THE UNITARITY PUZZLE AND PLANCK MASS STABLE PARTICLES

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Received 5 January 1987

We suggest that the ultimate remnant of an evaporating black hole is an infinitely degenerate particle at the Planck mass. Arguments are presented for the stability of these objects and their cosmological and theoretical implications are briefly discussed.

1. Semi-classical methods allow us to follow the "Hawking evaporation" [1,2] of black holes down to mass (length) scales of order $M_{\rm pl}\simeq 10^{19}$ GeV $\simeq 10^{-5}$ gr ($R_{\rm pl}=M_{\rm pl}^{-1}=G_{\rm N}M_{\rm pl}\approx 10^{-33}$ cm). When the Compton wavelength of the remaining system exceeds its Schwarzschild radius, a quantum treatment of the black hole motion (and of the gravitational field in general) is required.

Since a proper theoretical framework is presently lacking the eventual fate of the mini black hole seems an open question.

In the absence of any conservation law prohibiting the decay, one might expect a quick ($t\approx t_{\rm pl}\approx 10^{-43}$ s) decay into the quanta of the various elementary fields.

Such a decay is inconsistent with the quantum mechanical "unitarity" postulate (Pa): "The time evolution of any finite physical system is governed by a unitary operator in the Hilbert space of states." For Pa to be maintained a (practically) stable remnant whose mass, in turn, is shown to be near $M_{\rm pl}$ is required.

We propose that such objects – "planckons" – do indeed exist. Their phenomenology and cosmological impact is briefly discussed.

2. Let us consider an initial, quantum, state corresponding to a collapsing system which eventually becomes a black hole of mass M_0. A black hole of mass M has a "Hawking temperature"

$$T=M_{\rm pl}^2/8\pi M\,. \qquad (1)$$

During a time

$$\tau_{\rm bh}\approx M_0^2/M_{\rm pl}^3 \qquad (2)$$

the black hole emits Hawking radiation. The emitted quanta are an uncorrelated thermal ensemble and cannot be described by a single wave function. However, the unitarity postulate Pa is maintained by correlating each state ψ_n of the radiation with a corresponding state ϕ_n of the black hole so that the joint system has a well-defined overall wave function:

$$|\psi_{\rm tot}\rangle=\sum C_n|\psi_n^{\rm rad}\rangle\times|\phi_n^{\rm bh}\rangle\,. \qquad (3)$$

The $\psi_n \leftrightarrow \phi_n$ correspondence is particularly vivid when the Hawking process is viewed as pair creation in the strong gravitational field with one member of the pair emitted and the other staying near the horizon. ψ_n (ϕ_n) are corresponding states in the Fock spaces of the emitted (retained) fields respectively.

When the degrees of freedom of the black hole are integrated out the external radiation is indeed described by a density matrix,

$$\hat\rho_{\rm rad}=\sum_n |C_n|^2|\psi_n{\rm rad}\rangle\langle\psi_n{\rm rad}|\,, \qquad (4)$$

consistent with the statistical nature of the thermal radiation.

If the residual mini black hole disappears completely by decaying into ordinary particles we will be deprived of the large reservoir of states in the black hole which is required for the construction of the overall wave function eq. (3). The whole system will then be described by the density matrix, eq. (4), and

Pa, the unitarity postulate, will be violated.

Since in the example considered the initial and final states do not contain black holes, we have to give up even the weaker assumption that an S-matrix exists between arbitrary initial and final asymptotic states.

It would thus appear that in order to preserve the unitarity postulate Pa the residual mini black hole should not decay completely and a stable remnant – "planckon" – of mass $M \approx M_{\rm pl}$ should survive.

In principle, this remnant could convert directly into many,

$$N \simeq 5(M_0/M_{\rm pl})^2 , \tag{5}$$

quanta, N being the average number of quanta emitted during the whole process of the Hawking evaporation. In this case the N decay quanta could match those from the Hawking radiation into an overall wave function like in eq. (3).

