those losses could

[11] In this model there cannot be market sharing in equilibrium: at each date, a single platform attracts the whole population.

be recovered in future periods. Of course, if the market is static then no platform finds it optimal to win the market at negative prices because there is no way to recover the resulting losses. If follows that, in a static market, the focal platform has the upper hand even when it is of lower quality.

In a dynamic market, one could suppose that a higher-quality nonfocal platform is advantaged; after all, should it become focal, its higher quality yields greater profit than can be generated by a lower-quality platform. Hence the higher-quality platform has more incentive to invest in capturing the market than the lower-quality platform has to invest in defending its position. The focal platform anticipates this and strives to prevent the nonfocal platform from capturing the market.

3 Dynamic game with finite horizon

In this section we consider the case of a finite time horizon and show that there exists a unique subgame perfect Nash equilibrium. In this equilibrium, a high-quality but nonfocal platform wins the market in the first period and maintains its acquired focal position in all subsequent periods — provided that platforms care sufficiently about the future (i.e., δ is high and the time horizon is long) or that the quality gap between platforms A and B is sufficiently large. Otherwise, a low-quality platform that is focal in the first period will maintain that position thereafter.

Suppose that the time horizon is $T \geq 2$. We define the price $p_i^f(t)$ and the value function $V_i^f(t)$ as the equilibrium price and the expected future discounted profit, respectively, of platform i in period t when platform f is focal during that period. Our analysis will focus on the interesting case, where network effects are sufficiently large with respect to the quality gap, $\beta > |q_i - q_j|$, so that the focal platform wins a one-period game even with a lower quality than the nonfocal platform. Lemma 1 and Proposition 1 below describe a general solution for all values of β, q_A, and q_B. We focus on subgame perfect equilibria and solve the game by backward induction.

Consider the last period, $t = T$. Since in this period there is no future, the subgame equilibrium is identical to the one-period benchmark described in Section 2. When platform i is focal, it wins the market regardless of the quality gap; in this case, i earns $V_i^i(T) = q_i - q_j + \beta$ while the nonfocal platform j earns $V_j^i(T) = 0$. Observe that the focal platform's last-period profits are greater when it is the higher-quality platform; formally, $V_i^i(T) > V_j^j(T)$ when

$q_i > q_j$.

Continuing with our backward induction, we next turn to the period immediately before the last one, $t = T-1$. To facilitate notation, suppose that A is the focal platform in this period. Each platform takes into account that capturing the market in this period will render it focal in the next period and earn it an additional profit of $V_i^i(T)$. An individual buyer, however, cannot affect the identity of the focal platform, as it depends on the buyer's beliefs concerning the behavior of other consumers.

Consider first a subgame equilibrium in which A wins the market. The lowest price that the losing platform B is willing to charge at time $T-1$ is $p_B^A(T-1) = -\delta V_B^B(T)$. In order to convince buyers to join it, platform A must charge a price such that $q_A + \beta - p_A^A(T-1) = q_B - p_B^A(T-1)$. Notice that the buyer's expected payoff at time T does not affect the buyer's decision at time $T-1$, because future payoffs depend only on future network effects, i.e., on the buyer's beliefs concerning the decisions of other buyers and therefore do not depend on the buyer's choice of platform A or B. As we show in Section 5, the buyer's choice of the platform affects his future payoffs when we replace network effects with switching costs.

Setting $p_A^A(T-1) = q_A + \beta - q_B + p_B^A(T-1)$ is the best reply of platform A only if the profit $p_A^A(T-1) + \delta V_A^A(T)$ is nonnegative. We thus have the condition

$$q_A - q_B + \beta - \delta V_B^B(T) + \delta V_A^A(T) \geq 0. \tag{2}$$

The same analysis applies to a subgame equilibrium in which the nonfocal platform B wins in period $T-1$. In this equilibrium, the losing platform A charges a price $p_A^A = -\delta V_A^A(T)$ and the winning platform B must earn a positive profit at the best-reply winning price. So now we have the following condition:

$$q_B - q_A - \beta + \delta V_B^B(T) - \delta V_A^A(T) \geq 0. \tag{3}$$

Comparing conditions (2) and (3) reveals that, in $T-1$, exactly one of those subgame equilibria exists for any set of parameters (except for the degenerate case where profit is zero for both platforms). In this unique equilibrium, a focal platform A wins, with positive profit, in $T-1$ if

$$q_B - q_A < \frac{\beta}{1+2\delta}. \tag{4}$$

We can apply the same analysis to the case where platform B is focal simply by transposing q_A and q_B. Hence we conclude that the equilibrium is unique in period $T-1$ and has the following characteristics.

(i) If the quality differential is small (i.e., less than $\beta/(1+2\delta)$), then the focal platform wins the market. In this case, the value of the focal platform (we can suppose for convenience that the focal platform is A) can be written as

$$V_A^A(T-1) = (1+2\delta)(q_A - q_B) + \beta = V_A^A(T) + 2\delta(q_A - q_B), \tag{5}$$

which is lower than the period T corresponding value if and only if platform A is of lower quality than platform B.

(ii) If the quality differential lies between $\beta/(1+2\delta)$ and β, then the higher-quality platform wins the market.

Thus a nonfocal platform B can win the market in $T-1$ even when it could not in the one-period case (or, equivalently, in the final period of a dynamic game). The intuition for this result is that, since platforms care about the future and since B earns more profit (than A) from being focal in the last period, it follows that platform B's incentive to capture the focal position at $T-1$ could well be stronger than platform A's incentive to maintain its focal position. In particular, if δ is high enough and if the quality gap $(q_B - q_A)$ is sufficiently large, then a high-quality but nonfocal platform can win the market in $T-1$ and maintain its focal position in period T. In such cases, forward-looking platforms eliminate the inefficiency that might otherwise emerge because of the consumer coordination problem.

If $T > 2$, then there are periods before $T-1$ — namely, $t = 1, \ldots, T-2$. Suppose that platform A is focal in $T-2$ but that platform B is of sufficient quality to win in period $T-1$ despite not being focal (i.e., condition (4) does not hold). Then, in any subgame equilibrium in $T-2$, platform A sets a nonnegative price. This is because, when condition (4) does not hold, platform A would lose the market in $T-1$ even if it had won in $T-2$ and was still focal in $T-1$; formally, $V_A^A(T-1) = V_A^B(T-1) = 0$. We can easily verify that the value function for platform B is such that, in period $T-1$, the benefit $V_B^B(T-1) - V_B^A(T-1)$ is greater than the last period's benefit $V_B^B(T)$. As a consequence, platform B wins the market in period $T-2$ even though it is not focal. By a similar logic, platform A loses the market and sets price 0 for all periods before $T-2$. So when condition (4) does not hold, there is no subgame equilibrium in $t < T$ where platform A wins the market.

If condition (4) *does* hold and if a symmetric condition holds for platform B, then platform A wins or loses the market in $T-2$ according to whether it is focal or not. In this case we can replicate the reasoning for period $T-1$ by replacing $V_i^i(T)$ with $V_i^i(T-1)$ for

$i = A, B$. Hence the equilibrium analysis is the same and once again we find that the focal platform wins when the quality difference is small but that the higher-quality platform wins otherwise. The difference in this case between period $T-2$ and period $T-1$ lies in the quality differential threshold above which the higher-quality platform wins the market. In a subgame where platform A wins in $T-2$, it sets the highest price with which it can win, $p_A^A(T-2) = q_A - q_B + p_B^A(T-2) + \beta$, where $p_B^A(T-2) = -\delta V_B^B(T-1)$. Thus A wins in $T-2$ provided that

$$q_B - q_A < \frac{\beta}{1 + 2\delta + (2\delta)^2};$$

otherwise A loses. Note that the quality differential threshold is smaller for period $T-2$ than for period $T-1$. Applying this analysis recursively shows that the threshold decreases with the time horizon and may even vanish at some point. It follows that there is a maximal (possibly infinite) horizon short of which there exists a unique equilibrium where the focal platform wins in all periods irrespective of its identity, while the higher-quality platform wins if the time horizon is longer. Let T_A be shortest time horizon for which platform A does not win the market in the equilibrium, even if it starts focal. In general, T_A depends on the model parameters and may be an arbitrary number. If platform A is of higher quality than B or if the discount factor is small, then there is no finite T_A. In this case, for any T, platform A wins the market if it starts being focal. We can similarly find a T_B for platform B.

The following lemma characterizes how, for an arbitrary finite horizon, the equilibrium outcome depends on the parameters.

Lemma 1 (Subgame perfect equilibrium for arbitrary finite T) *For any set of parameters q_A, q_B, β, and δ, there exists a unique subgame perfect equilibrium for arbitrary finite T. In the equilibrium, the same platform wins the market in all periods. The winning platform's identity and its future discounted profit depend on the parameters as follows.*

(i) *If $|q_A - q_B| < \beta\frac{1-2\delta}{1-(2\delta)^T}$ then A wins every period because it is initially focal, and it earns a total profit of*

$$(q_A - q_B)\frac{1-(2\delta)^T}{1-2\delta} + \beta.$$

(ii) *If $q_A - q_B > \beta\frac{1-2\delta}{1-(2\delta)^T}$ then $T_B < T$. Platform A wins every period because its quality advantage is sufficient, and it earns*

$$(q_A - q_B + \beta)\frac{1-\delta^{T-T_B}}{1-\delta} + \delta^{T-T_B}\left((q_A - q_B)\frac{1-(2\delta)^{T_B}}{1-2\delta} + \beta\right).$$

(iii) *If* $q_B - q_A > \beta\frac{1-2\delta}{1-(2\delta)^T}$ *then* $T_A < T$. *Platform B wins every period because its quality advantage is sufficient, and it earns*

$$(q_B - q_A + \beta)\frac{1-\delta^{T-T_A}}{1-\delta} + \delta^{T-T_A}\left((q_B - q_A)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right) - 2\beta.$$

In each case, the losing platform earns zero profits.

Proof. See Appendix.

The main qualitative results of Lemma 1 are the following. First, the same platform wins the market in all periods, so the nonfocal platform wins the market in the first period or never. Second, nonfocal platform B wins the market only if it has quality advantage. Yet, platform A may win either because it has a quality advantage, or it can win despite offering lower quality, because it started with a focal position. The latter occurs when $0 < q_B - q_A < \beta\frac{1-2\delta}{1-(2\delta)^T}$, and results in an inefficient outcome. In all other cases, the higher-quality platform wins and so the equilibrium outcome is efficient. Observe that the set of parameters under which the equilibrium outcome is inefficient decreases as T and δ increase — that is, as the future becomes more important to the platforms.

Thus, competition over multiple periods yields an efficient equilibrium outcome for parameters such the lower-quality platform wins in a static model. In this sense, there is less inefficiency when the time horizon increases. One might therefore suppose that inefficiency would disappear altogether if the time horizon were extended to infinity. However that is not always the case, as the following proposition illustrates. In the proposition, we extrapolate the equilibrium outcome in Lemma 1 to the case where $T \to \infty$. For this purpose it is important to recognize that the ratio $\frac{1-(2\delta)^T}{1-2\delta}$ converges to $\frac{1}{1-2\delta}$ for $\delta < 1/2$, and converges to infinity for $\delta > 1/2$.

