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Observations on Conductance Quantization and Dephasing in Mesoscale Systems

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We consider here two problems of major recent interest in mesoscopic physics. We first review the adiabatic (slowly varying confining walls) approximation to the quantized conductance. We show that the corrections to this approximation are exponentially small in the smoothness parameter of the constriction. A condition for accurate quantization is given, based on the result that the reflections due to the sudden widening of the constriction (existing in real devices) would be highly suppressed if a small adiabatic widening and/or a potential barrier should precede the sudden widening. An interesting collimation effect associated with the adiabatic picture is briefly discussed. Next, we consider the problem of quantum interference in the presence of an environment, employing two approaches. One treats the problem from the point of view of the trace left by the interfering particle on its environment. The other regards the phase accumulation of the interfering waves as a statistical process, and explains the loss of interference in terms of uncertainty in the relative phase. The equivalence of the two approaches is proven for the general case. Some applications are discussed: Dephasing by coupling to a local spin, by photon modes in a cavity, including the difference between coherent and thermal states, and by electromagnetic fluctuations in metals.

§1. Introduction

Mesoscopic systems that are on the borderline between the microscopic and macroscopic domains have recently proven to be an ideal laboratory for testing some fundamental quantum mechanics issues. For a review, including the Aharonov-Bohm effect and conductance fluctuations, see ref. 1. Here we first consider a recent interesting experimental finding in narrow wires, explain it and predict how its accuracy can be increased substantially. We then discuss the connection between "dephasing" and "leaving a trace in the environment". We prove their general equivalence and discuss examples, including those of current experimental relevance.

We first review the theory behind the quantization of the conductance of ballistic point contacts[2,3] that was recently found experimentally. The two-terminal conductance of a narrow constriction is quantized in integer multiples of $e^2/\pi\hbar$ (including spin degeneracy). The experiments were done on a two dimensional electron gas (2DEG) in a GaAs–$Al_xGa_{1-x}As$ heterostructures for which the length of the constriction was smaller than the elastic and inelastic mean free paths. The constriction is regarded as a narrow strip, having its own n modes or "conduction channels", connecting two wide regions that in turn are connected with negligible resistance to electron reservoirs. It has been pointed out[1] that such a quantization follows from the two-terminal Landauer[4] formula $G=(e^2/\pi\hbar)\sum_{ij}T_{ij}$, for full transmission ($\sum_i T_{ij}=1$) in the conducting channels, i.e., for a ballistic constriction. In addition, to satisfy the assumptions of the Landauer formula, one needs[5] a reflectionless transmission between the constriction and the wide regions. Thus, an important condition for accurate quantization of G is that a wave moving along the constriction in one of the relevant n modes will be fully transmitted into the wide regions. There are two ways to obtain this condition:

(a) As pointed out by Glazman *et al.*,[6] such reflectionless transmission follows for an adiabatic constriction (smoothly varying confining walls). Landauer[7] remarked earlier that a tapered, adiabatic opening of the constriction to the wide regions might help the feeding of channels in the constriction (see also ref. 8).

(b) Here, we point out, based on the results of Szafer and Stone[9] (see also refs. 10–16), that approximately reflectionless transmission from the constriction to the wide regions can also be obtained for a **sudden** opening of the constriction. The condition for this to occur will be shown to be that the mode number n under consideration will be smaller than the maximum available mode number, n_a, at the sudden opening. For an opening of width 2d, n_a is the integral part of $(2dk_F/\pi)$. Since it will be shown that condition (a) is hard to satisfy globally,[17] i.e., for wide variations of d along the constriction, we propose two practical ways to achieve accurate quantization in practice:

(1) Adiabatic opening of the constriction from a minimum, d_0, to d, where further propagation is not adiabatic, but for $d/d_0 - 1 > O(1/n)$, the reflections will be small.

(2) Alternatively, a smooth potential barrier along the constriction will select a value of n which, when smaller than $n_a(d)$, will guarantee small reflections.

This lack of reflections will also serve as the theoretical explanation for the non-observability of resonances in the above-mentioned experiments. The lack of resonances (see also Escapa and Garcia[13]) is quite surprising in view of recent theoretical work done by Szafer and Stone,[9] Kirczenow,[10] and Tekman and Ciraci.[12]

Glazman *et al.*[6] showed that in the adiabatic limit there is no interchannel scattering, therefore $T_{ij}=\delta_{ij}$ (for transmitted channels), and the leading corrections to the reflections **within each channel** are exponentially small in some smoothness parameter. In this paper we also show that the leading corrections to the **interchannel** reflections are exponentially small in, basically, the same smoothness parameter as well.

