

# Dephasing of Interference by a Back Reacting Environment

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## Abstract

We discuss the dephasing of quantum interference due to the interaction of the interfering particle with its environment. We interpret this dephasing using two descriptions. One examines the way the interfering waves change their environment, and the other examines the quantum uncertainty in the phase shift induced by the environment. We demonstrate the equivalence of the two descriptions, and focus on the case in which the potential exerted by the environment on the interfering particle depends on the history of that particle.

## 1. Introduction

The two slit interference experiment is often used as a starting point in the description of quantum mechanics <sup>(1)</sup>. When the path of the interfering particle is not measured, says the moral of that experiment, an interference pattern is seen on the screen. In principle, such an experiment can use interfering objects of all scales, from microscopic electrons to macroscopic billiard balls. Yet, in practice, the first is experimentally observed, while the latter is not. The disappearance of quantum interference in the macroscopic scale is believed to be due to two reasons. First, the de-Broglie wave length of macroscopic objects is very small, and second, the interaction of those objects with their environment suppresses interference effects <sup>(2)(3)</sup>. In this work, and in earlier works on the subject <sup>(4)</sup>, we investigate the second reason. We do not discuss the first reason here, but only comment that recent experiments in mesoscopic systems have allowed the observation of interference effects even when the relevant wavelengths are very short <sup>(5)</sup>.

In our previous works, we have used two descriptions of the way the interaction of an interfering particle with its environment might suppress quantum interference. The first regards the environment as measuring the path of the interfering particle. When the environment has the information on that path, no interference is seen. The second description answers the question naturally raised by the first: How does the interfering particle "know", when it reaches the screen, that the environment has identified its path? We have proven that the two descriptions are equivalent. In the present work, we will review those two descriptions, and dwell on some subtleties arising when the interaction of the particle with its environment depends on its history, i.e., it is non-local in time.

As a guiding example, we consider an Aharonov-Bohm (A-B) interference experiment on a ring (see Fig. 1). The A-B effect has been proven to be a convenient way to observe interference patterns in mesoscopic samples, because it provides an experimentally easy way of shifting the interference pattern. This experiment starts by a construction of two electron wave packets,  $l(x)$  and  $r(x)$  ( $l, r$  stand for left, right), crossing the ring along two opposite sides. We assume that the two wave packets follow well defined classical paths,  $x_l(t), x_r(t)$ . The interference is examined after each of the two wave packets traverses half of the ring's circumference. Therefore, the initial wave function of the electron (whose coordinate is  $x$ ) and the environment (whose set of coordinates is denoted by  $\eta$ ) is:

$$\psi(t = 0) = [l(x) + r(x)] \otimes \chi_0(\eta) \quad (1.1)$$

At time  $\tau_0$ , when the interference is examined, the wave function is, in general,

$$\psi(\tau_0) = l(x, \tau_0) \otimes \chi_l(\eta) + r(x, \tau_0) \otimes \chi_r(\eta) \quad (1.2)$$

and the interference term is,

$$2 \operatorname{Re} \left[ l^*(x, \tau_0) r(x, \tau_0) \int d\eta \chi_l^*(\eta) \chi_r(\eta) \right] \quad (1.3)$$

Had there been no environment present in the experiment, the interference term would have been just  $2 \operatorname{Re}[l^*(x, \tau_0) r(x, \tau_0)]$ . So the effect of the interaction is to multiply the interference term by  $\int d\eta \chi_l^*(\eta) \chi_r(\eta)$ . The first way to understand this effect is seen directly from this expression, which is the scalar product of the two environment's states coupled to the two partial waves. At  $t = 0$  these two states are identical. During the time of the experiment, each partial wave has its own interaction with the environment, and therefore the two states become different.

Since the environment is not observed in the interference experiment, its coordinate is integrated upon, i.e. the scalar product of the two states is taken. When the two states do not overlap at all, the final state of the environment identifies the path the electron took. Quantum interference, which is the result of an uncertainty in this path, is then lost. Thus, interference is lost when the two interfering partial waves shift the environment into states orthogonal to each other, i.e., when the environment has the information on the path the electron takes.

The second explanation for the loss of quantum interference regards it from the point of view of how the environment affects the partial waves, rather than how the waves affect the environment. It is well known that when a static potential  $V(x)$  is exerted on one of the partial waves, this wave accumulates a phase (A system of units where  $\hbar = 1$  is applied.):

$$\phi = - \int V(x(t))dt \quad (1.4)$$

and the interference term is multiplied by  $e^{i\phi}$ . "A static potential" here, and throughout this paper, is a potential which is a function of the particle's coordinate and momentum only, and does not involve any other degrees of freedom. For a given particle's path, the value of a static potential is well defined. When  $V$  is not static, but created by environment degree(s) of freedom, its value is not well defined any more. The uncertainty in its value results from the quantum uncertainty in the state of the environment. Therefore,  $\phi$  is not definite, too. In fact,  $\phi$  becomes a statistical variable, described by a distribution function  $P(\phi)$ . (For the details of this description see ref. [4].) The effect of the environment on the interference is then to multiply the interference term by the average value of  $e^{i\phi}$ , i.e.,

$$\langle e^{i\phi} \rangle = \int P(\phi) e^{i\phi} d\phi \quad (1.5)$$

The averaging is done on the interference screen that shows a sum of many interference patterns, corresponding to different environment states. Since  $e^{i\phi}$  is periodic in  $\phi$ ,  $\langle e^{i\phi} \rangle$  tends to zero when  $P(\phi)$  is slowly varying over a region much larger than one period, i.e.,  $2\pi$ . Hence, when the uncertainty in the phase becomes of the order of the interference pattern, interference is lost. In the Feynman-Vernon terminology,  $\langle e^{i\phi} \rangle$  is the influence functional of the two paths taken by the two partial waves. This is, then, the second explanation for the loss of quantum interference.