We claim, however, that such a decay is extremely unlikely. The average wavelength of the final N quanta,

$$\bar{\lambda} \approx |\bar{p}|^{-1} \approx (M_{\rm pl}/N)^{-1} = NR_{\rm pl} , \tag{6}$$

is larger by a factor N than the size of the decaying system ($\approx R_{\rm pl}$). Thus, the "wave-function overlap" factor for each of the final quanta with the initial "interaction region" of size $R_{\rm pl}^3$ is

$$f = R_{\rm pl}^3/\lambda^3 \approx N^{-3} .$$

The rate for the simultaneous emission of N quanta is then supressed by

$$f^N \approx N^{-3N} \approx [5(M_0/m_{\rm pl})^2]^{-15(M_0/m_{\rm pl})^2} . \tag{7}$$

In the absence of the large N^{-3N} suppression, we expect for a Planck mass object a life time $\tau \approx \tau_{\rm pl} \approx 10^{-43}$ s. Eq. (7) suggests then, that once the mass of the initial black hole exceeds $(2–3)M_{\rm pl}$, the resulting residual system will be practically stable (lifetime > age of the universe).

The above estimate can be formalized by considering the effective local (up to the Planck scale) lagrangian which describes planckon$\to N$ light particles. Let the planckon be described by a local field $\Phi(x)$ and all the elementary particles ($e^\pm$, $\mu^\pm$, quarks, Z, W, gluons, Higgs,...) collectively by a scalar $\phi(x)$. (Since there is only a finite number of such fields, this will not change our qualitative conclusions.) The simplest candidate lagrangian is the one without derivatives,

$$L_{\rm eff}^{(0)} = G^{(N)}[\phi(x)]^N \Phi(x)(1/N!) . \tag{8}$$

It would yield a decay rate

$$\Gamma^{(0)}(\text{planckon} \to N \text{ particles}) \approx M_{\rm pl} |G^{[N]}|^2 I_N , \tag{9a}$$

with the N particle phase space integral:

$$I_N = (2\pi)^{-3N} \times \int \frac{d^3P_1}{2E_1} \cdots \frac{d^3P_N}{2E_N} \delta^4\left(\sum P_i - (M_{\rm pl}, \mathbf{0})\right) .$$

I_N is maximal when the masses of the final particles vanish and is then

$$I_N \approx M_{\rm pl}^{2(N-4)}/(2\pi)^{3N}(N!)^2 . \tag{9b}$$

On dimensional grounds we expect the coupling $G^{(N)}$ in (8) to scale with $M_{\rm pl}$ like

$$G^{(N)} = C^{(N)}/M_{\rm pl}^{N-3} . \tag{9c}$$

Combining (9a)–(9c) we find

$$\Gamma^{(0)}(\text{pl} \to N\,\phi\text{'s}) \approx M_{\rm pl}(C^{(N)})^2/(2\pi)^{3N}(N!)^3 \approx N^{-3N} \tag{10}$$

(the extra $N!$ is due to the identity of final states), which qualitatively is similar to (7), so long as $C^{(N)}$, does not diverge like $N!$ in which case the full effective lagrangian $\sum L^{(0)}(\Phi \to N) \cong \sum (M_{\rm pl})^{-N} \Phi \phi^N$ is too singular to serve as a consistent approximation for a local process.

The non-derivative lagrangian $L^{(0)}(N)$ describes the decay into S wave states only. In principle we should include derivative terms of the type

$$L_{\rm eff}^{(K)} \approx G^{(N,K)} \phi(x)\phi(x)...\phi(x)\nabla...\nabla\phi\phi... \times \nabla\nabla...\phi...\Phi(1/N!) \tag{11}$$

with altogether N ϕ's and K ∇'s. The reason for this is that due to the planckon's degeneracy each planckon should couple separately to an independent linear combination of terms of the form (11), so as to allow it to decay. On dimensional grounds (or from centrifugal barrier considerations) $G^{(N,K)}$ scales like $G^{(N,K)} \approx C^{(N,K)}/M_{\rm pl}^{N+K-3}$ so that the contribution of $L_{\rm eff}^{(k)}$ to $\Phi \to N\phi$'s decay width will be further suppressed by $(\bar{P}/M_{\rm pl})^K \approx (1/N)^K$ as compared with eq. (10). Since there are only $\sim \exp(\sqrt{N+K})$ terms of the type described in eq. (11), the inclusion of higher derivatives would not qualitatively modify the width $\Gamma(\mathrm{pl} \to N\phi)$ away from $\Gamma^{(0)}(\mathrm{pl} \to N\phi)$. This could happen only if $C^{(K,N)}$ increase as a function of K. However, in this case the effective lagrangian for the decay $\Phi \to N\phi$'s,