Next, we discuss the loss of quantum interference. This is probably the most prominent example for the suppression of quantum phenomena in the macroscopic world. It is commonly explained as caused either by the interfering particle failing to keep a definite phase along its trajectory or by the interfering particle changing the state of its environment.[18,19] In the present work we clarify the two explanations and show their equivalence. We show that the interference of two paths is destroyed when the environment contains information on the specific path the particle took, and that the relative phase of the two partial waves becomes completely uncertain at that time. The general principle is demonstrated in the case of an Aharonov-Bohm electron interference experiment. We examine two examples of electron-environment interactions — the interaction of the electron with well localized spins, and the interaction of the electron with the photon field in vacuum. We have been able to show that the above point of view is also valid for the dephasing caused by electromagnetic fluctuations in metals.[20,21] We explore the type of dephasing each of these interactions yield.

§2. The Adiabatic Picture

Let us begin by describing the quantized conductance in the adiabatic picture introduced by Glazman *et al.* Imagine a ballistic (disorder-free) 2D wire, or electronic waveguide, confining the electrons to the region $|y| \le d(x)$, yielding a smooth constriction. This is obtained by taking $d(x)$ to be a symmetric smooth function with $\lim_{x\to\pm\infty}=d$, $d(0)=d_0<d$ and changing slowly from d to d_0 over a scale $L \gg d, d_0, \lambda$ (λ being the electron wavelength). One now makes a Born-Oppenheimer type separation of the "slow" longitudinal variable x and the "fast" transverse one y. The y-problem is just a square well having energies

$$E_n(x)=\frac{\hbar^2}{2m}\left(\frac{n\pi}{2d(x)}\right)^2, \tag{1}$$

and wave functions

$$\phi(y)=\frac{1}{\sqrt{d(x)}}\sin\frac{n\pi}{2d(x)}(y-d(x))=|n_x(y)\rangle, \tag{2}$$

which satisfy the boundary condition $\phi(y=\pm d(x))=0\ \forall x$. For soft walls, $d(x)$ is

(half) the effective width of the wave function, including penetration beyond the classical turning points. As is familiar from the usual separation, $E_n(x)$ plays the role of an additional potential for the mode n. This potential depends on n and has the shape of a barrier with a maximum proportional to $(n/d_0)^2$. For a given E_F, a finite number of modes will have their energy above the barrier. This number, $n_{\max}$, is the integral part of $(2k_F d_0/\pi)$. In the adiabatic limit, tunneling below the barrier and reflections above it are negligible and the x-problem is well approximated by the WKB method. It follows that $n_{\max}$ modes have $T_{ij}=\delta_{ij}$ and all the others have zero transmission, hence $G=(e^2/\pi\hbar)n_{\max}$. This establishes quantization of G in this limit.

§3. Corrections to the Adiabatic Approximation

In order to discuss the corrections,[22] we shall use the scaled variable $u\equiv x/L$. Thus for $|u|>1$, d is a constant. Let us introduce the adiabatic basis by considering eq. (2) for every n. This basis spans the set of all relevant wave functions at every u. A general wave function ψ can be written as $\sum_{n=1}^{\infty} C_n(u)|n(u)\rangle$ which is written to emphasize the u dependence of the transverse wave functions. Inserting ψ into the Schrödinger equation, in terms of u, and projecting on mode m yields

$$C_m''+2\sum_n C_n'\left\langle m\left|\frac{\partial}{\partial u}\right|n\right\rangle+\sum_n C_n\left\langle m\left|\frac{\partial^2}{\partial u^2}\right|n\right\rangle -L^2C_m q_m^2+L^2\frac{2mE}{\hbar^2}C_m=0, \qquad (3)$$

where $q_m=(m\pi/2d)$ and $\langle m|\hat{A}|n\rangle$ means integration over y only. Grouping all the diagonal terms to the LHS and noting that $\langle m|(\partial/\partial u)|m\rangle=0$ gives:

$$C_m''+(k_mL)^2C_m=-\sum_{n\neq m}\left(2C_n'\left\langle m\left|\frac{\partial}{\partial u}\right|n\right\rangle+C_n\left\langle m\left|\frac{\partial^2}{\partial u^2}\right|n\right\rangle\right), \qquad (4)$$

with

$$k_m^2=\frac{2mE}{\hbar^2}-q_m^2+\frac{\left\langle m\left|\frac{\partial^2}{\partial u^2}\right|m\right\rangle}{L^2}. \qquad (5)$$

It turns out that k_m is the local wave number along the wire. We can see that the term q_m^2 and the matrix element serve as a potential barrier that each mode encounters, but this barrier is different for each mode. Thus we can define an A mode (Above barrier) as a mode for which k_m is real for all u, and a B mode (Below barrier) as a mode for which k_m is also imaginary in a certain interval of u. The asymptotic solutions to (4) are the well known WKB solutions that, far from the turning points ($k_m=0$), take the form

$$\Psi_m(u)=C_m^{(0)}(u)|m(u)\rangle =\frac{1}{\sqrt{k_m}}\exp\left(iL\int_{-1}^{u}k_m\,\mathrm{d}u\right)|m(u)\rangle. \qquad (6)$$

It is seen that the different modes decouple and therefore the adiabatic limit is obtained.

