ON THE ORIGIN OF THE UNIVERSE IN THE CONTEXT OF STRING MODELS ☆

Y. AHARONOV and A. CASHER

Physics Department, University of South Carolina, Columbia, SC 29208, USA
and School of Physics and Astronomy, Tel Aviv University, 69978 Ramat Aviv, Israel

Received 6 November 1985

Using some very general considerations, we propose that in the context of the string model the initial state of the expanding universe was a single string Some difficulties in the string picture are pointed out

It has been suggested [1,2] that the anomaly-free ten-dimensional superstring model provides a basis for the construction of a finite and consistent theory of quantum gravity unified with all other forces. A "theory of everything" should also explain the origin of the universe as a dynamical process. More specifically, well-defined initial conditions which develop into the adiabatically expanding hot universe of the standard model should replace the classical singularity.

In what follows, we make use of some of the general properties of the string model in order to identify this initial state. The considerations we use are mainly thermodynamical and we thus expect them to be independent of the detailed properties of the relevant model. The difficulties associated with the proposed scenario are related to the basic consistency of the string theory itself and will be briefly discussed at the end.

The most fundamental property of all string models (in any dimension, with or without compactifications and fermions) is the character of the particle mass spectrum:

$$\tfrac{1}{4}M^2(\{N_J\}) = \sum_{\{n_I\}} N_{n_I} n_I - C. \tag{1}$$

In eq. (1) $\{n_I\}$ are the integer valued frequencies, N_{n_I} is the occupation number of the indicated mode, I is an index which goes over the various Bose and Fermi coordinates, C is a normal-ordering finite constant, and we have put the string tension α' to be unity; the factor $\frac{1}{4}$ pertains to a closed string.

An immediate result of eq. (1) is the emergence of an exponential density of states:

$$\rho(M) \underset{M \gg 1}{\sim} kM^{-3}\exp(MT_c^{-1}), \tag{2}$$

where k and β_c depend on the model but are both O(1). T_c is of course the Hagedern temperature and provides an upper limit for the temperature achievable by any system in equilibrium with strings ($T_c = \pi^{-1}(3/D)^{1/2}$ for a bosonic string in D transverse dimensions).

Consider now a system composed of a large number N of strings whose average mass is m and whose average momentum (absolute value) is p, such that their total energy is M. The entropy of this system is thus

$$S(N,m,p) \sim N[\ln(p^d v) + T_c^{-1}m - 3\ln m], \tag{3}$$

where v is the average volume occupied by the center of mass of a string and d is the space dimension. Let us compare this entropy to that of a single string whose total mass is M:

$$S(M) \sim T_c^{-1}M - 3\ln M. \tag{4}$$

Hence,

$$S(M) - S(N,m,p) \sim \beta_c(M - Nm) - N\ln(p^d v) - 3(\ln M - N\ln m). \tag{5}$$

☆ Supported in part by the US–Israel Binational Science Foundation

Now $(M - Nm)$ is just the total kinetic energy of the N-system so that (neglecting $\ln M$, $\ln m$)

$$S(M) - S(N, m, p) \sim N\{\beta_c [(m^2 + p^2)^{1/2} - m] - \ln(p^d v)\} . \quad (6)$$

Suppose first that $m = 0$ and that v is determined by the condition that the N massless particles be in equilibrium. In that case (blackbody radiation):

$$p^d v \sim \# O(1) \quad (7)$$

(each massless particle occupies on the average a region comparable to its Compton-wavelength).

We now see that once p exceeds a critical value:

$$p \gtrsim p_c \sim T_c , \quad (8)$$

the entropy excess becomes positive. This means that the blackbody radiation at temperature $\sim T_c$ would stop being in equilibrium and would "collapse" into one string whose state of excitation is determined by the total energy of the massless strings.

Conversely, suppose we start with one excited string of mass $M \gg 1$. The entropy excess in eq. (6) depends on p in two ways: the string term is linear in p (for $p \gtrsim m$) while the kinetic energy phase-space varies logarithmically. Hence, a cascade process in which a fraction of the string internal energy is converted continuously to kinetic energies of lighter strings requires an exponential decrease in entropy and would be highly damped.