Proposition 1 (Subgame perfect equilibrium extrapolated for $T \to \infty$) *As* t *goes to infinity, the following statements hold.*

(i) *If* $|q_A - q_B| < \beta(1-2\delta)$ *or* $q_A = q_B$ *then platform A wins every period because it is initially focal, and it earns a total profit of*

$$V_A^A(1) = \frac{q_A - q_B}{1-2\delta} + \beta.$$

(ii) *If* $q_A - q_B > \max\{\beta(1-2\delta), 0\}$ *then platform A wins every period owing to its quality advantage, and it earns*

$$V_A^A(1) = \frac{q_A - q_B + \beta}{1-\delta}.$$

(iii) *If* $q_B - q_A > \max\{\beta(1-2\delta), 0\}$ *then platform B wins every period owing to its quality advantage and it earns*

$$V_B^A(1) = \frac{q_B - q_A + \beta}{1-\delta} - 2\beta.$$

The losing platform earns zero profits.

Proof. See Appendix.

The outcome of a subgame perfect equilibrium may be inefficient no matter how long the time horizon. When $0 < q_B - q_A < \beta(1-2\delta)$, platform A wins despite lower quality even under infinite time horizon. That being said, the problem of inefficiency due to excessive inertia arises less often as the time horizon increases in length. Furthermore, the inefficiency disappears if platforms care enough about the future — that is, when $\delta > 1/2$.

In the sections to follow, we explore some other reasons why the inefficient outcome may occur in equilibrium.

4 Markov perfect equilibria under infinite time horizon

In Proposition 1, we characterized an equilibrium of the infinite game by extrapolating the subgame perfect equilibrium of an arbitrary finite game. If the time horizon is infinite, there may be other equilibria as well. In this section we identify Markov perfect equilibria in the infinite game. Although the subgame perfect equilibrium identified in Proposition 1 is a Markov perfect equilibrium, there exist other Markov perfect equilibria that cannot be found by extrapolating any finite-game solution. These new equilibria often result in inefficient outcomes for the same parameters under which the equilibrium of Proposition 1 is efficient.

Every period t of the infinite game is characterized by the identity f_t of the focal platform, the state variable for that period. A Markov perfect equilibrium is characterized by the strategies of each platform in all possible states and by the outcome in each state. We consider three pure-strategy equilibrium outcomes: (i) platform A wins in both states, (ii) platform B wins in both states, and (iii) the focal platform wins.[12]

Next we characterize the strategies that support those equilibrium outcomes and identify the range of parameters under which each equilibrium exists. We define the value function V_i^f as the equilibrium expected discounted profit of platform i when platform f is focal.

Consider first the equilibrium outcome of platform A winning in both states. In this equilibrium, the value function for platform B is $V_B^B = V_B^A = 0$ because no customers adopt that platform. Platform B sets price $p_B^f = 0$, because it has no interest in winning with price $p_B < 0$, given that it cannot count on future profits to justify the investment required to capture the market. When A is focal, it optimally sets $p_A^A = q_A - q_B + \beta$; similarly, if platform B is focal then A sets price $p_A^B = q_A - q_B - \beta$ and B sets $p_B^B = 0$. If platform A were to set a higher price, then platform B would maintain its market dominance and secure nonnegative profits. Notice that as in the finite case, buyers only account for their current payoff because an individual buyer cannot affect the identity of the focal platform. Platforms, however, take into account the effect of their current prices on the future focal position, because platforms can convince all buyers to switch from one platform to the other.

We therefore have

$$V_A^A = q_A - q_B + \beta + \delta V_A^A \quad \text{and} \quad V_A^B = q_A - q_B - \beta + \delta V_A^A.$$

Moreover, incentive compatibility for platform A requires that

$$V_A^A \geq \delta V_A^B \quad \text{and} \quad V_A^B \geq 0.$$

It follows that this equilibrium exists whenever $q_A - q_B \geq \beta(1 - 2\delta)$. After a similar analysis for platform B, we arrive at our next lemma.

Lemma 2 *There is an equilibrium in which platform i wins in both states if $q_i - q_j \geq \beta(1 - 2\delta)$.*

Proof. See Appendix.

According to Lemma 2, a nonfocal platform B can capture the focal position and maintain it in all future periods provided $q_B - q_A \geq \beta(1 - 2\delta)$. This inequality holds when B's quality

[12] There is no equilibrium that supports the fourth possible pure-strategy outcome – namely, that the nonfocal platform wins.

is substantially higher than that of A, when platforms are very forward looking (so that δ is high), or when network effects (β) are weak. We remark that, for $\delta > 1/2$, the condition's left-hand side becomes negative, which means that an initially nonfocal platform B could win the market in every period even when $q_B < q_A$. Such an equilibrium involves *excess momentum* and is inefficient.

Similarly, platform A can dominate the market forever when $\delta > 1/2$, even if $q_A < q_B$, as long as $q_A - q_B \geq \beta(1 - 2\delta)$. This equilibrium involves *excess inertia* and is inefficient as well.

It is only when the time horizon is infinite that both equilibria with excess momentum and equilibria with excess inertia occur. The nature of these equilibria can be understood as follows. Suppose that both platforms expect A to behave aggressively in the future — in particular, to regain the focal position if it is ever lost. Then platform B has no reason to sacrifice profit in the current period because it expects no future gain from being focal. Faced with a weak competitor, will platform A decide to be aggressive as expected? If A is focal then the answer is clearly Yes. But suppose that A is not focal. Then the choice facing platform A is this: Should it make a sacrifice today (by setting its price $p_A = q_A - q_B - \beta$) in order to gain the benefit of network effects β in the next period, or should it wait and make that sacrifice tomorrow? If the discount factor is high enough, then the platform will opt for the sacrifice today. Hence, the beliefs that platform A always win are fulfilled. Such an equilibrium therefore involves a particular form of coordination failure among firms that does *not* arise in the finite-horizon game, even as $T \to \infty$, because such an aggressive strategy becomes not credible as the game's end approaches.

The remaining equilibrium to consider is one where the focal platform wins. Recall that p_i^f denotes the price of platform i when f is focal in such an equilibrium. Since the winning platform anticipates that it will be focal from the new period onward, we have value functions

$$V_i^i = \frac{p_i^f}{1-\delta} \quad \text{and} \quad V_i^j = 0.$$

The benefit of selling at a given date is $p_{it} + \delta V_i^i$. It follows that the minimal profit that platform i is willing to sacrifice today in order to capture the market is $-\delta V_i^i$. In such an equilibrium, the focal platform sets a price $p_i^i \leq q_i - q_j + \beta - \delta V_j^j$ because otherwise the competing platform would set a price above $-\delta V_i^i$ and win the market. Ruling out cases where $p_j < -\delta V_j^j$ because winning at this price would not be profitable for firm j,[13] we

[13] Allowing prices $p_j < -\delta V_j^j$ would not alter the existence conditions, but only the equilibrium profits.

obtain the equilibrium prices

$$p_i^i = q_i - q_j + \beta - \delta V_j^j \quad \text{and} \quad p_j^i = -\delta V_j^j.$$

In this equilibrium, the value functions solve the following equations:

$$(1-\delta)V_A^A + \delta V_B^B = q_A - q_B + \beta,$$
$$(1-\delta)V_B^B + \delta V_A^A = q_B - q_A + \beta,$$

yielding

$$V_A^A = \frac{q_A - q_B}{1-2\delta} + \beta \quad \text{and} \quad V_B^B = \frac{q_B - q_A}{1-2\delta} + \beta.$$

We can therefore draw the following conclusion:

Lemma 3 *There is an equilibrium where the focal platform wins in every state if* $\beta\,|1-2\delta| \geq |q_B - q_A|$.

Proof. For this to be an equilibrium, it is both necessary and sufficient that $V_A^A \geq 0$ and $V_B^B \geq 0$. ■

We arbitrarily designated A as the first period's focal platform; hence, for equilibria in which the focal platform wins in every state, platform A wins every period. If the condition stipulated in Lemma 3 is satisfied, then A can maintain its focal position in all future periods even when its quality is lower than that of platform B. This outcome is thus another instance of an inefficient *excess inertia* equilibrium.

To see the intuition behind that result, consider first the case of $\delta \leq 1/2$. As Lemma 3 shows, the equilibrium occurs in this case provided that both δ and the quality gap $(q_B - q_A)$ are sufficiently small. Suppose now that q_B increases; that increase has two effects on V_B^B. First, a *direct* effect is that, since $p_B^B = q_B - q_A + \beta - \delta p_A^B$, taking V_A^A as given, platform B can attract customers with a higher price p_B^B; this implies that V_B^B will increase. Second, a *strategic* effect is that, since $p_B^A = -\delta V_B^B$, platform A knows that even when it is focal, it will compete against a more aggressive platform B because the latter would gain more by capturing focality from A. This threat reduces V_A^A, which in turn increases V_B^B because a nonfocal A will not compete aggressively to capture the focal position. When $\delta < 1/2$, both the direct effect and the strategic effect increase V_B^B while reducing V_A^A. If the quality gap $q_B - q_A$ is sufficiently large, then V_A^A becomes negative, which implies that A cannot maintain its focal position when competing against a platform of higher quality. As δ increases, platform B cares more about future profits and so has mo re incentive to capture the market when it is not focal, as well as to maintain its market dominance when it is focal.

Now suppose that $\delta > 1/2$. Then the equilibrium is completely reversed. Now, if $q_B > q_A$ then $V_A^A > V_B^B$ and, as q_B increases, V_B^B declines while V_A^A increases. However, the equilibrium in this case relies on the rather unusual feature that platforms "overreact". In particular: as q_B increases, the direct effect increases V_B^B (just as when $\delta < 1/2$) but the strategic effect now works in the opposite direction and is stronger than the direct effect. To see how this works, suppose that platform B is focal and that q_B increases. Then the equilibrium holds when platform B expects that platform A, in response to the increase in q_B, will over-react by becoming very aggressive and reducing its price, p_A^B, thus in the opposite direction than in the case of $\delta < 1/2$. Here V_B^B increases because q_B increases (direct effect) but decreases because p_A^B decreases (strategic effect). The strategic effect outweighs the direct effect, so the overall effect is to reduce V_B^B while increasing V_A^A. In this equilibrium, however, platform A reduces its price p_A^B because it anticipates that, after becoming the focal platform, it will benefit from competing with a more efficient rival, another unusual feature of this equilibrium.

We remark that ruling out the possibility of overreaction prevents the equilibrium from adjusting to small changes in quality. So if $\delta > 1/2$ without overreaction, e.g., in the case of iterated best-response dynamics, a small change in q_B results in convergence to an equilibrium described in Lemma 2. In this sense, we can say that, when $\delta > 1/2$, an equilibrium where the focal platform wins in every state is unstable. The same cannot be said either for this equilibrium under $\delta \leq 1/2$ or, with any value of δ, for the equilibria described in Lemma 2.

Our next proposition summarizes the results of Lemmas 2 and 3.