$$L^{(N)} = \sum_k L^{(N,K)} ,$$

would cease to be local. The statement that the planckon has size $R_{\rm pl} \approx 1/M_{\rm pl}$, which was built into all our prior arguments would then become meaningless.

3. We next argue that if ordinary local field theory remains valid up to energy scales of order $M_{\rm pl}$, then, most plausibly, this will also be the mass [within factors O(1)] of the remnant planckon.

Since the semi-classical Hawking evaporation formula can be trusted once the mass of the black hole exceeds a few $M_{\rm pl}$ we are guaranteed that the planckons' mass is not much larger than $M_{\rm pl}$. It remains to show that it cannot be much smaller either.

The remnant system must have a large number of degrees of freedom to match those of the emitted Hawking radiation. Such a large (practically infinite) number of degrees of freedom (i.e., allowed quantum states) can in fact accumulate around the event horizon of a black hole.

The transformation [‡1] ($\hbar = c = G = 1$)

$$\mathrm{d}x' = \frac{\mathrm{d}r}{g_{00}} \equiv \frac{\mathrm{d}r}{1-2M/r} = \mathrm{d}r\, g_{rr} ,$$

$$\mathrm{d}t' = \mathrm{d}t \qquad (12)$$

maps the region $r \geqslant r_{\rm S} \equiv 2M$ of the Schwarzschild solution into $-\infty \leqslant x' \leqslant \infty$ with $x' = 0$ corresponding to $r = 2r_{\rm s}$. In the (x', t') variables we have an almost free one-dimensional wave equation along most of the x' axis with roughly equal (and infinite) number of states at $x' > 0$, i.e., in the outside region, and $x' < 0$, near the horizon. We next argue that quantum effects will prevent any system of mass m which is significantly lower then $M_{\rm pl}$ from collapsing into its Schwarzschild radius. Consider a collapsing spherical shell of (average) radius r and conjugate momentum P_r. The energy of the system

$$E = GM^2/r + P_r^2/2M , \qquad (13)$$

when subjected to the uncertainty constraint $rP_r \approx 1$, is minimized at the "Bohr radius" $r_0 = M_{\rm pl}^2/M^3$ which is $(M_{\rm pl}/M)^4$ times larger than the classical $r_{\rm sch.} = 2M/M_{\rm pl}^2$. This large ratio suggests that a sub-Planck mass system can, at best, tunnel with a very small probability into its Schwarzschild radius [and also justifies a posteriori the non-relativistic expression (13)].

Starting next with a pre-existing black hole we can imagine decreasing its mass continuously from $M \gg M_{\rm pl}$ to $M \ll M_{\rm pl}$. The center-of-mass coordinate of the black hole will spread over its Compton wavelength $\lambda = 1/M$ which eventually exceeds the classical event horizon radius $(2M/M_{\rm pl}^2)$ by a factor of $(M_{\rm pl}/M)^2$. These quantum mechanical fluctuations will thus completely destroy the classical event horizon.

This, in particular, quenches the Hawking radiation from the event horizon and prevents it from continuing to decrease the black hole's mass once $M \lesssim M_{\rm pl}$. All the above arguments notwithstanding we could still contemplate singular systems of sub-planckian masses M and infinite degeneracies. Such "particles" would be violently produced at energies $E \approx 2M$ and introduce "infinities" (related to the large degeneracy) into the propagators of all low-energy fields which would therefore require cutoffs at $2M$ below the Planck mass. This would contradict our assumption that $M_{\rm pl}$ is the only cutoff in te theory. In fact this lowering of the cutoff scale could amount to a redefinition of the Planck mass itself, since the graviton propagator would also be affected.

