A New Formulation of Quantum Mechanics

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Abstract

It is shown that the Schrödinger idea that considers a particle as an extended wave function is not wrong as is usually thought. The argument relies on a new method of measurement – the protective measurement – which measures the Schrödinger wave without disturbing it. However, to avoid paradoxes we have also to accept a new formulation of quantum mechanics which is based on two state vectors instead of one, the usual (history) state evolving toward the future and a second backward evolving (destiny) state.

I. The Standard Interpretation of Quantum Theory

When Schrödinger proposed his wave equation, there was much argument about the physical meaning of the wave function. While Schrödinger believed that the wave function for a single particle represents an extended object that was really moving in space, Born suggested that the wave function of a single particle has only a probabilistic meaning. That is, any experiment looking at a single particle will find that particle at only one location, but will never see it as an extended object. Only if we have an ensemble of particles, can we see the full implication of the wave function. For an ensemble the quantity $\psi^*(x)\psi(x)$ is proportional to the probability of finding the particle at the point x. We are able to infer the extended nature of a single particle only indirectly, for example, by analyzing a two slit experiment.

There are three general arguments usually presented as to why we can never see the wave function of a single particle. These arguments seem convincing, but we will later show why they are misleading.

1. In the laboratory we never see an extended object. If we make a measurement of an electron, we will always see it as a point on a

photographic plate, or a single track in a cloud chamber. It will always appear as a localized object, never as an extended object.

2. The second argument appeals to unitarity. Suppose we have two possible wavefunctions in the Schrödinger representation, ψ_1 and ψ_2. These are two different descriptions in space since, in general, the two functions are not orthogonal vectors in the Hilbert space. Suppose we now say that there is a measurement that can distinguish between the states ψ_1 and ψ_2 which are not orthogonal. That means there exists a measuring device with some state ϕ such that if the system is in state ψ_1 the state of the measuring device will go to ϕ_1, and if the system is in state ψ_2 the measuring device state will go to ϕ_2. To be able to distinguish between the two results, we must have ϕ_1 and ϕ_2 orthogonal. However, this violates unitarity since the initial states were not orthogonal.

   The usual argument is to have a large number of particles described by the same wavefunction ψ. That is, we start with a set $\psi_1(x_1)$, $\psi_1(x_2)$, ..., $\psi_1(x_N)$ and a set $\psi_2(x_1)$, $\psi_2(x_2)$, ...,$\psi_2(x_N)$. Using these two ensembles, we can distinguish between the two states, since the scalar product between any ψ_1 and ψ_2 is less than 1. The scalar product between the states of two sufficiently large ensembles of particles is essentially zero. Once again the statistical interpretation seems to be indicated.

3. The last argument is the most important since it forces us to adopt the two-vector formulation. Suppose at time t, there is a quantum particle whose wavefunction is non-zero in a large region. Let us assume there is an experiment which can determine that the particle is spread over this large region. We do this experiment and soon afterwards we do the usual experiment and find the particle localized at one position. If we were studying a charged particle, huge currents must flow to conserve charge. Otherwise there would be another frame of reference where the charge is not conserved. Thus the wavefunction cannot collapse infinitely fast. There is no way that an extended object can suddenly become a localized one.

We would like to be able to observe the full wave function. The wavefunction obeys the Schrödinger equation which tells us we have a vector in Hilbert space which evolves in a deterministic fashion. All the mystery in quantum mechanics occurs because we are told that we cannot observe the wavefunction. What we can observe is not what is described by the mathematics. The connection between what can be observed in the laboratory and what is described by the Schrödinger equation is only probabilistic. It would be beautiful if we could see the wavefunction directly.

II. Protective Experiments — An Example

The main argument for the reformulation suggested here is that there are experiments which protect the wavefunction so we can measure the wavefunction without destroying it. We call such experiments, protective experiments. Shelly Glashow suggested calling these protective experiments "in vivo" experiments. This is in analogy with biological experiments which preserve the life in a cell of small living objects. We shall consider below an example in which the protection is due to energy conservation.