Our statement of equivalence between the two explanations is then put into an equation,

$$\langle e^{i\phi} \rangle = \int d\eta \chi_i(\eta) \chi_r(\eta) \quad (1.6)$$

When the environment measures the path taken by the particle, it induces a phase shift whose uncertainty is of the order of  $2\pi$ .

## 2. Two examples of dephasing

We will now demonstrate the equivalence of the two descriptions on two simple examples, before we proceed to the more complicated case of an history-dependent interaction with the environment. First, we consider an environment composed of one heavy free particle. Initially, this particle is localized somewhere along the right arm of the ring, and its average momentum is zero. The coordinate and momentum of this particle are denoted by  $\eta$  and  $p_\eta$ , respectively (see Fig. 2). The particle's mass,  $M$ , is large enough so that  $\frac{\delta p_\eta}{M} \tau_0 \ll \delta\eta$ , where  $\tau_0$  is the duration of the experiment. Consequently, the particle's wave function does not change appreciably during the experiment, and its kinetic energy plays no role in its time evolution. So, our environment is composed now of one degree of freedom, with no significant dynamics of its own, but with an infinite dimensional Hilbert space. The interaction between the interfering electron and the environment is assumed to have the form

$$H_{int} = V(x - \eta) \quad (2.1)$$

where  $V$  is non zero within some short range of interaction.  $x$  here is the position of the interfering electron on the ring, while  $\eta$  is, e.g., the position of the environment particle on the plane of the ring (see Fig. 2).

As before, the experiment starts with the initial state

$$\left( l(x) + r(x) \right) \otimes \chi_0(\eta) \quad (2.2)$$

The left partial wave does not interact with the environment, so that its propagation does not affect the latter. The right partial wave interacts with the environment, and this interaction results in a momentum exchange between the right wave packet and the environment particle. When this momentum transfer is larger than the momentum uncertainty of the environment particle, this particle measures the path the electron traverses, and quantum interference can no longer be seen. On the other hand, the uncertainty in the environment particle position,  $\delta\eta$ , makes the potential  $V(x - \eta)$  uncertain, and then the shift in the right partial wave phase becomes uncertain, too. As the following paragraphs show, when the phase of the right partial wave becomes completely uncertain, and the interference pattern

becomes a sum of mutually canceling terms, the environment particle measures the electron's path.

The wave function at time  $\tau_0$  is,

$$\psi(\tau_0) = |l(\tau_0)\rangle \otimes \chi_0(\eta) + |r(\tau_0)\rangle \otimes \exp\left(-i \int_0^{\tau_0} V(x_r(t), \eta) dt\right) \chi_0(\eta) \quad (2.3)$$

The interference term then becomes

$$2\text{Re} \left[ l^*(x, \tau_0) r(x, \tau_0) \langle \chi_0(\eta) | e^{-i \int_0^{\tau_0} V(x_r(t), \eta) dt} | \chi_0(\eta) \rangle \right] \quad (2.4)$$

Hence, the whole effect of the environment on the interference term shows up in the expression,

$$\langle \chi_0(\eta) | e^{-i \int_0^{\tau_0} V(x_r(t), \eta) dt} | \chi_0(\eta) \rangle \quad (2.5)$$

As before, this expression can be interpreted in terms of the effect of the electron on the environment, or in terms of the effect of the environment on the electron. The first follows from looking at eq. (2.5) as a scalar product of two environment states, one coupled to the left partial wave, and the other coupled to the right partial wave. The second is a bit more subtle. For a given  $\eta$ ,  $\phi(\eta) \equiv \int_0^{\tau_0} V(x_r(t), \eta) dt$  is the phase shift the electron would have experienced, had the environment particle been in a position  $\eta$ . Eq. (2.5) is then just

$$\begin{aligned} & \int \chi_0^*(\eta) e^{i\phi(\eta)} \chi_0(\eta) d\eta \\ &= \int \left| \chi_0(\eta(\phi)) \right|^2 e^{i\phi} \frac{d\eta}{d\phi} d\phi \end{aligned} \quad (2.6)$$

The limits of the last integral depend on the function  $\phi(\eta)$ , and are not relevant for our present discussion. (In cases where several values of  $\eta$  lead to the same  $\phi$ , the right hand side of this equation has to include a summation over these values of  $\eta$ .) The last integral leads us to the second interpretation: this integral is the average value of  $e^{i\phi}$ , averaged over a probability distribution for the phase shift  $\phi$ . This distribution function, as seen in (2.6), is

$$P(\phi) \equiv \left| \chi_0(\eta(\phi)) \right|^2 \frac{d\eta}{d\phi}. \quad (2.7)$$

The effect of the environment on the electron, which would have been a well determined phase shift for a "classical" environment, becomes the average value of

all possible phase shifts when the quantum nature of the environment is taken into account.

When does the environment destroy the interference pattern? Again, there are two ways to answer this question, corresponding to the two interpretations discussed above. The interference term reduces to zero when the environment's state coupled to the right partial wave is orthogonal to the one coupled to the left partial wave, or, alternatively, the interference term reduces to zero when its average value, averaged over all possible states of the environment, is zero. By further inspection of (2.6), the second condition can be rephrased in terms of the phase distribution function  $P(\phi)$ . The environment multiplies the interference term by the factor given by (2.6), and this factor is,

$$\langle e^{i\phi} \rangle = \int P(\phi) e^{i\phi} d\phi = e^{i\langle\phi\rangle} \int P(\phi) e^{i(\phi-\langle\phi\rangle)} d\phi \quad (2.8)$$

where  $\langle\phi\rangle \equiv \int \phi P(\phi) d\phi$  is the average phase shift. If  $P(\phi)$  is a narrow distribution function, that is, if  $\langle\delta\phi^2\rangle \ll 1$ , equation (2.8) can be approximated,

$$\langle e^{i\phi} \rangle \approx e^{i\langle\phi\rangle} = e^{i \int_0^{\tau_0} \langle V(x_r(t)) \rangle dt} \quad (2.9)$$

The environment induces then a phase shift proportional to the expectation value of the potential. Hence, as far as the interfering particle is concerned the environment potential can, in this limit, be replaced by a static potential  $V(x)$ .