Proposition 2 (Markov perfect equilibria) *Suppose that platform A is focal in period $t = 1$. Then:*

(i) *for $q_B - q_A > \beta|1 - 2\delta|$, there exists a unique equilibrium in which platform B wins;*

(ii) *for $\beta(1 - 2\delta) < q_B - q_A < \beta|1 - 2\delta|$ (which holds only when $\delta > 1/2$), there exist multiple equilibria and, of these, there is one in which platform B wins;*

(iii) *for $q_B - q_A < \beta(1 - 2\delta)$, platform A wins in all equilibria.*

Proof. These statements follow from our assumption that A is initially focal and from Lemmas 2 and 3. ■

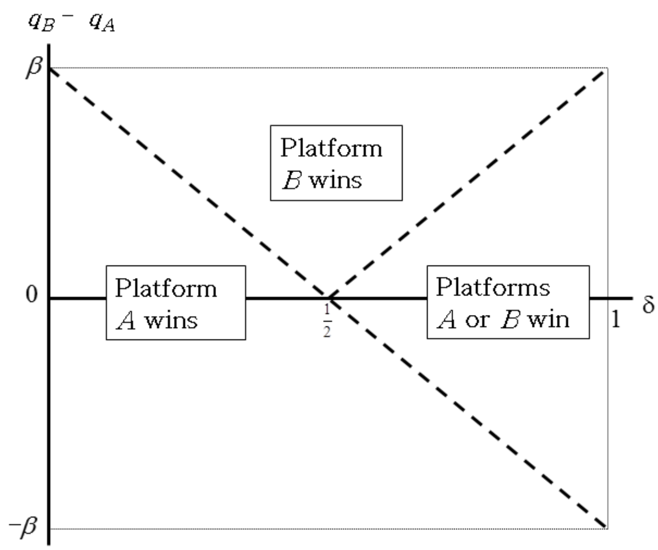

Figure 1: Equilibrium configuration

Figure 1 illustrates which platform is active in equilibrium, depending on the parameters. It shows that, if both the discount factor and the quality differential are low, then there is a unique equilibrium in which the focal platform A wins. Intuitively, for those parameter values the results of the dynamic game are the same as in the static game. For a positive quality differential $q_B - q_A$ and intermediate values of δ, there is a unique equilibrium in which the most efficient platform wins the market and maintains its position thereafter. However, for high discount factors and low quality differential there exist multiple equilibria: some in which A wins and some in which B wins. Observe that disregarding the Lemma 3 equilibria (because they are unlikely to emerge) would not restore efficiency of the equilibrium in this parameter region; the reason is that there are also two equilibria, including one in which the low-quality platf orm wins, identified by Lemma 2. In both of these equilibria, one platform expects to encounter low competitive pressure while the other renounces capturing the market because it expects to encounter high competitive pressure — and these expectations are self-fulfilling. In sum: if the discount factor is high, then each firm's prospect of capturing the focal position is not enough to outweigh the firms' (self-fulfilling) expectations of the competitive pressure each will face.

5 The distinction between network effects and switching costs

Network effects in our model may appear, at first blush, to play the same role as switching costs. In both cases, a consumer pays a cost when switching from one platform to the other, which can be either direct switching costs or indirect through the loss of network effects when switching to the nonfocal platform. This raises the question of what is the qualitative difference between switching costs and network effects, and how our results depend on our focus on network effects.

The key difference is that with switching costs, a consumer does not need to coordinate with others to maximize the (intertemporal) value of his consumption. This difference qualitatively affects the results. In this section we develop an alternative model in which β serves as switching costs instead of network effects. That is, a buyer incurs a switching cost when moving from one platform to the other, while the adoption decisions of other buyers do not affect his utility. Otherwise, the alternative model is as close as possible to our base model. In a one-period game, switching costs have the same consequences as focality with network effects and the equilibrium is identical to the one-period game in our base model. However, in a dynamic game with infinite horizon, the main results of Section 4 no longer hold in a model with switching costs. Therefore, the main conclusion of Section 4 is the result of network effects and cannot be replicated with switching costs.

Consider two platforms, A and B, that offer a base utility or quality, q_A and q_B, where for conciseness we assume that $q_B > q_A$. We abstract from network effects by assuming that there is one buyer.[14] If the buyer joined a platform in period t, he incurs a cost s when switching to the other platform at $t + 1$, where $s < q_A < q_B$. We focus the analysis on the case where at the beginning of the game, the buyer is "born" in platform A. This is equivalent to our base-model assumption that platform A is focal initially.

[14] This is equivalent to assuming that platforms can price discriminate between new and old buyers.

5.1 One-period game

Consider first a one-period game and an equilibrium in which platform A wins. In this equilibrium platform B charges $p_B^A = 0$, where p_i^f is the price of platform $i = \{A, B\}$ when at the beginning of the period the buyer is at platform f and therefore has to pay s to switch from f to the competing platform. In the context of switching cost, f denotes the "incumbent" platform.[15] This is analogous to f being a focal platform under network effects, where a buyer incurs disutility β if he decides to join alone the competing platform. Since the buyer starts the period on platform A, to win, platform A charges p_A^A such that: $q_A - p_A^A = q_B - s - p_B^A$. Hence, platform A earns $p_A^A = s - (q_B - q_A)$ and there is an equilibrium in which platform A wins if $s > q_B - q_A$. This condition is identical to the condition for the focal platform A winning under network effects, i.e., $\beta > q_B - q_A$, which may explain why, from first glance, network effects and switching costs may appear to drive the same results. This, however, does not hold in a dynamic game. In what follows, suppose that $s > q_B - q_A$ such that platform A has an "incumbency" advantage at $t = 1$.

5.2 Infinite time horizon

Consider now an infinitely repeated game. As before, at the beginning of $t = 1$ the buyer is "born" in platform A. The buyer and the two platforms discount future payoffs by δ.

Let's first consider a putative equilibrium equivalent to the Markov equilibrium under network effects in which platform A wins in all periods whether it is "focal" or not. In the context of switching costs, in this equilibrium, platform A attracts the buyer in every period regardless of whether the buyer starts the period on platform B and needs to pay s to switch to A, or the buyer starts the period on platform A and needs to pay s to switch to B. Define V_i^f as the discounted sum of payoffs of platform i from period t onwards, when the buyer is at platform f at the beginning of period t. Likewise, let U^f denote the discounted sum of the buyer's payoffs from period t onwards, when the buyer is in platform f at the beginning of period t.

In equilibrium, platform B charges $p_B^B = p_B^A = 0$ because platform B does not win the buyer when the buyer is initially at platform A, and loses the buyer if he is at platform B.

[15] Also in the terminology of Biglaiser, Crémer and Dobos (2013)

The price of platform A when the buyer starts the period on platform A, p_A^A solves

$$q_A - p_A^A + \delta U^A = q_B - p_B^A - s + \delta(q_A - p_A^B - s + \delta U^A). \tag{6}$$

The intuition for this condition is that in equilibrium, both platforms expect that even if platform B will win the incumbency position (that is, convince the buyer to switch from A to B), it will not be able to maintain it in the next period. Therefore, the buyer expects that if he will stay at platform A, he will earn $q_A - p_A^A$ at the current period and then continue to stay with platform A in the next period and earn U^A (the left hand side). If however the buyer switches (the right hand side), the buyer earns $q_B - p_B^A - s$ in the current period. In the next period, platform B will have an incumbency advantage on this particular buyer but the prices will be such that the buyer will find it optimal to switch back to platform A and will earn $q_A - p_A^B - s + \delta U^A$.

Notice that condition (6) is different than the analogous condition under pure network effects. The condition under network effects is

$$q_A - p_A^A + \beta + \delta U^f = q_B - p_B^A + \delta U^f\ ,$$

which reduces to $q_A - p_A^A + \beta = q_B - p_B^A$. This is because under network effects individual buyer's decision whether to join platform A or B does not affect the identity of the focal platform f in the beginning of the next period. Identity of f depends on the decisions of other buyers. And thus it is the same on both sides of the equation. In contrast, under switching costs, the identity of the incumbent platform depends solely on the buyer's last period decision which platform to join. Thus, in (6) we have δU^A if the buyer joins A, and $\delta U^B = \delta(q_A - p_A^B - s + \delta U^A)$ if he joins B.

In the case the buyer starts a period on platform B, platform A sets such p_A^B to attract the buyer, i.e., p_A^B solves

$$q_A - p_A^B - s + \delta U^A = q_B - p_B^B + \delta(q_A - p_A^B - s + \delta U^A).$$

The value functions given the prices are defined by

$$V_A^A = \frac{p_A^A}{1-\delta}, \quad V_A^B = p_A^B + \delta V_A^A \quad \text{and} \quad U^A = \frac{q_A - p_A^A}{1-\delta}.$$

Substituting the prices into the value functions yields:

$$V_A^A = -\frac{q_B - q_A}{1-\delta} + s, \quad V_A^B = -\frac{q_B - q_A}{1-\delta} - s \quad \text{and} \quad U^A = \frac{q_B}{1-\delta} - s.$$

Since $q_B > q_A$, the value V_A^B is negative. Thus, under switching cost there is no equilibrium in which platform A wins in both states.

Similarly, the value functions in the putative equilibrium in which platform B wins every period, whether it starts as an incumbent or not, are

$$V_B^B = \frac{q_B - q_A}{1-\delta} + s, \quad V_B^A = \frac{q_B - q_A}{1-\delta} - s \quad \text{and} \quad U^B = \frac{q_A}{1-\delta} - s.$$

Indeed, there is an equilibrium in which platform B wins in both states if and only if $q_B - q_A > s(1-\delta)$. In this equilibrium, the price that the incumbent platform B charges is $p_B^B = q_B - q_A + (1-\delta)s$. In an analogous equilibrium under network effects, i.e., platform B wins in both states, platform B as the focal platform charges $p_B^B = q_A - q_A + \beta$. Thus, for the same magnitude of switching cost and network effects, $\beta = s$, platform B charges lower prices when the cost represents switching cost than when it represents network effect. The price charged by platform B when it is not incumbent is $p_B^A = q_B - q_A - (1+\delta)s$, again lower than for same size network effects.

Next, consider a putative equilibrium equivalent to the Markov equilibrium under network effects in which the focal platform wins. In the context of switching cost, the incumbent platform wins over the buyer every period. In such a putative equilibrium, prices for the "non-incumbent" platform are

$$p_A^B = -\delta V_A^A \quad \text{and} \quad p_B^A = -\delta V_B^B.$$

The equilibrium prices of the incumbent solve

$$q_A - p_A^A + \delta U^A = q_B - p_B^A - s + \delta U^B,$$

$$q_B - p_B^B + \delta U^B = q_A - p_A^B - s + \delta U^A.$$

Since market expectations are that the incumbent platform wins, these expectations should also include the buyer. Therefore the buyer expects that if he switches from platform A to B, then the next period equilibrium prices will be such that he will find it optimal to stay with platform B forever.

Notice that again the difference between the two conditions above and the analogous conditions under network effects is that now the buyer can affect the identity of the platform with the incumbency advantage in the next period.

