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The issue of retrodiction in Bohm's theory

Y. Aharonov and D. Albert *Tel Aviv University and University of South Carolina*

Bohm's pathbreaking hidden variable theory of 1952[1] is often accused of artificiality and inelegance, and doubtless it is guilty of both. But to make such accusations, and to leave it at that, is to entirely miss the point. What Bohm was after in his theory was not elegance and not naturalness; Bohm's intentions were simply to produce a theory which, *whatever* its other characteristics, had *logically clear foundations.* It is or that clarity which Bohm's theory is highly and rightly praised.

We should like to point to one very straightforward example of that logical clarity here, one which is related to an ancient debate within quantum mechanics, and to some recent work of ours[2,3,4]. It concerns the question of retrodiction.

The question of retrodiction might be posed like this. Can we know more of *the past history of a quantum-mechanical* system than we can in principle predict about its future? Or, more time-symmetrically and more precisely, like this. Can we know more of quantum-mechanical systems within the interval *between* two complete measurements than can in principle be known about the past or the future of any single complete measurement?

The conventional quantum-mechanical answer, the answer which follows from von Neumann's non-time-reversal-symmetric collapse postulate, is 'No.' That postulate dictates that the quantum state of any system at any time is determined (via the equations of motion) entirely by the result of the most recent complete measurement of that system. *Upcoming* measurements, whatever they may be (according to this view), determine nothing whatever about the state of the system *now*; they, rather, produce information about the state of the system *subsequent* to their execution.

On the other hand, the probability that a given experiment, carried out within the interval between two other complete measurements on the same system, will produce a given result, is known to depend *symmetrically* on the results of the measurements at the beginning and at the end of that interval (see references 2–4). Albeit that all this can be derived from the non-time-symmetric formalism, the time-symmetry of the experimental probabilities suggests that the *correct* underlying description of the quantum-mechanical systems ought to be time-symmetric as well; that the results of experiments on *both* ends of a given time-interval ought to be regarded as producing information about the system *within* that interval.

This question, the reader is doubtless aware, has been and continues to be the subject of a long convoluted and mirky debate; but, within Bohm's theory (and this is the point of the present note), this question can be posed and answered definitively, and with stunning clarity. Can we know more of the past than of the present or the future? Bohm's answer is yes. A very simple example will suffice to make the point.

Suppose that a small impenetrable box is located at the point x_1, and that another such box is located at the point x_2. A single-particle system is prepared at time t_0 in the state:

$$|x_1\rangle + |x_2\rangle \equiv |\alpha\rangle; A|\alpha\rangle = \alpha|\alpha\rangle$$

(where $|x_1\rangle$ is a state wherein the particle is located within the box at x_1, etc.) by means of a measurement of some complete set of commuting observables A at that time, and that at some later time t_2 the particle is found to be in the box at x_2.

According to the conventional quantum-mechanical account, the state of such a particle as that within the interval $t_0 < t < t_2$ is $|x_1\rangle + |x_2\rangle$, and its position (within that interval) is undefined. The fact that any measurement of X within that interval would with certainty have produced the result $X = x_2$ has a very different explanation, within this account, than the fact that any measurement of A within that interval would with certainty have produced the result $A = \alpha$. According to the retrodictive (or, rather, the time symmetric) picture of references 2–4 both A and X are well-defined within the interval ($A = \alpha$ and $X = x_2$ there); within that picture it is in some sense the case that either of *two* quantum states (or, in some other sense, *both* of them) can be associated with the particle within that interval.

Within Bohm's account, all this is splendidly clear and definite. The quantum state, the *wave-function* of the particle within the interval $t_0 < t < t_2$, is certainly and unambiguously $|x_1\rangle + |x_2\rangle$, and the *position* of the particle is clearly and unambiguously x_2 (it is hoped, by the way, that the reader will find this somewhat perplexing; and this perplexity will serve as the reader's invitation to become familiar with Bohm's brilliantly clear, if inelegant and artificial, theory).

It ought to be pointed out that there will in general be many particulars about which Bohm's picture and the time-symmetric retrodictive one do *not* agree; but there is at least one profound generality about which they surely *do*: more can be known of the pasts of quantum systems than of their futures.

Acknowledgments

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