Suppose we are given a particle described by a known Hamiltonian with discrete, non-degenerate eigenstates. We are told that the particle is in a definite eigenstate and we are asked to measure its wavefunction. A particular example of this would be an electron in the ground state of a hydrogen atom. In the standard interpretation, we measure the energy of this state and say that this is all that can be known. However, quantum mechanics contains much more information than this. It tells us that there is a wavefunction at each position in space. This is an infinite amount of additional information for a single particle. We will now discuss how we can extract this information without disturbing the wavefunction.

Measurement in an ideal quantum-mechanical experiment has been described by von Neumann. We let $\mathbf{H}_0$ be the Hamiltonian of the free system. This could be the Hamiltonian of an electron in the atom where, for simplicity, we take the proton mass as infinitely large. We let $\mathbf{A}$ represent the quantity we wish to measure, and let q be a variable of the measuring device. Then, the Hamiltonian of the system is

$$\mathbf{H} = \mathbf{H}_0 + g(t)\, q\, \mathbf{A}. \qquad (1)$$

where $g(t)$ is an interaction parameter. We choose

$$g(t) = \frac{g_0}{2T} e^{-|t|/T} \qquad (2)$$

Here T is the effective time of the measurement and g_0 is a constant representing the strength of the coupling between the system and the measuring device.

There are two interesting limits. The first is the impulsive limit where we take $T \to 0$ and the other is the adiabatic limit where we take $T \to \infty$. The usual experiment is to take the impulsive limit in which the experiment lasts an extremely short time. In this case we can ignore $\mathbf{H}_0$, and the momentum conjugate to q will be changed by one of the eigenvalues of **A**. We are only able to get probabilities for this change and hence cannot measure the wavefunction.

In the adiabatic limit the experiment lasts a long time while the coupling between the measuring device and the particle becomes very weak and approaches zero. Surprisingly, even though the coupling goes to zero, we can still get information about the particle and we get this information without changing the wavefunction. Indeed, in the adiabatic limit the ground state wave function is the ground state of the full Hamiltonian during the full time of the measurement. The only thing that can change is the phase.

We will first look at an eigenstate of q and then at a superposition of eigenstates. For an eigenstate, the adiabatic limit becomes a normal perturbation problem. The energy goes to the original energy plus a correction that goes to zero, that is

$$\mathbf{E} = \mathbf{E}_0 + g(t)\, q\, \langle A \rangle \tag{3}$$

where $\langle A \rangle$ is the expectation value of **A** calculated with the original wavefunction. Now $\mathbf{E} - \mathbf{E}_0 \to 0$ but the total phase accumulated is

$$\int \mathbf{E}(t)\, dt = \int E_o\, dt + g_o\, q\langle A \rangle. \tag{4}$$

Since quantum theory is a linear theory, what is true for q as an eigenstate is true for a superposition of eigenstates. If we start with the measuring device in a superposition of q's there will be a different phase associated with each value of q. If p is the momentum conjugate to q, the change in p will be $\delta p = g_o\langle A \rangle$. So we can measure not only the eigenvalue of an operator, but the average of an operator in a given state.

We can also clearly make N simultaneous measurements with N measuring devices, each measuring a different $\mathbf{A}_n$. The Hamiltonian in this case is

$$H = H_o + g(t) \sum_{n=0}^{N} q_n \mathbf{A}_n. \tag{5}$$

If we choose the set of variables $\mathbf{A}_n$ to be the projection operators in different regions of space, the results for each $\mathbf{A}_n$ will be proportional to $\psi^*(x_n)\ \psi(x_n)$ at this point x_n and the entire set measures $\psi^*(x)\ \psi(x)$ in its full glory in all space.

III. Refutation of the Three Arguments of Section I

We must now show why the three seemingly very convincing arguments of Section I were misleading.