The value functions given the prices are defined by

$$V_A^A = \frac{p_A^A}{1-\delta}, \quad V_B^B = \frac{p_B^B}{1-\delta}, \quad U^A = \frac{q_A - p_A^A}{1-\delta} \quad \text{and} \quad U^B = \frac{q_B - p_B^B}{1-\delta}.$$

Substituting the prices into the value functions yields

$$V_A^A = -\frac{q_B - q_A}{1-\delta} + s, \quad V_B^B = \frac{q_B - q_A}{1-\delta} + s, \quad U^A = \frac{q_B}{1-\delta} - s \quad \text{and} \quad U^B = \frac{q_A}{1-\delta} - s.$$

Therefore, such an equilibrium exists if and only if $q_B - q_A < s(1-\delta)$. We summarize these findings in the following observation.

Observation *In the environment with switching costs,*

(i) *if $q_B - q_A < s(1-\delta)$, there is a unique Markov equilibrium in which platform A wins in all periods because it has the incumbency advantage at $t = 1$;*

(ii) *if $q_B - q_A > s(1-\delta)$, there is a unique Markov equilibrium in which platform B wins in all periods because it has a quality advantage.*

Figure 2 illustrates these results. Comparing this figure with Figure 1 shows that there are two differences between switching costs and network effects. First, the region in which platform A can win the market even when platforms are forward looking (the region where $\beta > \frac{1}{2}$ and $0 < q_B - q_A < \beta(2\delta - 1)$) vanishes under switching costs. Therefore, the main result of Section 4 — that platform A can exploit network effects in order to keep the focal position if δ is high — no longer holds if we reinterpret network effects as switching costs. The second difference is that the cutoff value of $q_B - q_A$ that distinguishes between the unique equilibrium in which platform A wins and the unique equilibrium in which platform B wins shift upwards from $q_B - q_A = \beta(1-2\delta)$ under network effects to $q_B - q_A = s(1-\delta)$ under switching costs.

These differences affect the inefficiencies that can occur in the market with switching costs and in the market with network effects. Excess inertia occurs in both markets. In the market with switching cost it occurs when $0 < q_B - q_A < s(1-\delta)$, i.e., for lower δ rather than for higher δ. In the market with network effects excess inertia occurs for low and for high δ. However, in the market with switching costs there is no equilibrium where excess momentum occurs, while such equilibrium exists in the market with network effects. That is, if platform A starts as the incumbent, and $q_A - q_B > 0$, platform B never wins under switching costs. However, under network effects, excess momentum in equilibrium is possible for high δ.

The intuition for these differences is the following. It is possible to think of network effects as indirect switching costs that other buyers inflict on an individual one. If all other buyers switch, an individual buyer "pays" switching costs if he does not switch. If they all stay, the buyer "pays" switching costs if he switches. As a consequence, with network effects, a buyer cannot affect the identity of the focal platform in the next period, which depends on market beliefs. In contrast, the regular switching costs are costs that the buyer inflicts on himself whenever the buyer chooses to switch from one platform to the other, regardless of the decisions of other buyers. Under switching costs the buyer can determine the identity of the platform with the incumbency advantage over himself in next period.

Therefore, switching costs affect our results in two ways. First, they eliminate the equilibria that arises due to excess momentum or excess inertia. These equilibria depend on the buyers' beliefs concerning the participation of other buyers, which do not play a role in the absence of network effects. This result highlights the role that network effects play in determining the identity of the dominant platform, and illustrates why it is important to study network effects in isolation from the standard switching costs. The second effect of switching costs is that they increase the threshold in the quality gap between the two platforms, from which onward platform B wins the market. Intuitively, with switching costs, the buyer is less willing to switch from platform A to platform B, as doing so grants platform B with an incumbency advantage over the buyer and reduces the buyer outside option of switching back to platform A.

6 Stochastic qualities

The analysis so far focused on the case where the qualities of the two platforms are constant for an infinite time horizon. We saw that, in any equilibrium, the same platform wins the market in all periods. Yet in many markets for platforms there is a shift in leadership every few years that results from improvements in technology. Therefore, in this section we consider the more realistic case in which platform qualities are stochastic. We demonstrate the existence of an equilibrium in which each platform has a positive probability of winning in each period. Our main finding is that when the distribution of qualities is dispersed, there

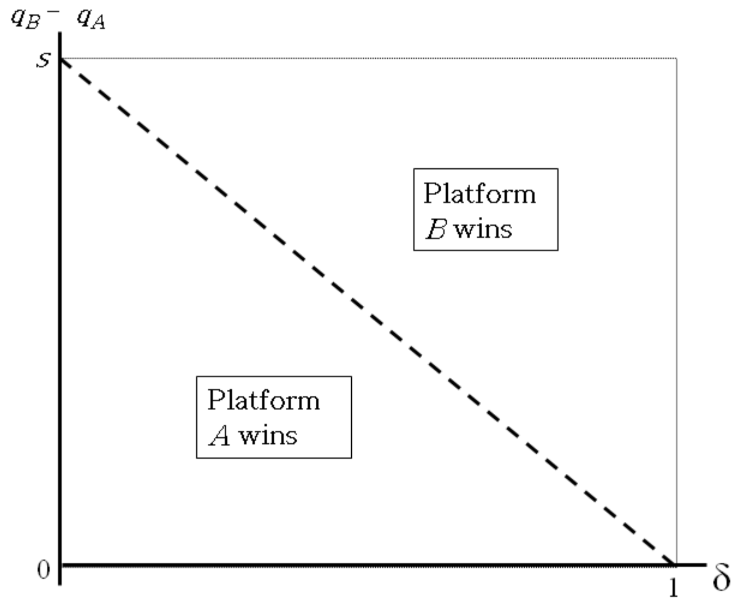

Figure 2: Equilibrium configuration with switching costs

is a unique Markov equilibrium, but in this equilibrium social welfare may decline with δ.

Suppose that the quality of each platform changes randomly from one period to the next. At the beginning of each period, each platform observes the realization of both its own quality and its competitor's quality for that particular period. Then the two platforms compete by setting prices.

We have shown in the previous sections that the equilibrium depends not on the absolute value of each platform's quality but rather on the difference between them. Hence we suppose, without loss of generality, that $Q \equiv q_B - q_A$ changes randomly in each period — with full support on the real line — according to a probability function $f(Q)$ with cumulative distribution function $F(Q)$. Our assumption of an infinite support ensures that there will be an equilibrium in which each platform can win the market with a positive probability.[16]

[16]This assumption is stronger than required because our results hold also when the support is finite,

Suppose that Q has a mean $\mu > 0$ such that, on average, platform B is of higher quality than platform A; the case of $\mu < 0$ is symmetric.

Let $\bar{Q}^A$ and $\bar{Q}^B$ denote equilibrium cut-offs such that, if platform A is focal in period t, it wins if $Q \leq \bar{Q}^A$ but otherwise cedes market dominance to platform B. Conversely, if platform B is focal in period t, then it wins if $Q \geq \bar{Q}^B$ but otherwise platform A wins.[17] This equilibrium has the feature that, when A is the focal platform, it will win in every period as long as $Q < \bar{Q}^A$. Then, once there is a realization with $Q > \bar{Q}^A$, platform B wins the market and becomes focal. Platform B will then maintain its focal position in future periods as along as $Q \geq \bar{Q}^B$, until there is a realization of q with $Q < \bar{Q}^B$, such that platform A wins back its focal position. The game repeats ad infinitum, with platforms "taking turns" at winning according to the realization of Q.

Let V_i^f denote the expected value function of platform i when platform f is focal. To solve for the equilibrium, suppose that platform A is focal in period t and that the quality difference has a particular realization Q. The lowest price that platform B is willing to charge in order to win the market is $-\delta V_B^B + \delta V_B^A$. This claim follows because B will earn the expected value V_B^B from becoming focal in the next period, and earn the expected V_B^A from remaining nonfocal. Given the price of platform B, the highest price that allows platform A to win the market is $p_A = \beta - Q - \delta V_B^B + \delta V_B^A$. Then A earns $p_A + \delta V_A^A$ if it does indeed win (i.e., when $Q \leq \bar{Q}^A$) or $0 + \delta V_A^B$ if it loses (i.e., when $Q > \bar{Q}^A$). Hence we can write

$$V_A^A = \int_{-\infty}^{\bar{Q}^A} (\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{Q}^A}^{\infty} \delta V_A^B f(q) dq.$$

Suppose now that platform A is nonfocal. Then the lowest price that platform B is willing to charge to maintain its focal position is $p_B^B = -\delta V_B^B + \delta V_B^A$. If A wins, it sets p_A^B that ensures that $-p_A^B \geq \beta - p_B^B + Q$, which writes as $p_A^B = -\beta - Q - \delta V_B^B + \delta V_B^A$. Then platform A earns $p_A^B + \delta V_A^A$ if it wins the market (i.e., when $Q \leq \bar{Q}^B$) or $0 + \delta V_A^B$ if it does not win (i.e., when $Q > \bar{Q}^B$). Therefore,

$$V_A^B = \int_{-\infty}^{\bar{Q}^B} (-\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{Q}^B}^{\infty} \delta V_A^B f(q) dq.$$

The cases of V_B^B and V_B^A are symmetric: platform B wins the market if $Q \geq \bar{Q}^B$ when it is focal or if $Q > \bar{Q}^A$ when it is not focal. Moreover, Q has a positive effect on B's profit. It

provided it is wide enough. However, assuming infinite support facilitates the analysis and allows us to avoid corner solutions.

[17]It is straightforward to see that any Markov equilibrium must have this form.

follows that

$$V_B^B = \int_{\bar{Q}^B}^{\infty} (\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{Q}^B} \delta V_B^A f(q) dq,$$

$$V_B^A = \int_{\bar{Q}^A}^{\infty} (-\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{Q}^A} \delta V_B^A f(q) dq.$$

Next consider the equilibrium $\bar{Q}^A$ and $\bar{Q}^B$. The equilibrium $\bar{Q}^A$ is such that, for $Q = \bar{Q}^A$, a focal platform A is indifferent between capturing the market or not, taking the equilibrium future value functions and the price of platform B as given. That is,

$$\beta - \bar{Q}^A - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Note that the condition for making the nonfocal platform B indifferent between winning and losing is equivalent to the condition just stated. Analogously, the equilibrium $\bar{Q}^B$ should be such that, for $Q = \bar{Q}^B$, a nonfocal platform A is indifferent between capturing the market or not, again taking the equilibrium future value functions and the price of platform B as given. Thus,

$$-\beta - \bar{Q}^B - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Once again, this condition is equivalent to the condition for making the focal platform B indifferent between winning and losing.

The last six equations define the equilibrium values of V_A^A, V_A^B, V_B^B, V_B^A, $\bar{Q}^A$ and $\bar{Q}^B$. In our next proposition, we use these equations to derive a sufficient condition for the equilibrium values of $\bar{Q}^A$ and $\bar{Q}^B$ to be unique.

Proposition 3 (Unique solutions to $\bar{Q}^A$ and $\bar{Q}^B$) *Suppose that $4\beta \max_q f(q) < 1$. Then there are unique equilibrium values of $\bar{Q}^A$ and $\bar{Q}^B$ as follows:*

(i) *if $\delta = 0$, then $\bar{Q}^A = \beta$ and $\bar{Q}^B = -\beta$;*

(ii) *$\bar{Q}^A - \bar{Q}^B = 2\beta$ for all δ.*

Proof. See Appendix.