The lowest order corrections to the asymptotic solution are obtained by iterating (4) once and give interchannel scattering. Let us assume an incident mode s from the right that is A type. The leading corrections will couple this mode to all other modes including the B type ones. The coupling to the B type modes is much more complicated and will be demonstrated elsewhere[22] to give similar results. The transmission and reflection amplitudes from the incident mode s to mode m take the following form

$$t_{ms}=\frac{1}{L}\int_{-1}^{1}\frac{1}{\sqrt{k_m}}\exp\left(iL\int_1^u k_m\,\mathrm{d}u\right)\frac{1}{\sqrt{k_s}} \times\exp\left(-iL\int_1^u k_s\,\mathrm{d}u\right)f_m^{(0)}(u)\,\mathrm{d}u, \qquad (7)$$

$$r_{ms}=\frac{1}{L}\int_{-1}^{1}\frac{1}{\sqrt{k_m}}\exp\left(-iL\int_1^u k_m\,\mathrm{d}u\right)\frac{1}{\sqrt{k_s}} \times\exp\left(-iL\int_1^u k_s\,\mathrm{d}u\right)f_m^{(0)}(u)\,\mathrm{d}u, \qquad (8)$$

with

$$f_m^{(0)}=\left(-2iLk_s\left\langle m\left|\frac{\partial}{\partial u}\right|s\right\rangle+\left\langle m\left|\frac{\partial^2}{\partial u^2}\right|s\right\rangle\right) \qquad (9)$$

(here both m and s are A type). In the large L limit, the integrands have quickly varying phases and analyzing them by steepest descent and stationary phase methods, one arrives at the result that **all** the corrections to t_{ms} and r_{ms} are exponentially small in L. The details of the calculation for both types of modes will be

presented in ref. 22.

The qualitative reason for the smallness of the corrections is the strong cancellation in the integrals due to the varying phases. We note that there is no qualitative difference between the interchannel reflections ($m \neq s$) and the intrachannel ones ($m=s$). This follows from eq. (8) where the phase term in the expression for r_{ms} depends on the sum of the two wave numbers $k_m + k_s$.

§4. Conditions for Accurate Quantization

We now demonstrate, based on the results of Szafer and Stone,[9] (see also refs. 10–16) that with a modest adiabatic widening of the constriction from d_0 to $d(\bar{x})$, $\bar{x}$ being the point where a sudden opening occurs, is enough to yield rather small reflections at $\bar{x}$. Szafer and Stone[9] have calculated the transmission and reflection probabilities in the sudden geometry numerically and showed that the exact numerical results can be very well approximated by a Mean Field Approximation (MFA) which they have introduced. In this approximation it turns out that the total transmission probability from mode s is given by:

$$T_s = \sum_i T_{is} = \frac{4K_s k_s}{(K_s + k_s)^2 + J_s^2}, \tag{10}$$

where

$$K_s + iJ_s = \frac{d(\bar{x})}{2\pi} \int_{q_{s-1}}^{q_{s+1}} \mathrm{d}q \sqrt{k_F^2 - q^2}. \tag{11}$$

The momenta k_s, q_{s-1} and q_{s+1} are calculated at $d(\bar{x}) \geq (n_{\max} + 1/n_{\max}) d_0$, J_s in (11) is zero and (10) takes the form:

$$T_s = \sum_i T_{is} = \frac{4K_s k_s}{(K_s + k_s)^2} = \frac{1}{1 + \dfrac{(K_s - k_s)^2}{4K_s k_s}}. \tag{12}$$

At $d(\bar{x})$, due to the adiabatic transport up to $\bar{x}$, the electrons will occupy only $n_{\max}$ channels out of the n_a available ones. The difference $K_s - k_s$ can be approximated by the second derivative $(1/4)(d^2 k_s / ds^2)$ and after some algebra we obtain the total reflections from channel s by:

$$R_s = 1 - T_s = \frac{1}{64} \left(\frac{n_a}{n_a^2 - s^2} \right)^4, \tag{13}$$

which is valid for $n_a \geq n_{\max} + 1$. If, for example, $n_{\max} = 10$ and $d(\bar{x}) = 1.2 d_0$ it follows that $n_a = n_{\max} + 2$ and the reflections for $s = n_{\max}$ (which are the largest) are $O(10^{-4})$. Thus, one may say that in order to get accuracy in the quantization of the constriction conductance, one needs that it will open adiabatically to a large enough (but still rather modest) width, so that the sudden opening beyond that width would not cause appreciable reflections.