On the other hand, when  $P(\phi)$  is very broad, i.e.  $\langle\delta\phi^2\rangle \gg 1$ , there is an appreciable probability for various values of phase shifts, extending over a range much larger than  $2\pi$ . Then, excluding pathological cases of extremely non-smooth distribution functions, the intensity of the interference pattern tends to zero. In this limit, the environment's potential cannot be approximated by a static potential, and the analysis of the experiment has to be done in terms of the system + environment. The experiment involves then a transition of the interfering electron from a pure state to a mixed state.

The equivalence between the two descriptions discussed above can also be understood in terms of the uncertainty principle. Suppose that the free particle's state is initially a minimal wave packet, i.e.,  $\delta\eta\delta p_\eta \sim 1$ . Suppose also that the interaction between the free particle and the interfering particle is smooth enough so that it can be expanded as,

$$V(x_r(t), \eta) \approx V(x_r(t), \eta = 0) + \nabla_\eta V(x_r(t)) \cdot \eta \quad (2.10)$$

where  $\eta = 0$  is chosen to be the center of the minimal wave packet. The zeroth order contribution to the expansion is just a static potential, which induces a well defined phase shift. By (2.3), the state of the environment (i.e., the free particle) after the interaction is

$$e^{-i \int_0^{\tau_0} \nabla_\eta V(x_r(t)) dt \cdot \eta} \chi_0(\eta) \quad (2.11)$$

so that the interaction shifts the free particle's momentum by  $\int_0^{\tau_0} \nabla_\eta V(x_r(t)) dt$ . When this momentum shift is larger than the uncertainty in the free particle's momentum,  $\delta p_\eta$ , the environment is shifted into a state orthogonal to its original one and the interference is destroyed. The condition for this destruction is, therefore,

$$\int_0^{\tau_0} \nabla_\eta V(x_r(t)) dt > \delta p_\eta \quad (2.12)$$

On the other hand, by looking at this loss of interference from the point of view of phase uncertainty, we find that the condition for this loss is,

$$\langle \delta \phi^2 \rangle^{\frac{1}{2}} = \int_0^{\tau_0} \nabla_\eta V(x_r(t)) dt \delta \eta > 1 \quad (2.13)$$

The assertion  $\delta \eta \delta p_\eta \sim 1$  yields the agreement of the two conditions. When the uncertainty in momentum is large, it takes a large momentum shift, i.e., a long interaction time, in order to shift the environment to a state orthogonal to its initial one. On the other hand, large momentum uncertainty yields small uncertainty in position. This, in turn, yields a well defined potential, and again, a long interaction time is needed for the phase to become uncertain. Thus, the equivalence between the two descriptions is a consequence of the momentum – position uncertainty in the minimal wave packet.

The above example of an environment lacks an important feature, commonly found in real situations, namely, the independent dynamics of the environment. These dynamics exist in the following example. The interference of two electron's trajectories  $x_r(t)$  and  $x_l(t)$  is examined in a cavity at zero temperature. The electron interacts with the electromagnetic vector potential. It is well known that such an interaction causes a relative phase shift of  $\frac{e}{\hbar c} \int (A(x_r(t), t) \cdot \dot{x}_r(t) - A(x_l(t), t) \cdot \dot{x}_l(t)) dt$ . The average value of the vector potential in vacuum is zero, therefore the mean phase shift is also zero. However, there are quantum fluctuations of the vector potential in the vacuum, and those will cause an uncertainty in the phase shift. The uncertainty will be,

$$\begin{aligned} \langle \delta \phi^2 \rangle = & \left( \frac{e}{\hbar c} \right)^2 \int dt \int dt' \left[ \vec{A}(x_r(t), t) \cdot \dot{\vec{x}}_r(t) - \vec{A}(x_l(t), t) \cdot \dot{\vec{x}}_l(t) \right] \\ & \times \left[ \vec{A}(x_r(t'), t') \cdot \dot{\vec{x}}_r(t') - \vec{A}(x_l(t'), t') \cdot \dot{\vec{x}}_l(t') \right] \end{aligned} \quad (2.14)$$

Now, let us look at the simple case in which the two partial waves follow a 1D harmonic oscillator trajectory in two opposite directions <sup>(6)</sup>, i.e.,

$$x_r(t) = -x_l(t) = R \sin \omega_0 t \quad (2.15)$$

Then, the phase uncertainty is,

$$\begin{aligned} \langle \delta\phi^2 \rangle &= \left(\frac{e}{\hbar c}\right)^2 \int dt \omega_0 R \cos \omega_0 t (A_x(R \sin \omega_0 t, t) + A_x(R \sin \omega_0 t, t)) \\ &\quad \times \int dt' \omega_0 R \cos \omega_0 t' (A_x(R \sin \omega_0 t', t') + A_x(R \sin \omega_0 t', t')). \end{aligned} \quad (2.16)$$

The correlation function  $\langle A_x(x, t) A_x(x', t') \rangle$  in vacuum is

$$\langle A_x(\vec{x}, t) A_x(\vec{x}', t') \rangle = \sum_{\alpha} \frac{\hbar c^2}{\Omega \omega_{\alpha}} e^{(i k_{\alpha} \cdot (\vec{x} - \vec{x}') - i \omega_{\alpha} (t - t'))} \left(1 - \frac{(k_{\alpha z})^2}{|k|^2}\right), \quad (2.17)$$

where  $\omega_{\alpha} = c k_{\alpha}$ , and  $\Omega$  is the cavity's volume.