The condition $4\beta \max_q f(q) < 1$ requires that the quality gap be sufficiently dispersed and that network effects not be too high. These conditions ensure that a nonfocal platform can always overcome its competitive disadvantage provided its realized quality is sufficiently high, and that there exist unique equilibrium values of $\bar{Q}^A$ and $\bar{Q}^B$. Proposition 3 also shows that, when evaluated at $\delta = 0$, the equilibrium cut-offs are $\bar{Q}^A = \beta$ and $\bar{Q}^B = -\beta$. It is intuitive that, when $\delta = 0$, the equilibrium is identical to the one-period benchmark in which a focal platform wins as long as its quality advantage outweigh the value of network effects.

We now study the effects of δ, β, and μ on the equilibrium values of $\bar{Q}^A$ and $\bar{Q}^B$. Toward that end, we make the simplifying assumption that $f(Q)$ is symmetric and unimodal around μ. That is: $f(\mu + x) = f(\mu - x)$; and $f(Q)$ is weakly increasing in Q for $Q < \mu$ and weakly decreasing in Q for $Q > \mu$. This is a sufficient but not a necessary condition for the results to follow. Those results may hold also when $f(Q)$ is neither symmetric nor unimodal — provided that $f(Q)$ places higher weights on positive than on negative values of Q, in which case B is more likely than A to be the focal platform in future periods. We also assume the uniqueness condition of Proposition 3, i.e., that $4\beta f(\mu) < 1$.

Proposition 4 (Effects of δ, β, and μ on $\bar{Q}^A$ and $\bar{Q}^B$) *Suppose that $f(.)$ is symmetric and unimodal around μ and that $4\beta f(\mu) < 1$. Then:*

(i) *Both $\bar{Q}^A$ and $\bar{Q}^B$ are decreasing in δ, and if $F(0) < 1/4$ then $\bar{Q}^A < 0$ when δ is sufficiently high.*

(ii) *Both $\bar{Q}^A$ and $\bar{Q}^B$ are decreasing in μ (holding constant the distribution of $Q - \mu$).*

(iii) *If $\delta < 1/2$, then $\bar{Q}^A$ is increasing and $\bar{Q}^B$ is decreasing in β; if $F(0) < 1/4$ and δ is close to 1, then $\bar{Q}^A$ is decreasing in β.*

Proof. See Appendix.

Figure 3 illustrates part (i) of Proposition 4. The figure reveals that an increase in δ need not increase the probability of the current period's higher-quality platform winning. To see why, consider first the case where platform B is focal. Then, at $\delta = 0$, the value is $\bar{Q}^B = -\beta$ and $\bar{Q}^B$ decrease with δ. Therefore, as δ increases, a focal B is more likely to win the market even when its quality realization is lower than platform A's, which implies that the probability of the "wrong" platform winning increases with δ. Now consider the case in which A is focal. Then, when δ is low, an increase in δ makes it less likely that a focal platform A will be able to maintain its focal position with quality realization that is lower than that of platform B, because $\bar{Q}^A$ is decreasing in δ. However, if δ is sufficiently

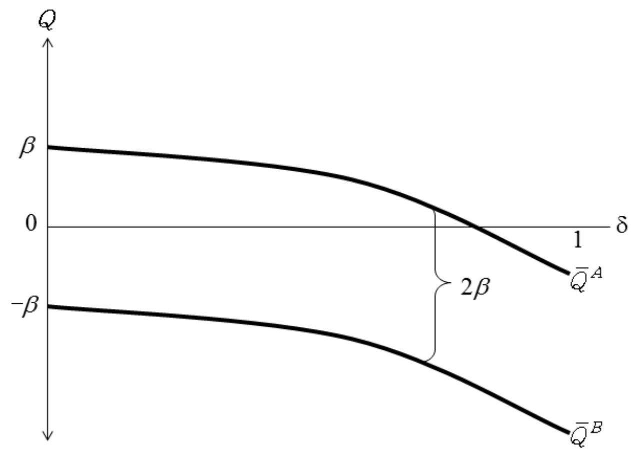

Figure 3: Effect of δ on $\bar{Q}^A$ and $\bar{Q}^B$ when $F(0) < 1/4$

high and if $F(0) < 1/4$, then $\bar{Q}^A$ crosses the horizontal axis, thus becoming negat ive and falling further below 0 as δ increases. In this case, platform A can lose the market even if it is focal and has higher quality realization than platform B (when $\bar{Q}^A < Q < 0$). Therefore, the probability of the "wrong" platform (i.e., one with lower quality realization) winning increases with δ when either A or B is focal.

The intuition underlying these results is as follows. Recall that a platform's expected profit depends on its current profit and the probability of maintaining its focal position in future periods. Since $\mu > 0$, we know that B is more likely to have higher quality realization than A in future periods. As δ increases, platform A takes into account that its chances of winning in future periods is lower and thus will have less incentive to compete aggressively in the current period. At the same time, platform B takes into account that it is more likely to win in future periods and so will have more incentive to compete aggressively in the current period. These effects increase B's competitive advantage over A even when the former has lower quality realization than the latter. If $F(0) < 1/4$, then μ is sufficiently high for platform B's competitive advantage to prevent platform A from capturing the market even when A is focal and offers a higher quality than does B.

These considerations also account for part (ii) of the proposition. As μ increases, it becomes more likely that platform B will have higher quality realization in future periods. Hence B's incentive to win in the current period increases, so both $\bar{Q}^A$ and $\bar{Q}^B$ decrease.

According to Proposition 4(iii), if δ is not too high then an increase in the strength of network effect makes it more likely that the focal platform wins. This result is similar to that in the one-period case. A stronger network effect increases the strategic advantage of being focal because it then becomes easier for the focal platform to attract consumers. But if δ is sufficiently high and if $F(0) < 1/4$, then a stronger network effect reduces the ability of focal platform A to retain its focal position. In this case we can see that an increase in the network effect increases the incentive of a nonfocal platform B to capture the market, because B is more likely than before to maintain its focal position, due to its higher expected quality.

We now turn our attention to social welfare. Our first question is whether social welfare is higher when platform B or rather when platform A is focal. Given that B is expected to be of higher quality than A, one could expect that social welfare is higher when platform B is focal. Yet according to Proposition 4, the probability of platform B winning despite platform A's higher quality realization increases with δ, a fact that may well offset the previous effect.

In order to investigate this issue, we normalize q_A to 0 and so $q_B = Q$. Let $\bar{W}^i$, $(i = A, B)$ denote the recursive expected social welfare when platform i is focal in period t; thus

$$\bar{W}^A = \int_{-\infty}^{\bar{Q}^A} (\beta + \delta \bar{W}^A) f(q) dq + \int_{\bar{Q}^A}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq,$$
$$\bar{W}^B = \int_{\bar{Q}^B}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq + \int_{-\infty}^{\bar{Q}^B} (\beta + \delta \bar{W}^A) f(q) dq.$$

Let $W^i = (1 - \delta)\bar{W}^i$ denote the one-period expected welfare. Our next proposition details the results from comparing W^A with W^B.

Proposition 5 (Effect of δ on social welfare) *Suppose that $f(.)$ is symmetric and unimodal around μ and that $4\beta f(\mu) < 1$. Then the following statements hold.*

(i) *When evaluated at $\delta = 0$, we have $W^B \geq W^A$; and W^A is increasing in δ while W^B is decreasing in δ.*

(ii) *There is a cut-off value δ' $(0 \leq \delta' \leq 1)$ such that $W^B > W^A$ for $\delta \in (0, \delta')$ and $W^A > W^B$ for $\delta \in (\delta', 1)$. A sufficient condition for $\delta' < 1$ is $F(0) < 1/4$.*

(iii) *When evaluated at $\delta = 1$, we have $W^A = W^B$.*

Proof. See Appendix.

Observe that the case where Q is distributed uniformly along a finite interval is a special case of the symmetric and unimodal distribution in which $f(Q)$ is constant. In this case $\delta' = 0$, so that $W_A > W_B$ for $\delta \in (0,1)$ and $W_A = W_B$ for $\delta = 0, 1$.

Part (i) of the proposition states that W_B is greater than W_A for low values of δ. In this case, the cut-offs $\bar{Q}^A$ and $\bar{Q}^B$ are close to their one-period levels: a focal platform A wins if $Q < \beta$, and a focal platform B wins if $Q > -\beta$. Because Q is more likely to be positive than negative, welfare is maximized when B starts out as the focal platform. But part (i) also shows that, for low values of δ, welfare W_B is decreasing in δ whereas W_A is increasing in δ. This follows because, as platforms become more patient, it becomes more likely that a focal platform B will win despite having quality realization lower than A, which reduces welfare. According to part (ii) of Proposition 5(ii), if platforms are sufficiently patient (i.e., if δ is sufficiently high) and if platform B is significantly more likely than platform A to be of higher quality ($F(0) < 1/4$), then social welfare is higher when A is the i nitial focal platform because otherwise B would have too much of a competitive advantage and so would win more often than it should. We remark that these results do not show that social welfare is maximized when platform A is focal in all periods. Rather, they imply that — in the first period only — it is welfare maximizing to start the dynamic game with platform A as focal even though platform B is of higher quality on average.

The results obtained so far suggest that social welfare in the one-period case might be greater than when platforms are patient. For the general distribution function, however, the comparison between social welfare evaluated at $\delta = 0$ and $\delta = 1$ is inconclusive. Hence in the following corollary we make the simplifying assumption that Q is uniformly distributed.

Corollary 1 (Welfare under uniform distribution) *Let Q be uniformly distributed along the interval $[\mu - \sigma, \mu + \sigma]$, and suppose that $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{\mu^2 + 6\mu\beta + \beta^2}$. Then*

$$\bar{Q}^A = \beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta} \quad \textit{and} \quad \bar{Q}^B = -\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}. \tag{7}$$

Moreover, $W^A|_{\delta=0} = W^B|_{\delta=0} > W^A|_{\delta=1} = W^B|_{\delta=1}$.

Proof. See Appendix.

This result is illustrated in Figure 4.

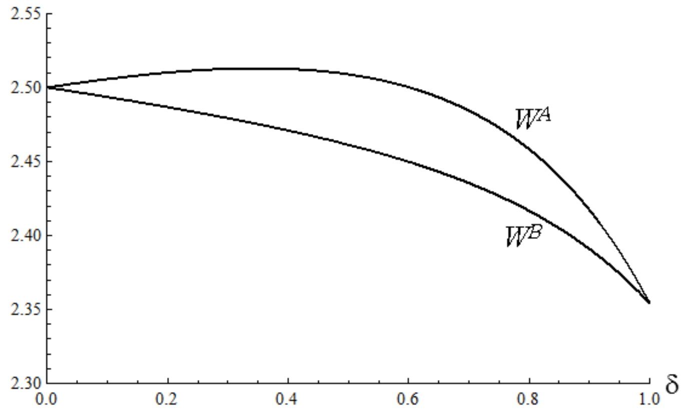

Figure 4: Effect of δ on welfare for a uniform distribution

7 Conclusions

In platform competition, offering the highest-quality product may not be enough to dominate the market. When there are network externalities, a platform's success depends not only on quality but also on consumers' beliefs that other consumers will adopt it. In a static model, a focal platform that has such a beliefs advantage may dominate the market despite offering lower quality; the result is an inefficient equilibrium. We ask whether this inefficiency can be eliminated in a dynamic game with a long time horizon.