We now turn to discuss the conditions for the breakdown of adiabaticity at some $d(\bar{x}) > d_0$, for gradually increasing $d(x)$ but with no sudden opening. This is relevant for realistic experimental devices. Let us consider the case where $d(x)$ far from the narrowest part d_0 is very large. Obviously, for given values of m and s and length L, $(k_m - k_s)L$ will become very small, for large enough d, and therefore the phase in (6) will not oscillate rapidly and the previous results will not be valid. In order to obtain some quantitative results on t_{ms} and r_{ms} in this large $d(x)$ regime we will first calculate the indefinite integrals, denoted by $t_{ms}(u)$ and $r_{ms}(u)$. We then obtain the transmission and reflection amplitudes for a given interval (u_0, u) by $t_{ms}(u) - t_{ms}(u_0)$ and $r_{ms}(u) - r_{ms}(u_0)$, respectively. We also make the assumption that k_m, k_s and $f_{\mathrm{m}}^{(0)}$ are approximately constant over the interval (u_0, u). It follows from the above considerations that:

$$|t_{ms}(x)| \approx \frac{16}{\pi} \frac{2dk_F d'}{\pi} \frac{ms}{(m^2 - s^2)^2}, \tag{14}$$

and

$$|r_{ms}(x)| \approx \frac{4}{\pi} \frac{\pi d'}{2dk_F} \frac{ms}{(m^2 - s^2)}, \tag{15}$$

where $m \neq s$ and d' is the derivative of d with respect to x, the non scaled variable. For $dk_s > 1$, $|r_{ms}|$ is much smaller than $|t_{ms}|$. The largest value x, x_c, for this approximation to be valid is such that $\max_m |t_{ms}| \leq O(1)$ and since the transmission is maximum for modes $m = s \pm 1$ we obtain the condition

$$d'(x_c) \leq \frac{\pi}{4} \frac{\pi}{2d(x_c)k_F}. \tag{16}$$

Note that already for $x \leq x_c$ adiabaticity is lost since we have strong interchannel scattering but at x_c the perturbative treatment used becomes invalid. This condition can also be ob-

tained by geometrical considerations. At a given $d(x)$ the electron has a determined transverse and longitudinal momentum and therefore can be viewed as moving at an angle $\theta_s(x)=\tan^{-1}(q_s/k_x)$. This angle depends only on $d(x)$ and since in the adiabatic limit the transverse quantum number s is conserved the electron adjusts its angle continuously as it propagates along the waveguide. The motion between the confining walls is ballistic and therefore in order to have a definite angle for every $d(x)$, the distance, Δx, the electron passes without changing its angle (i.e., the distance along the x direction the electron passes from the point it hits the lower wall to the point it hits the upper wall) must be such that d changes very little over that range. If we now consider deviations from the adiabatic limit and ask what is d' such that $\theta_s(x)$ will correspond to $d(x+\Delta x)$ with quantum number $s+1$, we obtain (16). Using condition (16) we obtain the total reflection probability, at x_c, from mode s to all other modes:

$$R_s=\sum_m |r_{ms}|^2 \approx |r_{s+1,s}|^2+|r_{s-1,s}|^2$$
$$\propto \left(\frac{\pi}{2d(x_c)k_F}\right)^4 s^2$$
$$=\left(\frac{d_0}{d(x_c)}\right)^4 \frac{s^2}{n_{\max}^4}, \qquad (17)$$

where we have used the relation, $k_F \cong (n_{\max}\pi/2d_0)$. Note that although the (interchannel) transmission probability is large, the reflections are small even for the maximum mode number $s=n_{\max}$. If the consriction continues to open up to larger d', the electrons will reach the point $\bar{x}$ where the geometrical-optics rays will be parallel to the confining walls and obviously from that point on the electrons will not be affected by the constriction any more. Therefore we expect that the above constriction will be equivalent to a consriction that is suddenly removed at $\bar{x}$. The condition for the local rays to be parallel to the constriction is

$$d' \geq \frac{q_s(\bar{x})}{k_F}. \qquad (18)$$

Although eq. (7) is not valid at $\bar{x}$, eq. (8) is, and we obtain the following dependence of R_s on the physical parameters at $\bar{x}$:

$$R_s \propto \left(\frac{d_0}{d(\bar{x})}\right)^4 \frac{s^4}{n_{\max}^4}. \qquad (19)$$

Note that the reflections at x_c are smaller than the reflections at $\bar{x}$ by more than an order of magnitude (i.e., for $n_{\max}=10$ the reflections at $\bar{x}$ are smaller by two orders of magnitude), although adiabaticity at x_c is already lost. Let us write eqs. (17) and (19) in a slightly different manner. At every $x>x_0$ (assuming adiabaticity for $x_0<x<x_c$), the electrons occupy only $n_{\max}$ channels out of the $n_a(x)$ available ones. Therefore

$$R_s(x_c)\propto \frac{s^2}{n_a(x_c)^4} \quad \text{and} \quad R_s(\bar{x})\propto \frac{s^4}{n_a(\bar{x})^4}. \qquad (20)$$

To achieve a reflectionless transmission, even for a constriction with constant width, d, it is clearly necessary to populate less channels than the maximal allowed number $n_a \cong (k_F 2d/\pi)$. This can obviously be achieved also by a potential barrier along the channel. This barrier should be long enough to be either fully transmitting or fully reflecting and somewhat smooth to avoid resonances. A smooth saddle-point potential, for example, is satisfactory. Due to coulomb effects, potential variations practically always exist in narrow channels.