Substituting this expression into the expression for  $\langle \delta\phi^2 \rangle$ , equation (2.14) we obtain,

$$\langle \delta\phi^2 \rangle = 4\omega_0^2 R^2 \int d\vec{k} \frac{\hbar c}{|k|} \left| \int_0^{\tau} dt \cos\left[\frac{1}{2} k_x (x_1(t) - x_2(t))\right] \cos \omega_0 t e^{i c |k| t} \right|^2 \left(1 - \frac{k_x^2}{|k|^2}\right) \quad (2.18)$$

Most of the contribution to this integral comes from  $|k| \approx \frac{\omega_0}{c} \pm \frac{1}{c\tau}$ . Since  $\omega_0 R \ll c$ , the dipole approximation is valid, i.e.,  $k_x (x_r(t) - x_l(t)) \ll 1$ . The expression for  $\langle \delta\phi^2 \rangle$  is then exactly the probability of a photon field at  $T=0$  to absorb a photon from an oscillating dipole of frequency  $\omega_0$ . Since the photon field is at  $T=0$ , the most effective way to leave a trace on it is to emit a photon. The time it takes for the phase to be completely uncertain is exactly the time it takes for the electron to emit a photon. Since the accelerations of the two partial waves are exactly opposite, the radiation emitted from each of them is in a phase shift of  $\pi$  relative to the other. The phase of the radiation contains the information about the path taken by the electron. Therefore, although the two partial waves emit the same amount of radiation (and they necessarily do, since in the dipole approximation they both feel the same potential fluctuations), the trace they leave on the environment must be different. This is seen in terms of the environment's wave function. The environment's wave function is, up to first order in the potential,

$$|\text{vac}\rangle \pm i \sum_k |a_k| |1 \text{ photon in } k \text{ mode}\rangle, \quad (2.19)$$



where the  $\pm$  sign refers to the environment state coupled to the first and second partial waves. The difference in sign reflects the phase difference in the radiation.

Had the two partial waves propagated with equal velocities in part of their trajectory, then for that part the dipole radiation would have caused uncertainty in each of the partial waves' phase, but not in their relative phase. The relative phase uncertainty would then have been accumulated by the quadropole interaction (next order in  $k \cdot r$ ). Then, the  $\cos[\frac{1}{2}k_z(x_1(t) - x_2(t))]$  in (2.14) should be replaced by  $[k_z(x_1(t) - x_2(t))]^2$ , yielding

$$\langle \delta\phi^2 \rangle = 4\omega_0^2 R^2 \int d\vec{k} \frac{\hbar c}{|k|} \left| \int_0^r dt [k_z(x_1(t) - x_2(t))] \cos \omega_0 t e^{ic|k|t} \right|^2 \left(1 - \frac{k_z^2}{|k|^2}\right). \quad (2.20)$$

In both cases, we conclude that the time it takes for the phase to get uncertain is just the time it takes the partial waves to emit photons that will identify them. By suitably placed reflecting mirrors one can isolate the photons of each path from that of the other path, thus creating a situation where any photon identifies its path.

### 3. Dephasing by a back reacting environment

As demonstrated above, the potential the environment exerts on the interfering partial waves might induce a shift and/or a reduction of the interference pattern. However, we haven't discussed yet what determines this potential. In particular, is the potential exerted by the environment on the interfering particle affected by the presence of that particle? Classically, the answer is negative for a "test particle", while it might be positive for a real particle. The formal generalization of the examples we have considered, given below, opens a way to discuss this question quantum mechanically. Consider a dynamical environment interacting with an interfering particle. The hamiltonian of the environment will be denoted by  $H_{env}(\eta, p_\eta)$ , while the interaction term is  $V(x_r(t), \eta)$ . For simplicity, we restrict ourselves to the case where only the right wave interacts with its environment.

Starting with the initial wave function, eq. (1.1), the wave function at time  $\tau_0$  is,

$$\psi(\tau_0) = l(\tau_0) e^{-iH_{env}\tau_0} \chi_0(\eta) + r(\tau_0) \hat{T} e^{-i \int_0^{\tau_0} (H_{env} + V) dt} \chi_0(\eta) \quad (3.1)$$

where  $\hat{T}$  is the time ordering operator. It is useful at this point to write  $\psi(\tau_0)$  in terms of  $\hat{V}_I(t) \equiv e^{iH_{env}t} V(x_r(t), \eta) e^{-iH_{env}t}$ , i.e., the potential  $V$  in the interaction picture. Note that  $\hat{V}_I(x, t)$  is an operator in the environment Hilbert space, with

parameters  $(x, t)$ . Its time evolution is not affected by the presence of the particle. Therefore,  $\hat{V}_I(x, t)$  represents the local potential exerted by the environment at  $(x, t)$ , in the absence of the interfering particle. Using  $\hat{V}_I$ ,  $\psi(\tau_0)$  can be written as

$$\psi(\tau_0) = l(\tau_0) \otimes e^{-iH_{env}\tau_0}\chi_0(\eta) + r(\tau_0) \otimes e^{-iH_{env}\tau_0}\hat{T}e^{-i\int_0^{\tau_0} V_I(x_r(t), t)dt}\chi_0(\eta) \quad (3.2)$$

Hence, the interference term is multiplied by

$$\begin{aligned} & \langle \chi_0 | e^{iH_{env}\tau_0}\hat{T}e^{-i\int_0^{\tau_0} (H_{env}+V)dt} | \chi_0 \rangle \\ & = \langle \chi_0 | \hat{T}e^{-i\int_0^{\tau_0} V_I(x_r(t), t)dt} | \chi_0 \rangle \end{aligned} \quad (3.3)$$

Obviously, this is the scalar product of the two environment state coupled to the two partial waves. The description in terms of the statistical properties of the phase stems from the observation that equation (3.3) is the expectation value of the time evolution operator, which is a unitary transformation. Being a unitary transformation, the time evolution operator can be represented as  $e^{i\hat{\phi}}$ , where  $\hat{\phi}$ , naturally defined as the phase operator, is hermitian.