In a model with long but finite time horizon we find that, indeed, the better platform wins and the efficient outcome is achieved when the future matters. More specifically, a higher-quality entrant can overcome the incumbent's network effect advantage. The future matters when both the time horizon is long and the discount factor is high. But if the discount factor is low, platforms are less concerned about the future and so the competition more nearly resembles a static game; in this case, inefficiency may persist even for a time horizon extended to infinity. We conclude that for a finite horizon social welfare is (weakly) increasing in the extent to which platforms are forward-looking, because forward orientation makes it more likely that consumers will be served by the higher-quality platform. Intuitively, a high-quality platform has more to gain by being focal in the game's final period than does the low-quality platform, which means that it will have more incentive to compete aggressively in early periods toward the end of capturing (or retaining) the focal position.

Once we modify the model to capture more realistic features, we find new sources of inefficiency even if the discount factor is high. A finite time horizon entails that platforms know when the last period occurs; if that is not known, then it is better to model it as an infinite horizon. Markov equilibria in the infinite game replicate those in the finite-horizon game extended to infinity. For high discount factors, however, additional and inefficient Markov equilibria arise in which the lower-quality platform dominates the market in all periods. These inefficient equilibria do not emerge in an alternative model in which switching costs replace network effects, because then each buyer's decision does not relay on the beliefs regarding the decisions of other buyers.

We also consider a scenario where the platforms' qualities change stochastically from period to period, which allows each platform to win any period with some probability. Here, the more the platforms are forward looking the less likely it is that even a high-quality platform will overcome its nonfocal position. This is because, if one platform is of higher quality on average than the other, then dynamic considerations give it more incentive (than in a static setting) to capture the market or to maintain its focal position, even if in the current realization it has lower quality. At the extreme, it is possible for a focal platform with higher quality realization to lose its dominance — provided that platforms are sufficiently forward looking. This finding indicates that, when qualities are stochastic, social welfare may decline as platforms become less myopic.

Our paper considers homogeneous consumers, which raises the question of how these results might be affected by the presence of heterogeneous consumers. When consumers differ in their valuations for different platforms, a focal position becomes less important for consumers. Armstrong (2006), for example, considers a continuum of consumers that differ in their preferences for two competing platforms. He shows that if the two platforms are sufficiently horizontally differentiated then for given platforms' prices, there is a unique allocation of consumers, such that each platform has a positive market share. Jullien and Pavan (2014) reach the same conclusion assuming that there is enough dispersion in beliefs about platforms' ability to attract consumers. Halaburda and Yehezkel (forthcoming) show that the importance of focality is decreasing in the extent to which consumers are loyal to a specific platform.

Applying the intuition behind these three papers, it is reasonable to expect that increasing consumer heterogeneity reduces the effect of focality on platform profits; hence platforms will be less inclined, in that case, to compete in the current period so as to secure a future focal position.

Of course, real-life consumers are heterogeneous. Nevertheless, our motivating examples reveal that, in many markets for platforms, an important role is played by consumers' coordination problems and by platform focality. As this paper addresses the effect of dynamic considerations on the focal position of platforms, our assumption of homogeneous consumers provides us with a tractable model for determining the net effect of that market position.

Our model also abstracts both from the presence of an installed base. Many markets with network effects are influenced by this factor, which constitute an additional force capable of driving excess inertia and resulting in an equilibrium where the lower-quality platform dominates for extended periods. Nonetheless, we abstract from installed base so that we can highlight the role of coordination problem as a driving force of excess inertia. While any market that exhibits network effects is affected not only by customer expectations but also by installed base, those markets are not all affected to the same extent. In the market for video-game consoles, for example, excess inertia is indeed likely driven by consumer expectations. New generations of the platforms are clearly distinguished from the previous ones by technological jumps, and backward compatibility seldom has limited appeal. In other markets, such as smartphones and computer operating systems, an installed base may play a more important role. These differences may explain the more frequent leadership changes observed in the market for video-game consoles compared with the market for computer operating systems. Even so, expectations and thus focality affect the market dynamics for the latter types of environments, too. We have demonstrated that, independently of other factors, it may lead to excess inertia and hence to reduced social welfare.

Appendix

Proof of Lemma 1

Let $\Pi_i^f(T)$ be the total discounted profit of platform i when platform f is focal at date $t = 1$ and there are T periods.

To win in $t = 1$, the focal platform A needs to set $p_{A1} \leq p_{B1} + q_A - q_B + \beta$, and set such p_{A1} that would force $p_{B1} \leq -\delta\Pi_B^B(T-1)$. That is, platform A wins when it sets

$$p_{A1} \leq p_{B1} + q_A - q_B + \beta = -\delta\Pi_B^B(T-1) + q_A - q_B + \beta,$$

in which case it earns

$$\Pi_A^A(T \mid A \text{ wins in } t = 1) = q_A - q_B - \delta\Pi_B^B(T-1) + \delta\Pi_A^A(T-1). \tag{8}$$

Notice that calculated in such a way the profit under the condition of winning may be negative. Then, the optimal action is to cede the market and earn no profit. Therefore, the profit from unconditionally optimal actions is $\Pi_i^f(T) = \max\{\Pi_i^f(T| \, i \text{ wins in } t = 1), 0\}$. Using similar logic,

$$\begin{aligned}\Pi_B^A(T \mid B \text{ wins in } t = 1) &= q_B - q_A - \beta - \delta\Pi_A^A(T-1) + \delta\Pi_B^B(T-1) \\ &\equiv -\Pi_A^A(T \mid A \text{ wins in } t = 1).\end{aligned} \tag{9}$$

Let $\hat{\Pi}_i^f(T) \equiv \Pi_i^f(T \mid i \text{ wins in } t = 1)$. Then $\Pi_i^f(T) = \max\{\hat{\Pi}_i^f, \delta\Pi_i^j(T-1)\}$. Notice that $\Pi_i^f(T)$ is bounded by 0, while $\hat{\Pi}_i^f(T)$ is not.

Suppose that $\hat{\Pi}_i^i(k) > 0$ for both $i = A$, B and $k = 1, \ldots, T-1$. Then from (8) we obtain[18]

$$\begin{aligned}\hat{\Pi}_i^i(T) &= q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T-1) + \delta\hat{\Pi}_i^i(T-1) \\ &= (q_i - q_j)\sum_{k=1}^{T}(2\delta)^{k-1} + \beta = (q_i - q_j)\frac{1-(2\delta)^T}{1-2\delta} + \beta.\end{aligned} \tag{10}$$

[18]This follows from applying the same formulas recursively in

$$\hat{\Pi}_i^i(T-1) - \hat{\Pi}_j^j(T-1) = 2(q_i - q_j) + 2\delta[\hat{\Pi}_i^i(T-2) - \hat{\Pi}_j^j(T-2)] = 2(q_i - q_j)\sum_{k=1}^{T-1}(2\delta)^{k-1}.$$

The fraction $\frac{1-(2\delta)^T}{1-2\delta}$ is positive and increasing with T. Therefore, $\hat{\Pi}_i^i(T)$ is also monotonic. When $q_i - q_j > 0$, then $\hat{\Pi}_i^i(T)$ is positive and increasing. Conversely, when $q_i - q_j < 0$ then $\hat{\Pi}_i^i(T)$ is decreasing and when $q_i - q_j < -\beta\frac{1-2\delta}{1-(2\delta)^T}$, it may even be negative.[19] And once it is negative, it stays negative for all larger T.

Now, suppose $(q_i - q_j)\frac{1-(2\delta)^T}{1-2\delta} + \beta < 0$. By the monotonic properties of $\frac{1-(2\delta)^T}{1-2\delta}$, it may only happen for $q_i < q_j$, and there exists $T_i \leq T$ such that $(q_i - q_j)\frac{1-(2\delta)^{T_i}}{1-2\delta} + \beta < 0$ and either $(q_i - q_j)\frac{1-(2\delta)^{T_i-1}}{1-2\delta} + \beta > 0$ or $q_i - q_j + \beta < 0$. In the latter case, $T_i = 1$. That is, T_i is the shortest time horizon for which it is not worth capturing the market. For time horizon T_i and shorter, $\hat{\Pi}_i^i(T)$ for $i = A, B$ can be calculated using (10) — but not for longer horizons.

Lemma 4 *If $\hat{\Pi}_i^i(T) < 0$ then, for all $T' > T$, $\hat{\Pi}_i^i(T') < 0$.*

Proof. Suppose $T_i > 1$. By definition of T_i, $\hat{\Pi}_i^i(T_i - 1) > 0$ (and given by (10)), and

$$\hat{\Pi}_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i - 1) + \delta\hat{\Pi}_i^i(T_i - 1) < 0. \tag{11}$$

Now $\hat{\Pi}_i^i(T)$ for $T > T_i$ can no longer be calculated using (10). We need to apply (9) directly:

$$\hat{\Pi}_i^i(T_i + 1) = q_i - q_j + \beta - \delta\Pi_j^j(T_i) + \delta\Pi_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i)$$

since $\Pi_j^j(T_i) = \hat{\Pi}_j^j(T_i)$ and $\Pi_i^i(T_i) = 0$.

By properties of (10), $\hat{\Pi}_j^j(T_i) > \hat{\Pi}_j^j(T_i - 1)$. From $\hat{\Pi}_i^i(T_i) < 0$, we have $\delta\Pi_j^j(T_i - 1) > q_i - q_j + \beta + \delta\hat{\Pi}_i^i(T_i - 1) > q_i - q_j + \beta$. Thus $\delta\hat{\Pi}_j^j(T_i) > q_i - q_j + \beta$ and $\hat{\Pi}_i^i(T_i + 1) < 0$ — and so forth for each $T > T_i$. ■

Thus, for $T > T_i$, $\Pi_i^i(T) = 0$. Moreover, $\Pi_j^j(T) = \hat{\Pi}_j^j(T)$ also can no longer be calculated using (10). Applying (9) directly:

$$\Pi_j^j(T_i + 1) = q_j - q_i + \beta + \delta\Pi_j^j(T_i),$$
$$\Pi_j^j(T_i + 2) = q_j - q_i + \beta + \delta(q_j - q_i + \beta) + \delta^2\Pi_j^j(T_i).$$

More generally, for any $T > T_i$ we have

$$\begin{aligned}\Pi_j^j(T) &= (q_j - q_i + \beta)\sum_{t=1}^{T-T_i}\delta^{t-1} + \delta^{T-T_i}\Pi_j^j(T_i)\\ &= (q_j - q_i + \beta)\frac{1-\delta^{T-T_i}}{1-\delta} + \delta^{T-T_i}\left((q_j - q_i)\frac{1-(2\delta)^{T_i}}{1-2\delta} + \beta\right).\end{aligned}$$

[19] This also implies that one of the $\Pi_i^i(T)$ must be positive. A negative $\hat{\Pi}_i^i(T)$ for some T implies $q_i - q_j < 0$, and $q_j - q_i > 0$ implies $\hat{\Pi}_j^j(T) > 0$ for all T.