4. The evaporation of black holes leads to two distinct issues. One is that of the "naked singularity" which remains at the location of the black hole once its Schwarzschild radius has shrunk all the way to zero. It has recently been suggested that a black hole may transform into a string in the last stages of its

[‡1] See e.g. ref. [2].

evaporation [3] [‡2]. More generally it is argued that the standard concept of spacetime continuum is modified at scales $r \lesssim R_{\rm pl}$ and the naked singularity issue may thus be resolved. However, so long as quantum mechanics holds the second major difficulty – namely, the evolution of a pure state into a mixed state – is not resolved. Indeed this applies for string models as the authors suggesting transition from black hole to strings [3] note.

We should emphasize that it is certainly possible to avoid stable planckons if we give up the quantum unitarity postulate (Pa) [4]. Indeed attempts have been made to formulate a generalization of quantum mechanics based on density matrices but at the present time the consistency of such schemes is not proven. In fact, it has been noted that dynamical transitions from pure to mixed states may cause violations of energy–momentum conservation [5].

Let us recall that the continuous β-decay spectrum motivated on the one hand speculations on the violation of sacred physical principles (energy–momentum conservation) at "short" (i.e., nuclear) distances, and on the other hand the hypothesis that a new stable particle, the neutrino, should exist. Hopefully the analogous planckon resolution of the present puzzle will turn out to be equally interesting (and correct!).

5. Planckons could "descend" from black holes with initial electric, color, weak charges, etc. In this case we will have very strong (electro, ...)-static fields around the planckons with self mass $O(\alpha m_{\rm pl})$. These planckons will get rid of these charges via pair creation in the very strong electric (etc.) fields with subsequent repulsion of that member of the pair with the same sign charge and capture of the other. Thus we expect the planckons to be completely neutral with respect to any interaction (of range $r \geqslant R_{\rm pl}$, i.e., *all* interactions) and have only gravitational interactions. The cross sections for mutual collisions or collisions with protons and nuclei,

$$\sigma \approx R_{\rm pl}^2/\beta^2 \approx 10^{-66}/\beta^2 \ ({\rm cm}^2) \tag{14}$$

(with $c\beta$ the relative velocity), are extremely small, rendering the detection of planckons impractical.

[‡2] In ruling out naked singularities it appears that string models strengthen our arguments.

Thus, planckons could be the ideal [‡3] cold matter candidates for the missing mass both cosmologically and at smaller scales. A necessary condition for this scenario is the existence of a consistent dynamical mechanism which creates the required density and fluctuation spectrum of planckon at the end of the quantum era [‡4].

6. Since planckons in our view are the ultimate quanta at the Planck scale they would certainly have far reaching implications for "quantum gravity". In this connection we may formulate a consistency condition for the coupling of planckons to low-momentum gravitons (or other quanta): since planckons are massive objects they certainly act as a source for a long-range gravitational field. Using special relativity, locality and quantum mechanics (on distance scales larger than the Planck length), we could therefore imagine creating a pair of virtual planckons by low-momentum gravitons. If the amplitude for this were non-vanishing a divergence in the graviton propagator would ensue (due to the infinite planckon degeneracy). Hence, the Planck-scale fundamental theory should lead to planckon "form factors" which vanish when the momentum transfer is timelike. Note that the above does not conflict with locality on scales which are larger than the Planck length.

Finally, we may conjecture that due to their infinite degeneracy planckons would dominate all other quantum fluctuations at the Planck scale and would thus provide a natural cutoff at the Planck momentum for quantum field theory [‡5].

We are grateful to L. Susskind for many helpful discussions and we thank Al Mueller for an exhaustive discussion. This work was supported in part by NSF grants PHY-8408265 and ISP-80-11451, and by the US–Israel Binational Science Foundation.

[‡3] See e.g. Turners' review in ref. [6].

[‡4] For a model of the quantum era which is based on a self-consistent production and decay of black holes see ref. [7].

[‡5] We should mention that particles of about Planck mass – "maximons" – have been considered by Markov [8].

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