Hence, the expression in equation (3.3) becomes,

$$\langle \chi_0 | e^{i\hat{\phi}} | \chi_0 \rangle \quad (3.4)$$

and  $\hat{\phi}$  is implicitly defined by,

$$e^{i\hat{\phi}} = \hat{T}e^{-i\int_0^{\tau_0} dt \hat{V}_I(x_r(t), t)} \quad (3.5)$$

The time ordering operator in the implicit definition suggests an investigation of the commutation relation

$$[\hat{V}_I(x_r(t), t), \hat{V}_I(x_r(t'), t')] \quad (3.6)$$

An immediate mathematical observation is that when this commutation relation is zero for all  $0 < t, t' < \tau_0$ , i.e., during the interference experiment, the time ordering operator can be ignored, and equation (3.5) is easily translated into an explicit expression for the phase. This expression is,

$$\hat{\phi} = - \int dt \hat{V}_I(x(t), t) \quad (3.7)$$

i.e., the instantaneous rate at which a partial wave accumulates its phase is given by the local potential the environment exerts on it at that instant. When the

commutation relation in equation (3.6) is non-zero for parts of the right path, the translation of equation (3.5) into an explicit definition of the phase is not trivial. In particular,

$$\dot{\phi}(t) \neq V_I(\mathbf{x}_r(t), t) \quad (3.8)$$

Physically, we expect equation (3.8) to be true when the potential the environment exerts on the interfering particle (say, an electron) differs from the local potential  $V_I$ , and the difference arises from a back reaction of the environment to the presence of the interfering electron. The local potential  $V_I$ , being local in time, does not allow present electron – environment interaction to modify the potential the electron will feel in the future. Hence, it does not allow a back reaction of the environment to the presence of the interfering electron. We will therefore demonstrate now the equivalence between the condition

$$[\hat{V}_I(\mathbf{x}_r(t), t), \hat{V}_I(\mathbf{x}_r(t'), t')] = 0 \text{ for all } 0 < t, t' < \tau_0 \quad (3.9)$$

and the statement that the potential exerted on the electron is not affected by the presence of the electron. We will demonstrate this equivalence for an electromagnetic-type electron–environment interactions, such as interactions with photons, phonons and other electrons.

Consider an interfering electron interacting with the (not necessarily free) electromagnetic field,  $A(\mathbf{x}, t)$ . The potential in the interaction picture is then

$$-\frac{e}{c} \dot{\mathbf{x}}_r(t) \cdot A_I(\mathbf{x}_r(t), t) \quad (3.10)$$

where the time evolution of  $A_I(\mathbf{x}_r(t), t)$  is determined by the independent dynamics of the field (independent of the presence of the interfering particle). The commutation relation in equation (3.9) is then proportional to the commutation relation

$$[A_I(\mathbf{x}, t), A_I(\mathbf{x}', t')] \quad (3.11)$$

at  $\mathbf{x} = \mathbf{x}_r(t), \mathbf{x}' = \mathbf{x}_r(t')$ . This commutation relation satisfies the same equation of motion as does  $A_I(\mathbf{x}, t)$ , i.e., a wave equation, with the initial conditions,

$$[A_I(\mathbf{x}, t), A_I(\mathbf{x}', t)] = 0 \text{ for all } \mathbf{x}, \mathbf{x}' \quad (3.12)$$

$$[A_I(\mathbf{x}, t), \dot{A}_I(\mathbf{x}', t)] \propto i\delta(\mathbf{x} - \mathbf{x}') \quad (3.13)$$

Therefore, the value of the commutation relation in equation (3.11) is just the amplitude of an electromagnetic excitation created at  $\mathbf{x}', t'$  to be present at  $\mathbf{x}, t$ .

When this amplitude is zero for all points  $(x_r(t), t)$  and  $(x_r(t'), t')$  the local potential expression for the phase (equation (3.7) ) is exact. But, when that amplitude is zero there is also no way the potential exerted on the electron at  $(x_r(t), t)$  can be affected by the electromagnetic interaction the electron has experienced before, at previous points along its path. Therefore, there can be no back reaction. Such is, e.g., the case of a *free* electromagnetic field. For a free electromagnetic field, the commutation relation in equation (3.11) is non-zero only when  $(x, t)$  is on the light cone of  $(x', t')$ , while the motion of the partial wave is, of course, inside the light cone. Hence, the phase accumulated by the right partial wave is given by the integrated local potential (equation (3.7) ). And, indeed, the potential exerted by a free electromagnetic field on the interfering partial wave is not affected by the partial wave's history. The interfering electron might emit photons along its path, thus changing the state of the field, but these photons travel faster than the partial wave, so that it can never re-encounter them. Hence, the potential it feels along its path is independent of the existence of *those* photons. On the other hand, if the field is not free, e.g., in the presence of mirrors or electric charges, photons emitted by the electron might be reflected, re-encounter the electron, and thus change the potential it feels.

Our conclusion at this point is then that if the potential exerted on a partial wave along its path is independent of its history, then the phase accumulated by that partial wave is given by the integrated local potential (equation (3.7) ). This is an important conclusion since it allows the use of equation (3.7) in many circumstances. In particular, when the interfering electron is coupled to a thermal bath, the short memory time of the bath usually justifies the no-back-reaction assumption. The back reaction might be important when the interfering electron shifts the bath significantly out of equilibrium for a long enough time scale.

What happens when there is a significant back reaction of the environment? In particular, do the fluctuations in the *local* potential,  $\hat{V}_I$ , have any relevance to the dephasing of interference? We first give a positive, somewhat surprising, answer to the second question, and then discuss the first. Regarding the dephasing of interference in a back-reacting environment, we show that the fluctuations in the local potential can be used for a determination of a lower limit to the intensity of the interference term, i.e., a lower limit to

$$\left| \langle \chi_0 | \hat{T} e^{-i \int_0^{\tau_0} V_I(x_r(t), t) dt} | \chi_0 \rangle \right| \quad (3.14)$$

A sufficient, though not necessary, condition for interference to be seen, is that this lower limit is not too small, compared with unity.