Notice that for the case when $T_i = 1$, $\Pi_j^j(T)$ reduces to $(q_j - q_i + \beta)\frac{1-\delta^T}{1-d}$.

Now, using those properties of $\Pi_i^i(T)$, for $i = A, B$, we can consider following cases.

(i) $|q_A - q_B| < \beta\frac{1-2\delta}{1-(2\delta)^T}$
Then both $\hat{\Pi}_A^A(k)$ and $\hat{\Pi}_B^B(k)$ are positive for all $k = 1, \dots, T$. Since platform A is focal in $t = 0$ and $\hat{\Pi}_A^A(T)$ is positive, the platform never cedes the market and its profit is $\Pi_A^A(T) = \hat{\Pi}_A^A(T) = (q_A - q_B)\frac{1-(2\delta)^T}{1-2\delta} + \beta$ (by (10)).

(ii) $q_A - q_B > \beta\frac{1-2\delta}{1-(2\delta)^T}$
That is, $(q_B - q_A)\frac{1-(2\delta)^T}{1-2\delta} + \beta < 0$, and thus, by the foregoing arguments, there exists a $T_B < T$. This means that $\hat{\Pi}_B^B(T) < 0$; that is, platform B would not find it worthwhile to win the market even if it was focal, given A's quality advantage. Platform A wins the market, but the prices it charges and profit depend on T_B, as derived earlier:

$$\Pi_A^A(T) = (q_A - q_B + \beta)\frac{1-\delta^{T-T_B}}{1-\delta} + \delta^{T-T_B}\left((q_A - q_B)\frac{1-(2\delta)^{T_B}}{1-2\delta} + \beta\right).$$

(iii) $q_B - q_A > \beta\frac{1-2\delta}{1-(2\delta)^T}$
Now there exists a $T_A < T$. That is, $\hat{\Pi}_A^A(T) < 0$; in other words, it is not worthwhile for platform A to defend the market in $t = 1$, given the quality advantage of platform B. Then platform B wins the market in $t = 1$, becomes the focal platform and keeps the market for the rest of the time horizon. To win the market, in $t = 1$, platform B sets $p_{B1}^A = q_B - q - A - \beta$, while platform A sets $p_{A1}^A = 0$. In the next period, platform B is the focal platform with quality advantage and with $T - 1$ period time horizon. Thus, the discounted total profit is as that of platform A in case (ii), with relabeling the platforms and length of the time horizon. That is:

$$\begin{aligned}
\Pi_B^A(T) &= q_B - q_A - \beta + \delta\left[(q_B - q_A + \beta)\frac{1-\delta^{T-1-T_A}}{1-\delta} + \delta^{T-1-T_A}\left((q_A - q_B)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right)\right] \\
&= (q_B - q_A + \beta)\frac{1-\delta^{T-T_A}}{1-\delta} + \delta^{T-T_A}\left((q_B - q_A)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right) - 2\beta.
\end{aligned}$$

This completes the proof of Lemma 1.

Proof of Proposition 3

Directly from the formulas for V_A^A, V_A^B, V_B^B, V_B^A, and conditions for $\bar{Q}^A$ and $\bar{Q}^B$, we obtain

$$\bar{Q}^A - \bar{Q}^B = 2\beta.$$

Moreover,

$$
\begin{aligned}
V_A^A &= \int_{-\infty}^{\bar{Q}^A} (\bar{Q}^A - q) f(q)\, dq + \delta V_A^B, \\
V_A^B &= \int_{-\infty}^{\bar{Q}^B} (\bar{Q}^B - q) f(q) dq + \delta V_A^B = \frac{1}{1-\delta} \int_{-\infty}^{\bar{Q}^B} (\bar{Q}^B - q) f(q)\, dq
\end{aligned}
$$

and

$$
\begin{aligned}
V_B^B &= \int_{\bar{Q}^B}^{+\infty} (q - \bar{Q}^B) f(q)\, dq + \delta V_B^A, \\
V_B^A &= \frac{1}{1-\delta} \int_{\bar{Q}^A}^{+\infty} (q - \bar{Q}^A) f(q)\, dq.
\end{aligned}
$$

The optimality condition is then

$$
\bar{Q}^A = \beta - \delta V_B^B + \delta V_B^A + \delta V_A^A - \delta V_A^B,
$$

which can be rewritten as

$$
\bar{Q}^A = \beta + \delta \phi(\bar{Q}^A), \tag{12}
$$

where

$$
\begin{aligned}
\phi(\bar{Q}^A) = & \int_{\bar{Q}^A}^{+\infty} (q - \bar{Q}^A) f(q)\, dq + \int_{-\infty}^{\bar{Q}^A} \big(\bar{Q}^A - q\big)\, f(q)\, dq \\
& - \int_{-\infty}^{\bar{Q}^B} (\bar{Q}^B - q) f(q)\, dq - \int_{\bar{Q}^B}^{+\infty} (q - \bar{Q}^B) f(q)\, dq.
\end{aligned}
$$

Integrating by parts yields

$$
\phi(\bar{Q}^A) = -2\beta + 2 \int_{\bar{Q}^A - 2\beta}^{\bar{Q}^A} F(q) dq. \tag{13}
$$

Now we have

$$
\begin{aligned}
\phi'(\bar{Q}^A) &= 2\big(F(\bar{Q}^A) - F(\bar{Q}^A - 2\beta)\big), \\
\phi(-\infty) &= -2\beta, \\
\phi(+\infty) &= 2\beta.
\end{aligned}
$$

These properties imply that $\bar{Q}^A > \beta + \delta\phi(\bar{Q}^A)$ for $\bar{Q}^A = \infty$ and that $\bar{Q}^A < \beta + \delta\phi(\bar{Q}^A)$ for $\bar{Q}^A = -\infty$. Hence there is a unique solution to $\bar{Q}^A$ if $\bar{Q}^A - \beta - \delta\phi(\bar{Q}^A)$ is increasing in $\bar{Q}^A$ i.e. if $\delta\phi'(\bar{Q}^A) < 1$. We observe that $\delta\phi'(\bar{Q}^A) < 1$ when

$$2\delta \max_q \big(F(q) - F(q - 2\beta)\big) < 1.$$

In this case, the equilibrium is unique. This is the case for all δ and if $4\beta \max_q f(q) < 1$.

Finally, notice that when evaluated at $\delta = 0$, the solution to $\bar{Q}^A = \beta + \delta\phi(\bar{Q}^A)$ is $\bar{Q}^A = \beta$.

This completes the proof of Proposition 3

Proof of Proposition 4

Proof of part (i): Since $\bar{Q}^A = \beta + \delta\phi(\bar{Q}^A)$,

$$\frac{\partial \bar{Q}^A}{\partial \delta} = \frac{\phi(\bar{Q}^A)}{1 - \delta\phi'(\bar{Q}^A)}.$$

From the proof of Proposition 3, if $4\beta f(\mu) < 1$ then $1 - \delta\phi'(\bar{Q}^A) > 0$. To see that $\phi(\bar{Q}^A) < 0$ for all $\bar{Q}^A \leq \beta$, suppose first that $\bar{Q}^A < \mu$. Then

$$\phi(\bar{Q}^A) = -2\int_{\bar{Q}^A - 2\beta}^{\bar{Q}^A} \left(\frac{1}{2} - F(q)\right) dq < 0,$$

where the inequality follows because symmetric and unimodal distribution (SUD) implies that for all $Q < \mu$, $F(Q) < 1/2$. Next, consider $\mu < \bar{Q}^A \leq \beta$. Then:

$$\phi(\bar{Q}^A) = -2\int_{\bar{Q}^A - 2\beta}^{\mu - (\bar{Q}^A - \mu)} \left(\frac{1}{2} - F(q)\right) dq - 2\int_{\mu - (\bar{Q}^A - \mu)}^{\mu + (\bar{Q}^A - \mu)} \left(\frac{1}{2} - F(q)\right) dq < 0,$$

where the first term is negative because $\bar{Q}^A > \mu > 0$ and SUD implies that $F(\mu - (\bar{Q}^A - \mu)) < F(\mu) = \frac{1}{2}$ and the second term equals 0 because SUD implies that $F(\mu + x) - \frac{1}{2} = \frac{1}{2} - F(\mu - x)$. Since $\phi(\bar{Q}^A) < 0$ we have $\frac{\partial \bar{Q}^A}{\partial \delta} < 0$, and since $\bar{Q}^B = \bar{Q}^A - 2\beta$ it follows that $\frac{\partial \bar{Q}^B}{\partial \delta} < 0$.

Next, $\bar{Q}^A < 0$ if

$$0 > \beta + \delta\phi(0),$$

which holds for δ large if

$$-\beta > \phi(0) = -2\beta\big(1 - 2F(-2\beta)\big) + \int_{-2\beta}^{0} (-2q) f(q)\, dq = -2\beta + 2\int_{-2\beta}^{0} F(q)\, dq$$

or if

$$\beta > 2\int_{-2\beta}^{0} F(q)\, dq.$$

This inequality holds for all β if $F(0) < 1/4$.

Proof of part (ii): Let $F(Q;\mu)$ denote the $F(Q)$ given μ. We have:

$$\frac{\partial \bar{Q}^A}{\partial \mu} = \frac{2\int_{\bar{Q}^A-2\beta}^{\bar{Q}^A} \left(\frac{\partial F(Q;\mu)}{\partial \mu}\right) dq}{1-\delta\phi'(\bar{Q}^A)} < 0,$$

where the inequality follows because SUD implies that $F(q;\mu)$ is decreasing in μ.

Proof of part (iii): We have:

$$\frac{\partial \bar{Q}^A}{\partial \beta} = \frac{1-2\delta+4\delta F(\bar{Q}^A-2\beta)}{1-\delta\phi'(\bar{Q}^A)} > 0,$$

where the inequity follows because $1-2\delta+4\delta F(\bar{Q}^A-2\beta) > 0$ if $\delta < \frac{1}{2}$. Since $\bar{Q}^B = \bar{Q}^A - 2\beta$, it follows that

$$\frac{\partial \bar{Q}^B}{\partial \beta} = -\left[\frac{1+\delta(2-4F(\bar{Q}^A))}{1-2\delta(F(\bar{Q}^A)-F(\bar{Q}^A-2\beta))}\right] < 0,$$

where the inequality follows because the numerator in brackets is positive when $\delta < \frac{1}{2}$ (since $F(\bar{Q}^A) < 1$) and because the denominator is positive when $\delta < \frac{1}{2}$ (since $F(\bar{Q}^A) - F(\bar{Q}^A - 2\beta) < 1$). When $F(0) < 1/4$ and $\delta = 1$, we have

$$\left.\frac{\partial \bar{Q}^A}{\partial \beta}\right|_{\delta=1} = \frac{-1+4F(\bar{Q}^B)}{1-\phi'(\bar{Q}^A)} < \frac{-1+4\frac{1}{4}}{1-\phi'(\bar{Q}^A)} = 0,$$

where the inequality follows because $F(\bar{Q}^B) < F(0) < 1/4$.

This completes the proof of Proposition 4.