This may be part of the reason for the observability of conductance quantization in the existing experiments. Examining the quantization by mode selection with a potential barrier should therefore have the additional advantage of accurate quantization for adiabatic barriers. An interesting feature of the channel selection is the possibility[23] of focussed emission of electrons from the sudden termination. Corrections to the adiabatic approximation are also relevant for the series addition of constriction conductances.[24,25]

The corrections to the adiabatic approximation have very recently been considered by Payne.[26] He obtains the result (as in eq. (18)) that the adiabatic approximation always breaks down for large enough width for any rate of change of the latter. His discussion of quantization does not emphasize the lack of reflections, and the reversibility in space is invoked to explain quantization with more than one constriction. We argue, however, that the lack of reflection is crucial in this picture. Elec-

trons that are relected and absorbed by a reservoir, are reemitted with a random phase and reversibility is lost.

§5. A Spin Model for Dephasing, Generalization

We will start by examining an interference experiment done in an ideal Aharonov-Bohm (A-B) ring. This experiment starts at $t=0$ with an electron wave packet formed at the entrance point A with 2 components, one heading to the right, and the other one to the left. We will denote them by $|r(t=0)\rangle$ and $|l(t=0)\rangle$, respectively. Both of these components have the same momentum magnitude, P. After a time $T=\pi R/(P/m)$ has passed the two components reach the exit point B, diametrically opposite to A. T is short enough so that the spreading of the two wave packets can be ignored. Then, the probability of finding the electron at the point B is:

$$|\Psi(B, t=T)|^2=|r(B, T)+l(B, T)|^2 = |r(B, T)|^2+|l(B, T)|^2 + 2\,\mathrm{Re}\,[r^*(B, T)l(B, T)]. \tag{21}$$

The result of dephasing is a reduction of the ratio between the interference term (the third one) and the classical terms (the first two ones.)

To illustrate the nature of that dephasing let us look at such an A-B ring with one scatterer. This scatterer plays the role of an environment, i.e., a degree of freedom which is not examined in the interference experiment. The scatterer will be a spin 1/2 which is coupled to the electron by the interaction (σ is the scatterer spin):

$$\begin{cases} V_0\sigma_z & \text{when the electron is in the region of interaction,} \\ 0 & \text{outside this region.} \end{cases}$$

The scatterer is located somewhere along the left arm, so that the left partial wave interacts with it along a region $l \ll R$, while the right partial wave does not interact with it. (The interaction may be thought of as an Ising-like interaction with the fixed z component of the electron's spin.)

Now, if at $t=0$ the spin, σ, is in an eigenstate of σ_z then the scattering is elastic. It causes the left component of the electron a phase shift, which multiplies the interference term by a phase factor. However, this phase shift can be cancelled by applying an A-B flux ϕ inside the ring; it will not affect the h/e oscillations of the conductance.

But if at $t=0$ the spin is, *e.g.*, in the state $|\sigma_x=+1\rangle$, the scattering is inelastic, and it causes dephasing. This dephasing cannot, as we shall shortly see, be cancelled by an A-B flux, and it reduces the h/e oscillations. Let us analyze such a situation in detail:

At $t=0$ the system's (electron and spin) wave function is

$$(|r(t=0)\rangle+|l(t=0)\rangle\otimes(|\sigma_z=+1\rangle + |\sigma_z=-1\rangle). \tag{22}$$

Under the semi-classical approximation, to first order in V, the wave function evolves in time to

$$|l(t)\rangle(|\sigma_z=+1\rangle \exp(-iV_0\tau) + |\sigma_z=-1\rangle \exp(iV_0\tau)) + |r(t)\rangle(|\sigma_z=+1\rangle+|\sigma_z=-1\rangle), \tag{23}$$

where $|r(t)\rangle$, $|l(t)\rangle$ describe the evolution in time of the right/left partial waves in the absence of the scatterer and $\tau=l/(P/m)$ is the time the left partial wave spends in the region of interaction. The probability of finding the electron at the point B will then be:

$$|\Psi(B, T)|^2=|r(B, T)|^2+|l(B, T)|^2 + \cos(V_0\tau)2\,\mathrm{Re}\,[r^*(B, T)l(B,T)], \tag{24}$$

i.e., the interference term is reduced by the factor $\cos(V_0\tau)$. An A-B flux, which multiplies $r^*(B, T)l(B, T)$ by $\exp(i\,2\pi\phi/\phi_0)$ cannot compensate for this reduction.