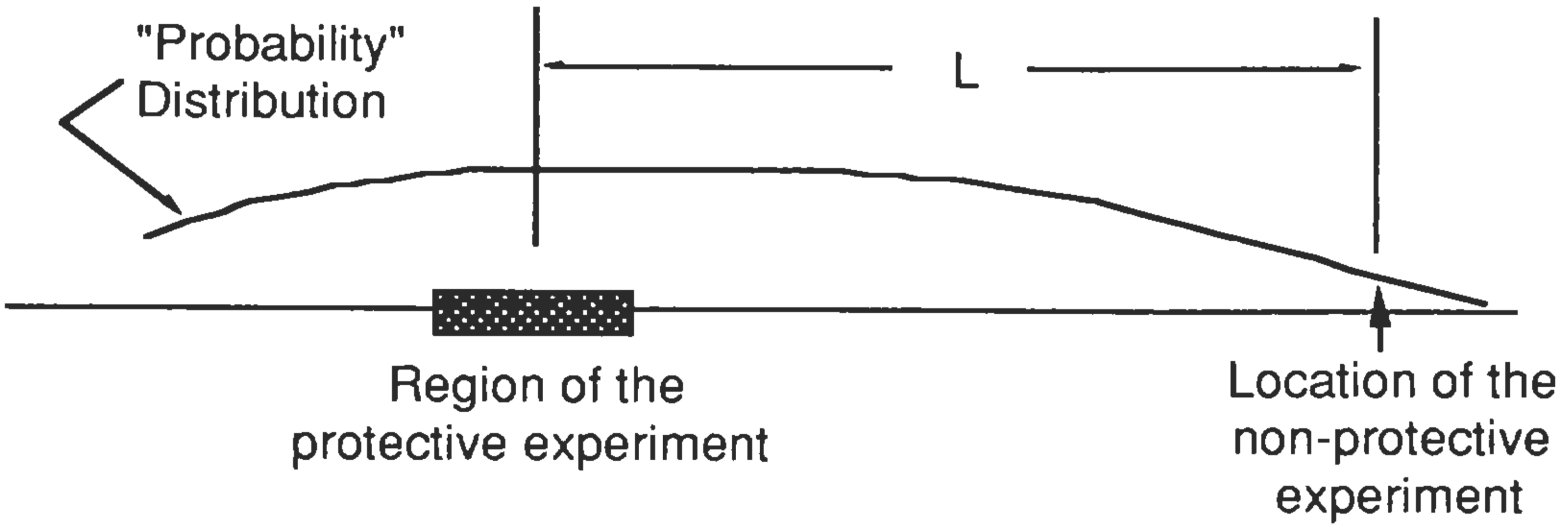
1. The first argument is easily discounted. The previous experiments were simply not the right experiments. Up to now, we have only designed experiments that would "kill" the wavefunction by looking for a localized particle. These were not "in vivo" experiments. In analogy with our biological example, if you do the wrong experiment on an organism, you will kill it. We previously were doing the wrong experiment on ψ and thus "killed" it.

2. The unitarity issue is resolved in an interesting way. The only states that were protected are nondegenerate eigenstates of the Hamiltonian. These eigenstates are orthogonal to each other, so no contradiction with unitarity arises. If we try to measure superpositions of two such states the system will collapse to one state or the other. We are still able to see the state in its full glory, but we see only one state out of the set of completely orthogonal nondegenerate eigenstates of the Hamiltonian. If we want to see other states such as a superposition of eigenstates, we must find a different protection since conservation of energy does not preserve them.

   The issue is not to think of measurement as just determining what we don't know. The real issue of measurement theory is determining what can manifest itself. If we have an electron passing through two slits, we can measure its wave function for a single particle and see the full glory of the interference spectrum. It is only necessary to devise the right protection.

3. Complete resolution of the third argument will be presented in the next section but it is interesting and fruitful to consider what happens, if while performing a protective measurement in one region of space, a usual position measurement is performed at some other location and finds the particle there. Can we violate causality and send signals faster than light? The answer is no. As an example suppose we have an

electron in the ground state of the hydrogen atom as shown in the figure.



We are doing our protective experiment in the vicinity of the proton and find the wave function density corresponding to the ground state. While we are doing our experiment some other physicist makes a non-protective measurement at a large distance L away from us and finds the whole particle there. This contradicts the outcome of our protective experiment. To avoid possibility of casual connection between these experiments, we must complete our protective experiment in a time T less than L/c. For finite time experiment we no longer can be sure that the electron remains in the ground state. There is a finite probability of exciting the state, which goes like e^{-ET}. This is the probability to make a mistake. On the other hand, the probability to find the particle at location L is

$$e^{-L\sqrt{2mE}}$$

where m is the mass of the particle and **E** is the binding energy. The only way to violate causality is to have a binding energy greater than $2mc^2$. We have a nice result. There is no way to consistently describe a single particle in relativistic quantum theory if the binding energy exceeds $2mc^2$.