To obtain the lower limit, we write the environment state coupled to the right wave as a superposition of the initial state  $|\chi_0\rangle$  and a state  $|\xi\rangle$  orthogonal to  $|\chi_0\rangle$ , with amplitudes  $A_{\parallel}, A_{\perp}$ , respectively:

$$\hat{T}e^{-i\int_0^{\tau_0} V_I(x_r(t), t) dt} |\chi_0\rangle = A_{\parallel} |\chi_0\rangle + A_{\perp} |\xi\rangle \quad (3.15)$$

where  $|A_{\parallel}|^2 + |A_{\perp}|^2 = 1$  and  $|\chi_0\rangle, |\xi\rangle$  are both normalized. Obviously,  $A_{\parallel} = \langle \chi_0 | \hat{T}e^{-i\int_0^{\tau_0} V_I(x_r(t), t) dt} |\chi_0\rangle = \langle \chi_0 | e^{i\phi} | \chi_0\rangle$ .  $|A_{\parallel}|$  is a measure for the intensity of the interference term, so that as long as  $|A_{\perp}| \ll 1$  interference is observed.

On the other hand, the time ordered product can be broken into an ordered product of infinitely many exponents,

$$\hat{T}e^{-i\int_0^{\tau_0} V_I(x_r(t), t) dt} = \prod_{k=1}^N e^{-\frac{i\tau_0}{N} V_I(x_r(t_k), t_k)} \equiv \prod_{k=1}^N e^{-iV_I^k \delta t} \quad (3.16)$$

where  $t_k \equiv \frac{k}{N}\tau_0$ , and the limit of  $N \rightarrow \infty$  is taken. For each instantaneous operator  $e^{-iV_I^k \delta t}$  we define instantaneous amplitudes  $a_{\parallel}^k, a_{\perp}^k$  such that,

$$e^{-iV_I^k \delta t} |\chi_0\rangle = a_{\parallel}^k |\chi_0\rangle + a_{\perp}^k |\xi^k\rangle \quad (3.17)$$

where again  $|\xi^k\rangle$  is a normalized state, orthogonal to  $|\chi_0\rangle$ . Now, since  $|a_{\parallel}^k| \leq 1$ , and  $|\langle \xi^k | \xi^{k'} \rangle| \leq 1$ , equations (3.15) - (3.17) imply that

$$|A_{\perp}| \leq \sum_{k=1}^N |a_{\perp}^k| \quad (3.18)$$

In the limit of  $N \rightarrow \infty$ ,  $a_{\parallel}^k$  and  $a_{\perp}^k$  can be expressed in terms of moments of  $V_I^k$ . Substituting the expression for  $a_{\perp}^k$  in equation (3.18), we get,

$$|A_{\perp}| \leq \sum_{k=1}^N \langle \delta(V_I^k)^2 \rangle^{\frac{1}{2}} \delta t = \int_0^{\tau_0} dt \langle \delta V_I(x_r(t), t)^2 \rangle^{\frac{1}{2}} \quad (3.19)$$

Therefore, a sufficient condition for the observation of interference is that the fluctuations in the *local* potential along the partial wave's path do not sum up to unity. If this condition is fulfilled, no additional fluctuations due to a back reaction potential can dephase the interference.

An interesting application of this conclusion is given in the following problem. A particle is located at the top of a symmetric potential "hill" (more quantitatively:

The particle's initial wave function  $\psi(x)$  is a minimal wave packet of width  $\delta x$ , centered around  $x = 0$ , and symmetric, i.e.,  $\psi(x) = \psi(-x)$ . The potential is, say,  $V(x) = V_0 e^{-\left(\frac{x}{a}\right)^2}$ , with  $a \gg \delta x$ . See Fig. 3) An external force whose magnitude is  $F$ , but its direction is unknown, is applied on the particle for a duration  $\tau$ , in which the particle's wave function does not significantly spread. For  $t > \tau$ , is there any measurement that can identify the direction of  $F$ ? Classically, the answer is, of course, positive, and it doesn't matter if  $F$  is infinitesimal. The particle falls down to the right or to the left, according to the direction of  $F$ . Quantum mechanically, however, a measurement can distinguish between two states only if they are orthogonal. And, as we have seen above, if the phase fluctuations induced by the uncertainty in the applied potential are much smaller than unity, the particle on the "hill" (corresponding to the "environment" in the discussion above) is not shifted to a state orthogonal to its initial state. This is true, even though the unstable nature of the initial state tends to "amplify" the effect of the external force. The applied potential is here,

$$V = -Fx \quad (3.20)$$

so that the phase uncertainty it induces is,

$$\delta\phi = \delta V\tau = F\delta x\tau \quad (3.21)$$

Thus, as long as  $F\tau \ll \frac{1}{\delta x}$ , no measurement, not even at  $t \rightarrow \infty$  will identify the direction of  $F$ .

We conclude our discussion of the back reaction of the environment with a simple, exactly solvable, problem, in which the phase shift due to the back reaction can be pointed out. We go back to the one dimensional interference experiment, in which two electron partial waves follow two opposite harmonic oscillator's trajectories (Eq. (2.15)). This oscillator's frequency is  $\omega_0$ . This time we turn off the coupling to the electromagnetic potential, and consider an environment composed of one phonon mode, whose frequency is  $\Omega \gg \omega_0$ , and its coupling to the electron is linear. The hamiltonian of the environment is the free hamiltonian of a phonon mode, i.e.,

$$\hat{H}_{env} = \frac{p^2}{2} + \frac{1}{2}\Omega^2 q^2 \quad (3.22)$$

and the interaction hamiltonian is

$$V = Cqx \quad (3.23)$$

In the above  $p, q$  are the phonon's momentum and position, respectively, and  $x$  is the interfering particle's position. The interaction can be represented in the interaction picture as,

$$\hat{V}_I(x_r(t), t) = Cx_r(t) \left( q \cos \Omega t + \frac{p}{\Omega} \sin \Omega t \right) \equiv Cx_r(t)q_I(t) \quad (3.24)$$