This result can be interpreted in two alternative ways:

a. The left partial wave changed the state of the spin

This can be seen by noting that in (23) the left partial wave rotated the spin by an angle $2V_0\tau$, as expected for a spin put for a time τ in a magnetic field V_0. In the case of $V_0\tau=\pi/2$ the wave function is

$$|r(T)\rangle|\sigma_x=+1\rangle+|l(T)\rangle|\sigma_x=-1\rangle, \tag{25}$$

and the interference is completely destroyed. In this case the spin acts like a measuring

device — it measures the arm of the ring the electron went through. Since the origin of this quantum interference is the uncertainty about the path of the electron, this measurement destroys it.

b. **The left partial wave accumulated a "phase uncertainty" of $\pm V_0\tau$**

In this interpretation, we look *only at the electron* and write its wave function as:

$$e^{-i\phi}|r(T)\rangle + e^{i\phi}|l(T)\rangle, \tag{26}$$

where ϕ, the electron's phase, is now a statistical variable whose distribution function is

$$P(\text{phase}) = \begin{cases} 0.5 \text{ for the phase to be } -V_0\tau, \\ 0.5 \text{ for the phase to be } +V_0\tau. \end{cases} \tag{27}$$

When the interference term is calculated, it is obtained as a function of the phase, and then averaged over the phase distribution function. The maximum phase uncertainty is $\pm\pi/2$, and it is exactly this value of $V_0\tau$ which destroys the interference.

One conclusion is, therefore, that the physical process of dephasing by one scatterer can be described by *either* of the two alternative and equivalent descriptions: (a) by different electron partial waves leaving the environment in states different from each other, or (b) by one (or some) of the electron's partial waves accumulating phase uncertainty. By looking at another experiment in the same system we will see that the important phase uncertainty in our second interpretation is the uncertainty in the relative phase of the two partial waves at the time of interference. In that experiment, interference is examined at the point A after a time $2T = 2\pi R/(P/m)$, when each partial wave completes a full circle. (This type of interference gives rise to $h/2e$ oscillations of the conductance). In our system, each of these partial waves interacts once with the spin scatterer. Hence, the wave function at $t = 2T$ is (assuming $V_0\tau = \pi/2$, for brevity):

$$(|r(2T)\rangle + |l(2T)\rangle)|\sigma_x = -1\rangle, \tag{28}$$

i.e., the two partial waves leave an identical trace on the environment, and the interference is not affected. The phase of *each* partial wave has a distribution function as in (27) but the two phases are correlated. The $|\sigma_z = +1\rangle$ component induces a phase shift of $-\pi/2$ in both of the partial waves, and the $|\sigma_z = -1\rangle$ component induces a phase shift of $+\pi/2$ in both waves. The relative phase has no uncertainty at all.

The foregoing example can be generalized to the following statement: When an interference pattern of a particle is examined after the particle has interacted with an environment, then:

a. The effect of this interaction on the interference term can be described by either one of the two *equivalent* descriptions:

1. The interference term is multiplied by the scalar product of the two environment states that are coupled to the two partial waves.

2. The relative phase of the partial waves, ϕ, becomes a statistical variable, described by a probability distribution, and the interference term is multiplied by the average value of $e^{i\phi}$.

b. Thus, the interference is lost when,

1. The two partial waves shift the environment into states orthogonal to each other.

or, equivalently,

2. The average value of $e^{i\phi}$ is zero, excluding pathological cases, which happen when the uncertainty in the phase, i.e., $\langle \Delta\phi^2 \rangle$ is much larger than 1.

c. When $\langle \Delta\phi^2 \rangle \ll 1$, the environment's potential can be approximated by a single particle (possibly time dependent) effective potential $V(x, t)$. Obviously, such a potential cannot dephase the interference. This has been amply confirmed by recent works on mesoscopic condensed matter systems.[27] When the changes induced by the interfering particle in the state of the environment do not significantly change the potential exerted by the environment on the particle (i.e., when the interfering particle does not feel a "back reaction" of the environment), this static potential is,

$$V(x, t) = \int d\{\eta\} \chi^*(\{\eta\}, t) V(x, \{\eta\}) \chi(\{\eta\}, t) \tag{29}$$

where x is the particle's coordinate, $\{\eta\}$ are the environment's coordinates and $\chi(\{\eta\}, t)$ is the environment's state as it evolves in the absence of the particle.

This is usually the case when the environ-

ment is a many body environment.[21]

This general statement is valid regardless of the particle-environment coupling strength. Its full proof will be given in a subsequent paper.[21] The essential ingredient of the proof is the observation that if the environment interacts only with, say, the left partial wave then the scalar product mentioned above is

$$\langle\chi|\hat{T}\,e^{i\int_0^T V_l(x_l(t),t)\,dt}|\chi\rangle, \tag{30}$$

where $\hat{T}$ is the time ordering operator, $V_l(x_l(t), t)$ is the particle-environment interaction in the interaction picture, and $x_l(t)$ is the left partial wave's trajectory. Equation (30) is just the expectation value of the time evolution operator of the environment, given that the interfering particle takes the left path. Since the time evolution operator is a unitary transformation, it can be represented as $e^{i\phi}$, where ϕ is an hermitian operator. Then, the scalar product of the two environment states equals the expectation value of $e^{i\phi}$.