We conclude that for a given volume occupied by the system, the free energy as a function of the number and composition of the strings has two degenerate minima: one at the single-string configuration, and another, at the "blackbody point" where the initial string has decayed into massless particles at temperature $\sim T_c$. The two points are separated by a barrier since continuous local deformations of the parameters decrease the entropy, so that the decay must be viewed as a sudden transition from a coherent metastable state to a thermal many-particle state. We propose that such a transition indeed signals the appearance of a "universe" composed of massless particles at the Hagedorn temperature T_c. Once this initial state has been formed, "ordinary" physics takes over and Einstein's equations would force the universe to expand adiabatically.

The above process contains two arbitrary parameters, namely the volume V occupied by the initial string, and the total mass M. Phase space consideration do not suffice to determine the relation between the two and some dynamics is needed. The basic assumption underlying the string model is that the main characteristics of the states are well approximated by perturbations theory around free strings. If we neglect the zero-point motion of the string (more on this point later) then its size is determined by the state of excitation:

$$\langle (\boldsymbol{x} - \boldsymbol{x}_{CM})^2 \rangle = \frac{1}{2} \sum_n \frac{N_n}{n} . \quad (9)$$

For purposes of illustration let us assume the "original" string to be in a state wherein a given mode ν is excited L times. Then

$$M \sim (L\nu)^{1/2}, \quad R \equiv \langle X^2 \rangle^{1/2} \sim (L/\nu)^{1/2} . \quad (10,11)$$

We find therefore

$$L \sim MR, \quad \nu \sim MR^{-1} . \quad (12)$$

The blackbody formula decrees that $p \sim T_c$ and hence $N \sim MT_c^{-1}$ and

$$V = R = (L/\nu)^{d/2} = Nv \simeq NT_c^{-d} = MT_c^{-d-1} . \quad (13)$$

We thus have

$$\nu \sim T_c^{(d+1)/d} M^{(d-1)/d}, \quad L \sim M^{(d+1)/d} T_c^{-(d-1)/d} . \quad (14)$$

Depending on whether the transition takes place before or after spontaneous compactification, $d = 10$ or 4. In both cases we are left with only one arbitrary parameter (M) which determines the initial state. In the real universe we know that $N \gtrsim 10^{90}$. If $\alpha' \leqslant G^{-1}$ (G = Newton's constant) then $M > 10^{90}$ and L and ν would also be huge according to eq. (14). We conclude that the hot big bang could have originated from an exceedingly rare explosive decay of a single highly-excited free string.

Clearly in order to test the above hypothesis the presumed process should be better understood dynamically. In particular, the resulting fluctuation spectrum should be investigated and compared with galaxy forming spectra. We do however expect that in the context of the string model the preceding scenario is a natural one and should supersede inflation as the primary origin of entropy. The spatial correlations in the initial single-string state would then resolve the "horizon" and "flatness" puzzles of the standard Robertson–Walker model.

A further problem raised by the mechanism is its relation to spontaneous compactification. The latter is a phase transition which could occur before, after, or during the transition and any detailed properties would depend crucially on this issue.

Finally, we briefly address ourselves to the basic consistency of the scenario. This issue is related to some of the fundamental difficulties of the string model itself. First, we have assumed that the string occupies a finite volume V determined by its classical size. This assumption does not hold true for a quantum string, which the zero-point motion renders infinite!

$$\langle X^2 \rangle_0 \sim \sum_{n=1}^{\infty} \frac{1}{n} . \tag{15}$$

This infinity cannot be cut-off in any way without destroying reparametrization invariance and is related to the lack of consistent off-mass-shell observables. Rather than go into this issue in detail we simply raise it here as a question.

Another puzzling issue which renders the perturbative treatment of strings questionable emerges when eqs. (1) and (9) are taken seriously. Assuming that $\alpha' \lesssim G^{-1}$ we see that all string states except for a finite number of low-mass ones satisfy

$$M/R \gg G^{-1} , \tag{16}$$

which means that in four dimensions almost every string resides within its Schwarzschild radius. Thus, it is doubtful whether the effects of virtual multigraviton states may be treated as a perturbation on free strings. It is therefore far from obvious that the spectral (eq. (2)) and geometrical (eq. (9)) relations which define the string character of the system are actually self-consistent. Note however that according to the preceding thermodynamical considerations the emission of any virtual particles in high excited strings may be highly suppressed.

This work was partially supported by an NSF grant PHY 8408265.

References

[1] M.B. Green and J.H. Schwarz, Phys. Lett. 149B (1984) 117.
[2] E. Witten, Phys. Lett. 149B (1984) 351.