Since the phonon mode is in the ground state,  $\langle q \rangle = 0$ , and  $\delta q = \langle q^2 \rangle^{\frac{1}{2}} = \frac{1}{\sqrt{\Omega}}$ . The motion of the electron partial wave, being slow relative to the phonon mode's frequency, shifts adiabatically the equilibrium point of the phonon mode, without exciting it. Thus, the mode's wave function at time  $t$  is the ground state of an oscillator centered at  $-\frac{Cx_r(t)}{\Omega^2}$ . In the beginning and the end of the partial wave's path, when  $\frac{C|x_r(t)|}{\Omega^2} \ll \frac{1}{\sqrt{\Omega}}$ , i.e.,  $|x_r(t)| \ll \frac{\Omega^{\frac{3}{2}}}{C}$ , the mode's state overlaps with its initial state. In the intermediate stage, the phonon mode's state is nearly orthogonal to its initial state. It has the information on the electron's path, and this information results in a back reaction potential. Therefore, we expect the interference pattern, examined after the electron (and, essentially, also the phonon mode) has returned to its initial point, to show an extra phase, accumulated during that intermediate period. Using the commutation relation of potentials at two different points:

$$[\hat{V}_I(x(t), t), \hat{V}_I(x(t'), t')] = i \frac{C^2}{\Omega} x(t)x(t') \sin \Omega(t - t') \quad (3.25)$$

we can derive an explicit expression for the phase  $\hat{\phi}$ , out of the implicit definition (equation (3.5)). This expression is:

$$\begin{aligned} \phi &= - \int_0^{\tau_0} dt Cx_r(t)q_I(t) - \frac{1}{2} \int_0^{\tau_0} dt \int_0^t dt' \frac{C^2}{\Omega} x_r(t)x_r(t') \sin \Omega(t - t') \\ &\equiv \phi_0 + \phi_{br} \end{aligned} \quad (3.26)$$

The first term in equation (3.26) is the phase accumulated due to the local potential. Its rate of accumulation is  $\dot{\phi}_0 = Cx_r(t)q_I(t)$ , the local potential. Being proportional to  $q_I(t)$ , this rate has a zero average value, and an uncertainty  $\delta\dot{\phi}_0 = Cx_r(t)\delta q_I(t) = Cx_r(t)\frac{1}{\sqrt{\Omega}}$ . The second term is the back reaction contribution. As generally expected for a back reaction, this contribution is of second order in the coupling constant, and it is non-local in time. In this special example, the back reaction contribution is a C-number, so that it does not contribute to the phase uncertainty, i.e., to dephasing. Substituting the right path,  $x_r(t) = R \sin \omega_0 t$

(equation (2.15) ), and taking the adiabatic limit,  $\frac{\omega_0}{\Omega} \rightarrow 0$ , we find that the back reaction contribution to the phase is,

$$\phi_{br} = -\frac{C^2 R^2}{2\Omega^2} \int_0^{\tau_0} dt \sin^2 \omega_0 t \quad (3.27)$$

and the rate at which it is accumulated is,

$$\dot{\phi}_{br} = -\frac{C^2 R^2}{2\Omega^2} \sin^2 \omega_0 t = -\frac{C^2}{2\Omega^2} x_r^2(t) \quad (3.28)$$

This rate becomes larger as  $x_r^2(t)$  increases, i.e., as the phonon mode is shifted far from its equilibrium point. In particular, for  $x_r(t) \gg \frac{\Omega^{\frac{3}{2}}}{C}$ , this rate is higher than the uncertainty in the local potential, which is  $Cx_r(t)\langle q^2 \rangle^{\frac{1}{2}}$ . As long as the environment state significantly overlaps its initial state,  $|\dot{\phi}_{br}| \ll \delta\dot{\phi}_0$ . Once the environment state is orthogonal to its initial one, most of the phase accumulation rate is due to the back reaction.

A generalization of the above picture to a general back reacting environment should be done in a careful way, since some of the above conclusions originate in the simple nature of the example. In general, back reaction of the environment might contribute also to the phase uncertainty. It is only in our simple example that  $\phi_{br}$  is a C-number. However, while  $\delta\phi_0$  is of second order in the interaction potential,  $\delta\phi_{br}$  is of higher order, since  $\phi_{br}$  by itself is of second order.

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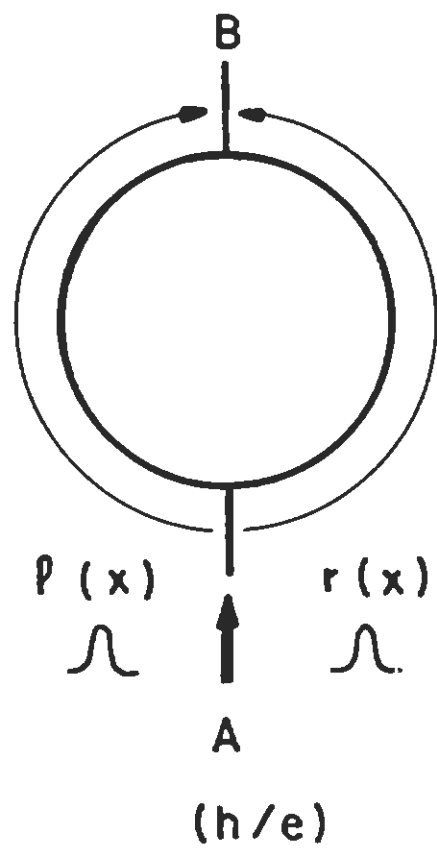


Fig.1: Schematics of interference experiments in A-B rings: Each partial wave traverses half the ring, and the interference is examined at the point B. This kind of interference gives rise to  $\frac{h}{e}$  oscillations of the conductance.