Therefore, the phase, ϕ is a statistical variable whose distribution function is related to $|\chi|^2$. When the environment's back reaction is negligible, the phase is,

$$\phi=\int_0^T V_I(x_l(t), t)\,dt. \tag{31}$$

§6. Interaction with Cavity Modes

A second illustration to the general statement is found in the following example. The interference of two electron's trajectories $x_1(t)$ and $x_2(t)$ is examined in a cavity at zero temperature. The electron interacts with the electromagnetic vector potential. It is well known that such an interaction causes a relative phase shift of $(e/\hbar c)\int\{\boldsymbol{A}(x_1(t), t)\cdot\boldsymbol{v}(x_1(t))-\boldsymbol{A}(x_2(t), t)\cdot\boldsymbol{v}(x_2(t))\}\,dt$. ($\boldsymbol{v}$ the electron's velocity). The average value of the vector potential in vacuum is zero, therefore the mean phase shift is also zero. However, there are quantum fluctuations of the vector potential in the vacuum, and those will cause an uncertainty in the phase shift. The uncertainty will be,

$$\langle\Delta\phi^2\rangle=\left(\frac{e}{\hbar c}\right)^2\int dt\int dt'[\boldsymbol{A}(x_1(t), t)\cdot\boldsymbol{v}(x_1(t))-\boldsymbol{A}(x_2(t), t)\cdot\boldsymbol{v}(x_2(t))]$$
$$\times[\boldsymbol{A}(x_1(t'), t')\cdot\boldsymbol{v}(x_1(t'))-\boldsymbol{A}(x_2(t')\cdot\boldsymbol{v}(x_2(t'))]. \tag{32}$$

Now, let us look at the simple case in which the two partial waves follow a 1D harmonic oscillator trajectory in two opposite directions,[28] i.e.,

$$x_1(t)=-x_2(t)=R\sin\omega_0 t,$$
$$v_1(t)=-v_2(t)=\omega_0 R\cos\omega_0 t. \tag{33}$$

Then, the phase uncertainty is,

$$\langle\Delta\phi^2\rangle=\left(\frac{e}{\hbar c}\right)^2\int dt\,\omega_0 R\cos\omega_0 t\{A_x(R\sin\omega_0 t, t)+A_x(R\sin\omega_0 t, t)\}\times\int dt'\,\omega_0 R\cos\omega_0 t'\{A_x(R\sin\omega_0 t', t')+A_x(R\sin\omega_0 t', t')\}. \tag{34}$$

The correlation function $\langle A_x(x, t)A_x(x't')\rangle$ in vacuum is

$$\langle A_x(\boldsymbol{x}, t)A_x(\boldsymbol{x}', t')\rangle=\sum_\alpha\frac{\hbar c^2}{\Omega\omega_\alpha}e^{i\boldsymbol{k}_\alpha\cdot(\boldsymbol{x}-\boldsymbol{x}')-i\omega_\alpha(t-t')}\left(1-\frac{(k_{\alpha x})^2}{|k|^2}\right), \tag{35}$$

where $\omega_\alpha=ck_\alpha$, and Ω is the cavity's volume.

Substituting this expression into the expression for $\langle\Delta\phi^2\rangle$ (32) we obtain,

$$\langle\Delta\phi^2\rangle=4\omega_0^2R^2\int d\boldsymbol{k}\,\frac{\hbar c}{|k|}\left|\int_0^\tau dt\cos\left\{\frac{1}{2}k_x(x_1(t)-x_2(t))\right\}\times\cos\omega_0 t\,e^{ic|k|t}\right|^2\left(1-\frac{k_x^2}{|k|^2}\right). \tag{36}$$

Most of the contribution to this integral comes from $|k|\approx(\omega_0/c)\pm(1/c\tau)$. Since $\omega_0 R\ll c$, the dipole approximation is valid, i.e., $k_x(x_1(t)-x_2(t))\ll 1$. The expression for $\langle\Delta\phi^2\rangle$ is then exactly the probability of a photon field at $T=0$ to absorb a photon from an oscillating dipole of frequency ω_0. Since the photon field is at $T=0$, the most effective way to leave a trace on it is to emit a photon. The time it takes for the phase to be completely uncertain is exactly the time it takes for the electron to emit a photon. Since the accelerations of the two partial waves are exactly opposite, the

radiation emitted from each of them is in a phase shift of π relative to the other. The phase of the radiation contains the information about the path taken by the electron. Therefore, although the two partial waves emit the same amount of radiation (and they necessarily do, since in the dipole approximation they both feel the same potential fluctuations), the trace they leave on the environment must be different. This is seen in terms of the environment's wave function. The environment's wave function is, up to first order in the potential,

$$|vac\rangle \pm i \sum_k |a_k|\,|1 \text{ photon in } k \text{ mode}\rangle, \quad (37)$$

where the $\pm$ sign refers to the environment state coupled to the first/second partial wave. The difference in sign reflects the phase difference in the radiation.