IV. The Two-Vector Reformulation of Quantum Theory

To resolve violation of Lorentz covariance in the wavefunction collapse problem we must reformulate quantum mechanics. It is possible to do this by using the two-vector formulation. The two-vector formulation can be described as follows. Suppose we have a region of space where an experiment is performed. For example, in a scattering experiment we start with an incoming state, call it ψ_1 and allow this prepared state to interact and produce a set of outgoing states corresponding to different outcomes. We want to select only particles that go into a particular outgoing state ψ_2. In classical physics, if we had a well-defined incoming state there would be only one outgoing state. In quantum mechanics there will be an ensemble of outgoing states. This allows us to define new quantum ensembles that do not have an analogy in classical physics. These are called pre-selected and post-selected ensembles. They are characterized by giving two boundary conditions on the particles. These are the boundary conditions at the start of the experiment and the boundary conditions at the end of the experiment. That is, if an incoming state ψ_1 can produce the following set of outgoing states ψ_2, ψ_3, etc., then we form separate ensembles for those experiments that produce the pair (ψ_1,ψ_2) and those that produce (ψ_1,ψ_3), etc.

This suggests characterizing each quantum particle in the pre-selected and post-selected ensemble by two states. Each quantum particle is described at any instant by two vectors that we will call the history vector and the destiny vector. This concept will enable us to explain how a distribution that was extended in space can suddenly be replaced by a distribution that is peaked near a given position.

What we measure is not $\rho(x) = \psi^*(x)\, \psi(x)$ but the density

$$\rho_{12}(x) = \frac{\psi^*_2(x)\, \psi_1(x)}{\int_{-\infty}^{\infty} \psi^*_2(x')\, \psi_1(x')\, dx'}$$

where ψ_1 is the history vector and ψ_2 is the destiny vector. In all protective experiments what is measured is not the average of either of these states but the above combination. In the usual non-protective experiments, the history vector and the destiny vector were the same, so this distinction was not obvious.

Let us consider again the paradoxical situation of the argument 3. Let the initial state ψ_1 be a superposition of the two localized states. The final state ψ_2 is one of these localized states. We might obtain it by just looking and not finding the particle in another place. The paradox is how the particle "jumps" instantaneously to the first location just by not observing it in the second location. The way out is that the particle was in the first location during the whole period between the two measurements! Indeed, the two vector density $\rho_{12}(x)$ is non-zero only when both ψ_1 and ψ_2 are not zero, i.e., only in the first location. Similarly we can resolve the problem of how an extended particle becomes localized. The product of an extended particle and a localized particle is always a localized particle. It is localized all the time.

We resolve argument 3 by thinking of a quantum system as being described by two vectors, the history vector and the destiny vector, rather than by one vector. We no longer violate causality since the description depends also upon what happens later, not just upon what has happened. If you change your mind about what you will measure, the destiny vector must be changed all the way back to the beginning just as we would have to change the history vector if we had decided to perform a different experiment. This is analogous to the Einstein-Podolsky-Rosen (EPR) experiment. In EPR, we have already learned that if we take a single system that is already correlated to another system, and make a measurement on one of the systems, it immediately changes the state of the other system. In an ensemble, the probability distribution remains unchanged, so we cannot use this to send information faster than light. In the same way here, the future state changes the present for an individual quantum system; but it doesn't change the probability distribution for an ensemble. Therefore, it cannot be used to transfer information backwards in time.

Conclusion

We have described a new type of experiment, the protective measurement, through which we can observe the extended wavefunction of a single particle in its full glory. This reality of the wavefunction strongly supports a new interpretation of quantum mechanics, the 2-vector formulation, in which there are 2 vectors describing a quantum system, the usual vector propagating from the past and a second one propagating backwards from the future. We show how this interpretation resolves the arguments given against the observability of the wavefunction of a single particle.

Acknowledgments

The following took part in developing this new formulation of quantum theory: Susan D'Amato, David Albert, Lev Vaidman, Sandu Popescu, Jeeva Anandan, Benny Resnik, and Daniel Rohrlich. This research was supported in part by grant U25/91-1 of the Basic Research Foundation (administrated by the Israel Academy of Science) and by NSF grant PHY-8807812.