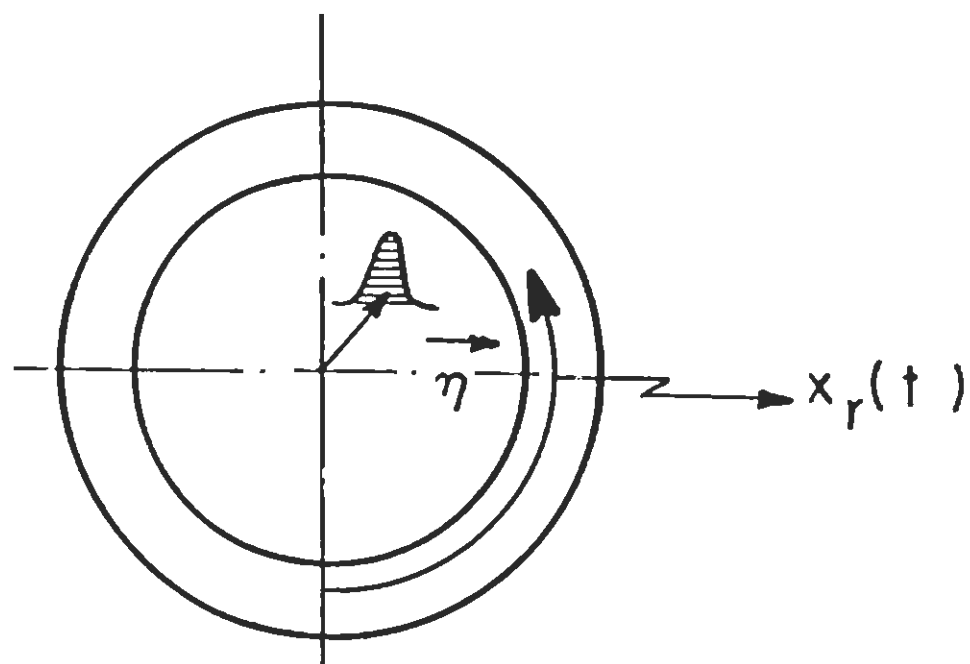


Fig.2: The right path  $x_r(t)$  interacts with an environment composed of one heavy free particle, whose coordinate is  $\vec{\eta}$ .

MAGNETIZATION BY SPIN TUNNELING ON THE TOTAL  
MACROSCOPIC QUANTUM TUNNELING OF THE TOTAL

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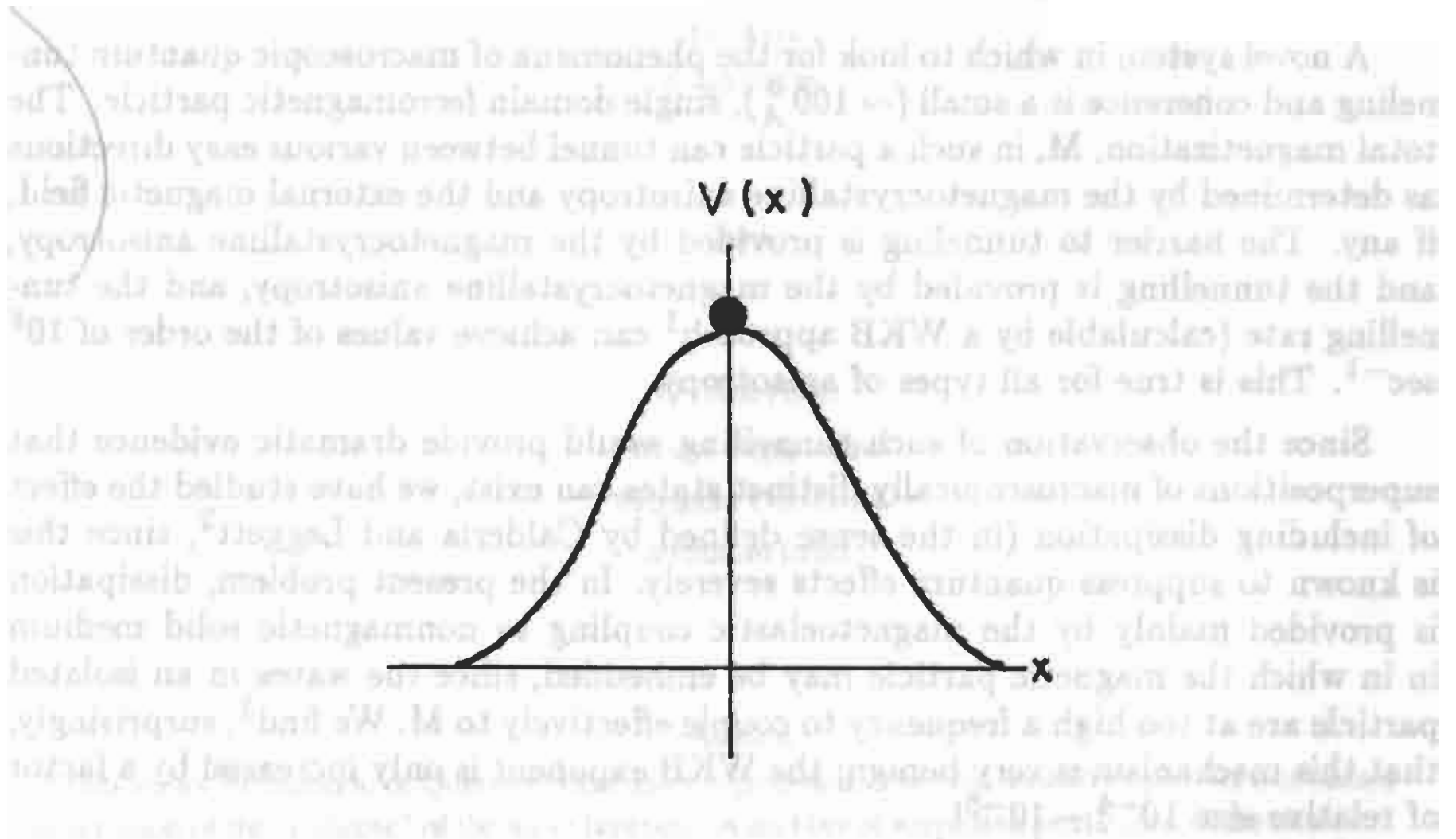


Fig.3: A particle is initially localized on the top of a potential hill.

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