Had the two partial waves propagated with equal velocities in part of their trajectory, then for that part the dipole radiation would have caused uncertainty in each of the partial waves' phase, but not in their relative phase. The relative phase uncertainty would then have been accumulated by the quadrupole interaction (next order in $\boldsymbol{k}\cdot\boldsymbol{r}$). Then, the $\cos[(1/2)k_x(x_1(t)-x_2(t))]$ in (32) should be replaced by $[k_x(x_1(t)-x_2(t))]^2$, yielding

$$\langle \Delta\phi^2\rangle = 4\omega_0^2 R^2 \int \mathrm{d}k \, \frac{\hbar c}{|k|} \left| \int_0^\tau \mathrm{d}t \right.$$
$$\left. \times \{k_x(x_1(t)-x_2(t))\} \cos\omega_0 t \, \mathrm{e}^{ic|k|t} \right|^2$$
$$\times \left(1 - \frac{k_x^2}{|k|^2}\right). \quad (38)$$

In both cases, we conclude that the time it takes for the phase to get uncertain is just the time it takes for the partial waves to emit photons that will identify them. By suitably placed reflecting mirrors one can isolate the photons of each path from that of the other path, thus creating a situation where any photon identifies its path.

An intersting issue is whether the dephasing due to photons/phonons is different in a thermal equilibrium vs. a coherent state. A calculation along the lines developed here shows that a coherent state with any amplitude achieves the same dephasing as a thermal state at zero temperature. With increasing temperatures, the thermal state dephasing becomes faster (in distinction with the amplitude independence in the coherent case). It is interesting that exchange of one oscillator quantum is enough to effect dephasing in the thermal case, even when the fluctuations in the number of photons in the state are much larger than one photon. This is due to the random phases of the components of the wave function in this case.

It is worth noting at this point that by replacing the correlation function $\langle A(x,t)A(x',t')\rangle$ in vacuum (in (32)) by the corresponding correlation function in metals and requiring $\langle \Delta\phi^2(\tau)\rangle = 1$ one immediately reproduces the results obtained by Altshuler *et al.* for the phase breaking time at low dimensions for weak localisation.[20-21] Those results which were obtained by solving the equation of motion of the cooperon have received convincing experimental proof.[29,30] They have been treated in the quasiclassical approximation in ref. 31, in the weak localization centres.

§7. Discussion of Dephasing

The two examples given above, i.e., dephasing by an interaction with a spin and dephasing by an interaction with the photon field, demonstrate different possible features of the dephasing processes. First, and simplest, is the question of a transfer of energy. As seen from these examples, dephasing might involve an exchange of energy between the electron and the environment (the photon field case), but it certainly does not have to (as in the spin case). Second is the question of the range of interaction. The interaction with the spin is limited in range, whereas the interaction with the photon field is extended. There are some interesting questions concerning each of the two cases.

Let us first look at the local interaction case, the spin example. In reality, the spin is coupled to many other degrees of freedom, i.e., a bath. Once the interaction with the spin is over, the spin relaxes to equilibrium with the bath. Can this relaxation wipe out the trace left (in our example) by the left partial wave on the spin? The answer is no, and it is based upon a general statement:

A trace left by an electron on the environment can be wiped out only by the same electron interacting again with the environment.

Internal interactions of the environment cannot wipe out this trace.

The proof of this statement follows simply from unitarity. The scalar product of two states that evolve in time under the same hamiltonian does not change in time. Therefore, if the state of the system (electron + environment) after the electron environment interaction is over is

$$|r(t)\rangle \otimes |\chi_{env}^{(1)}\rangle + |l(t)\rangle \otimes |\chi_{env}^{(2)}\rangle \qquad (39)$$

then the scalar product $\langle \chi_{env}^{(1)}(t) | \chi_{env}^{(2)}(t) \rangle$ does not change in time. The only way to change it is by another interaction of the electron with the same environment. Such an interaction keeps the product $\langle \chi_{env}^{(1)}(t) | \chi_{env}^{(2)}(t) \rangle \otimes \langle r(t) | l(t) \rangle$ constant, but changes $\langle \chi_{env}^{(1)}(t) | \chi_{env}^{(2)}(t) \rangle$. The interference will be retrieved only if the orthogonality is transferred from the environment wave functions to the electronic wave functions, which are not traced in the experiment.

The statement that was proved here is useful in many interference experiments. For example, in neutron Bragg scattering, the neutron might flip the spins of the scattering nuclei in the lattice. Those spins interact with the lattice in a time scale T_1, the spin lattice interaction time. Our statement then shows that the ratio between T_1 and the time of the experiment is of no relevance to the experiment. It is only the ratio of T_1 to the neutron-nuclei interaction time that matters.

The local interactions discussed above are, in fact, scattering processes. As such, it is sometimes simple to analyse the trace they leave on the environment. For example, a question has been recently raised[31] about possible dephasing by high frequency Einstein oscillators in their ground states. Reference 31 considered conduction electrons in a metal that do not have enough energy to excite such an oscillator. Now, since scattering processes conserve energy, and since the ground state of a harmonic oscillator is nondegenerate, there is no possible trace the electron can leave on the oscillators. Therefore, those processes do not dephase.

The situation is different in case the interaction is still on when the interference is examined. A discussion of this case will be given in a subsequent paper.[21]

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