Proof of Proposition 5

Solving for W^A and W^B, we obtain

$$W^A = \beta + \frac{(1-\delta+\delta F(\bar{Q}^B)\int_{\bar{Q}^A}^{\infty} qf(q)\, dq + \delta(1-F(\bar{Q}^A))\int_{\bar{Q}^B}^{\infty} qf(q)\, dq}{1-\delta F(\bar{Q}^A)+\delta F(\bar{Q}^B)},$$

$$W^B = \beta + \frac{\delta F(\bar{Q}^B)\int_{\bar{Q}^A}^{\infty} qf(q)\, dq + (1-\delta F(\bar{Q}^A))\int_{\bar{Q}^B}^{\infty} qf(q)\, dq}{1-\delta F(\bar{Q}^A)+\delta F(\bar{Q}^B)}.$$

First consider W^A. Solving the derivative of W^A with respect to δ and then evaluating at $\delta = 0$ yields

$$\begin{aligned}\frac{\partial W^A}{\partial \delta}\bigg|_{\delta=0} &= (1 - F(\bar{Q}^A))\bigg(\int_{\bar{Q}^B}^{\infty} qf(q)\,dq - \int_{\bar{Q}^A}^{\infty} qf(q)\,dq\bigg) - f(\bar{Q}^A)\bar{Q}^A\frac{\partial \bar{Q}^A}{\partial \delta} \\ &= (1 - F(\beta))\int_{-\beta}^{\beta} qf(q)\,dq - f(\beta)\beta\frac{\partial \bar{Q}^A}{\partial \delta},\end{aligned}$$

where the equality follows from the substitutions $\bar{Q}^A = \beta$ and $\bar{Q}^B = -\beta$. By our assumption of SUD, $\int_{-\beta}^{\beta} qf(q)\,dq \geq 0$ (proof available upon request), implying the the first term is nonnegative. Since Proposition 4 shows that $\bar{Q}^A$ is decreasing in δ, the second term is positive implying that $\frac{\partial W^A}{\partial \delta}|_{\delta=0} > 0$.

Next consider W^B. Solving the derivative of W^B with respect to δ and then evaluating at $\delta = 0$, we have

$$\begin{aligned}\frac{\partial W^B}{\partial \delta}\bigg|_{\delta=0} &= -F(\bar{Q}^B)\bigg(\int_{\bar{Q}^B}^{\infty} qf(q)\,dq - \int_{\bar{Q}^A}^{\infty} qf(q)\,dq\bigg) - f(\bar{Q}^B)\bar{Q}^B\frac{\partial \bar{Q}^B}{\partial \delta} \\ &= -F(-\beta)\int_{-\beta}^{\beta} qf(q)\,dq + f(-\beta)\beta\frac{\partial \bar{Q}^B}{\partial \delta},\end{aligned}$$

where the equality follows from the substitutions $\bar{Q}^B = -\beta$ and $\bar{Q}^A = \beta$. Again by our assumption of SUD, $\int_{-\beta}^{\beta} qf(q)dq \geq 0$, implying the the first term is nonpositive. Since Proposition 4 shows that $\bar{Q}^B$ is decreasing in δ, the second term is also negative implying that $\frac{\partial W^B}{\partial \delta}|_{\delta=0} < 0$.

Now we consider the gap $W^B - W^A$:

$$W^B - W^A = \frac{(1-\delta)(\int_{\bar{Q}^B}^{\infty} qf(q)dq - \int_{\bar{Q}^A}^{\infty} qf(q)dq)}{1 - \delta F(\bar{Q}^A) + \delta F(\bar{Q}^B)} = \frac{(1-\delta)}{1 - \delta F(\bar{Q}^A) + \delta F(\bar{Q}^B)} M(\bar{Q}^A),$$

where

$$M(\bar{Q}^A) = \int_{\bar{Q}^A - 2\beta}^{\bar{Q}^A} qf(q)dq.$$

Since $1 \geq F(q) \geq 0$ and $0 \leq \delta \leq 1$, it follows that $\text{sgn}(W^B - W^A) = \text{sgn}(M(\bar{Q}^A))$.

Consider first $\delta = 0$ such that $\bar{Q}^A = \beta$. Then, SUD implies $M(\beta) = \int_{-\beta}^{\beta} qf(q)dq \geq 0$ and $W^B - W^A \geq 0$. Second, consider $\delta = 1$. Then $W^B - W^A = \frac{0}{1}M(\bar{Q}^A)$, where $M(\bar{Q}^A)$ is finite; hence $W^B - W^A = 0$.

Next, we turn to $1 > \delta'$. We distinguish between two case, $F(0) < 1/4$ and $F(0) > 1/4$, which will be analyzed in turn.

Case 1: $F(0) < 1/4$. In this case, Proposition 4 implies that there is a cutoff, δ'' where δ'' is the solution to $\bar{Q}^A = 0$, such that $\bar{Q}^A > 0$ for $\delta \in [0, \delta'')$ and $\bar{Q}^A < 0$ for $\delta \in (\delta'', 1]$. For all $\delta \in [\delta'', 1]$, $M(\bar{Q}^A) < 0$ because $Q < 0$ for all $Q \in [\bar{Q}^A - 2\beta, \bar{Q}^A]$. For $\delta \in [0, \delta'')$, $M(\bar{Q}^A)$ is decreasing with δ. To see why:

$$\frac{\partial M(\bar{Q}^A)}{\partial \delta} = [\bar{Q}^A f(\bar{Q}^A) - (\bar{Q}^A - 2\beta) f(\bar{Q}^A - 2\beta)] \frac{\partial \bar{Q}^A}{\partial \delta}.$$

The term inside brackets is positive for all $\delta \in [0, \delta'']$ because $\bar{Q}^A \geq 0$ and $f(q) > 0$ and because $\bar{Q}^A \leq \beta$ implies that $\bar{Q}^A - 2\beta \leq \beta - 2\beta = -\beta < 0$. Since $\bar{Q}^A$ is decreasing in δ, we have $\frac{\partial M(\bar{Q}^A)}{\partial \delta} < 0$.

To summarize, $M(\bar{Q}^A) \geq 0$ for $\delta = 0$, $M(\bar{Q}^A)$ is decreasing with δ for $\delta \in [0, \delta'')$, and $M(\bar{Q}^A) < 0$ for $\delta \in (\delta'', 1]$. Hence there is a unique cutoff $\delta' < \delta''$ such that $M(\bar{Q}^A) > 0$ for $\delta \in [0, \delta')$ and $M(\bar{Q}^A) < 0$ for $\delta \in (\delta', 1]$. Since $\operatorname{sgn}(W^B - W^A) = \operatorname{sgn} M(\bar{Q}^A)$, this implies that $W_B > W_A$ for $\delta \in [0, \delta')$ and $W_B < W_A$ for $\delta \in (\delta', 1)$.

Case 2: $F(0) > 1/4$. In this case, $\bar{Q}^A > 0$ at $\delta = 1$. Notice that $M(\bar{Q}^A)$ is decreasing with δ for all $\delta \in [0, 1]$ (the proof that $\frac{\partial M(\bar{Q}^A)}{\partial \delta} < 0$ requires only that $\bar{Q}^A > 0$ which holds in Case 2 for all $\delta \in [0, 1]$). However, unlike Case 1, now $M(\bar{Q}^A)$ at $\delta = 1$ can be either positive or negative. It will be positive if $\bar{Q}^A$ at $\delta = 1$ is sufficiently higher than 0, in which case $M(\bar{Q}^A) > 0$ for all $\delta \in [0, 1]$ and so $W_B > W_A$ for all $\delta \in [0, 1)$. In this case $\delta' = 1$. Note that $M(\bar{Q}^A)$ can be negative at $\delta = 1$ if $\bar{Q}^A$ at $\delta = 1$ is sufficiently close to 0, in which case at $\delta = 1$; then $M(\bar{Q}^A) < 0$ and so $W_B > W_A$ for $\delta \in [0, \delta')$ and $W_B < W_A$ for $\delta \in (\delta', 1)$, as in Case 1.

Remark on uniform distribution. If the distribution is uniform, then $M(\bar{Q}^A) = 0$ at $\delta = 0$ and $M(\bar{Q}^A) < 0$ otherwise. This implies that $W_A > W_B$ for all $\delta \in (0, 1)$ and $W_A = W_B$ otherwise.

This completes the proof of Proposition 5.

Proof of Corollary 1

Substituting $F(Q) = \frac{Q+\sigma}{2\sigma}$ into (13) yields (7). To ensure that $\bar{Q}^B > \mu - \sigma$, we need σ to be large enough that $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{\mu^2 + 6\mu\beta + \beta^2}$. Observe that this assumption implies that $\sigma > 2\beta$. The recursive expected social welfare functions are then

$$\bar{W}^A = \int_{\mu-\sigma}^{\beta-\frac{2\delta\mu\beta}{\sigma-2\delta\beta}} (\beta+\delta\bar{W}^A)\frac{1}{2\sigma}\,dq + \int_{\beta-\frac{2\delta\mu\beta}{\sigma-2\delta\beta}}^{\mu+\sigma} (\beta+q+\delta\bar{W}^B)\frac{1}{2\sigma}\,dq,$$

$$\bar{W}^B = \int_{-\beta-\frac{2\delta\mu\beta}{\sigma-2\delta\beta}}^{\mu+\sigma} (\beta+q+\delta\bar{W}^B)\frac{1}{2\sigma}\,dq + \int_{\mu-\sigma}^{-\beta-\frac{2\delta\mu\beta}{\sigma-2\delta\beta}} (\beta+\delta\bar{W}^A)\frac{1}{2\sigma}\,dq.$$

Therefore,

$$\begin{aligned}
W^A &= (1-\delta)\bar{W}^A \\
&= \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + \frac{\mu(4\delta^2\beta^2(2\beta-3\sigma) - \sigma^2(\mu+2\sigma) + \delta\beta\sigma(5\mu-4\beta+10\sigma)))}{(\delta\beta-\sigma)(\sigma-2\delta\beta)^2}\right), \\
W^B &= (1-\delta)\bar{W}^B \\
&= \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + 2\mu + \frac{(\mu(8(-1+\delta)\delta^2\beta^3 + \delta\beta(5\mu-4(-1+\delta)\beta)\sigma - \mu\sigma^2))}{(\delta\beta-\sigma)(\sigma-2\delta\beta)^2}\right).
\end{aligned}$$

The gap $W^A - W^B$ can now be written as

$$W^A - W^B = \frac{2(1-\delta)\delta\mu\beta^2}{(\sigma-\delta\beta)(\sigma-2\delta\beta)}.$$

As $\sigma > 2\beta$ (by assumption), $W^A - W^B > 0$ for all $0 < \delta < 1$ and $W^A - W^B = 0$ for $\delta = 0$ and $\delta = 1$. Moreover:

$$W^A|_{\delta=0} - W^A|_{\delta=1} = \frac{\mu^2\beta^2(2\sigma-\beta)}{\sigma(\sigma-\beta)(\sigma-2\beta)^2} > 0;$$

where the inequality follows because, by assumption, $\sigma > 2\beta$ and $\mu > 0$.

This completes the proof of Corollary 1.

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