

The Schrödinger wave is observable after all!

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Abstract

It is shown that it is possible to measure the Schrödinger wave of a single quantum system. This provides a strong argument for associating physical reality with a quantum state of a single system in sharp contrast with the usual approach in which the physical meaning of a quantum state is related only to an ensemble of identical systems. An apparent paradox between measurability of a quantum state of a single system and the relativistic causality is resolved.

1. INTRODUCTION

The Schrödinger wave is the most important tool in quantum theory. However, when one tries to associate a physical reality with the Schrödinger wave of a single particle one runs into serious difficulties, and it is generally believed that this task is impossible. Many accept the interpretation in which the Schrödinger wave is only a mathematical tool for calculating probabilities of (real) experiments. Let us list some of the difficulties with identifying the Schrödinger wave as physical reality.

i) We have never seen a quantum state of a single particle in a laboratory. Although the wave is often spread over a region of space, we never see a particle simultaneously in several distinct locations.

ii) There is an argument that we will also never see the Schrödinger wave in the laboratory. Assume that we can “see” the quantum state. Then, it seems, we can distinguish among different states. But the possibility of distinguishing between nonorthogonal states contradicts the unitarity of quantum theory. Indeed, the scalar products between any pair of quantum states do not change during unitary time evolution. But the alleged measuring procedure changes this scalar product to zero.

iii) The collapse of the wave during the measurement contradicts Lorentz covariance [1].

If, however, the Schrödinger wave represents only a mathematical tool, no such difficulties arise: we should not “see” mathematical objects in a laboratory and there are no limits on the rate of change of a mathematical function.

In this letter we will show that the density $\Psi^*\Psi$ and the current $\frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi)$ of the Schrödinger wave are observable even for a single particle and, therefore, do represent physical reality. The usual quantum measurements referred to in (i) alter the Schrödinger wave, and therefore they are not adequate for our purpose. We will use special *protective* measurements. Protective measurements allow us to measure the density and the current of the Schrödinger wave without changing it. In some cases energy conservation provides protection for the state, while in other cases we have to add a special protection procedure.

The second argument is certainly correct in the claim that unitarity prevents us from distinguishing among nonorthogonal states, but it only implies that there is no *universal* procedure for observing different states; it allows, however, for the possibility that for any state there is an appropriate measuring procedure. In this work we shall show that this is indeed so.

The last problem is the most serious one. Assume that we start with a particle in a superposition of being in two separate boxes, and then find it in one of them: How did half of the wave moved instantaneously from one box to another? Recently, we have developed a novel approach which helps to solve this difficulty [2]. Here, however, we will only touch this issue by considering the apparent paradox between the causality requirement and our Schrödinger wave measurements. Our solution also yields an answer to the recent proposal for superluminal communication [3].

2. MEASUREMENTS OF NONDEGENERATE EIGENSTATES

Let us start by discussing the simple case in which the system itself supplies the protection of the state and no artificial protection is needed. This is the case of a discrete nondegenerate eigenstate of energy. Now, every sufficiently slow and weak measurement, i.e., any measurement with adiabatic interaction, will not destroy the state of the particle. And, if the measurement is long enough, it can provide any desired precision.

The first example is a measurement of the ground state of the electron in the hydrogen atom. The state $\Psi(\mathbf{x})$ can be chosen real and positive aside from the trivial time dependent part. Therefore, it is enough to measure the expectation values of projection operators P_{V_n} on small regions V_n where

$$P_{V_n}(\mathbf{x}) \equiv \begin{cases} 1, & \text{if } \mathbf{x} \in V_n; \\ 0, & \text{if } \mathbf{x} \notin V_n. \end{cases} \quad (1)$$

The expectation value of P_{V_n} yields the value of $|\Psi|^2$ times the volume of the region V_n , and since Ψ is not negative we obtain Ψ everywhere in space.

In order to measure P_{V_n} we will use the standard von Neumann measuring procedure, but instead of an instantaneous interaction we will make it long and adiabatic.

The interaction Hamiltonian will be

$$H = g(t)pP_{V_n}, \tag{2}$$

with $g(t) = 1/T$ for a period of time T and smoothly going to zero before and after. The initial state of the measuring device will be chosen so that the canonical conjugate p of the pointer variable q will be effectively bounded (say, a Gaussian centered at zero). For $g(t)$ smooth enough we obtain an adiabatic limit in which the electron cannot jump from one energy eigenstate to another. For bounded p , in the limit of $T \rightarrow \infty$, the interaction Hamiltonian goes to zero. The energy of the eigenstate shifts by an infinitesimal amount given by first-order perturbation theory:

$$\delta E = \langle H_{int} \rangle = \frac{\langle P_{V_n} \rangle p}{T}. \tag{3}$$

In the limit $T \rightarrow \infty$ the energy eigenstate remains unchanged. Although the interaction vanishes in this limit, its duration increases accordingly such that the measuring device yields the desired result.

$$e^{-\frac{i}{\hbar} p \langle P_{V_n} \rangle}. \tag{4}$$

Thus, the state of the pointer position q is shifted during the interaction by the expectation value $\langle P_{V_n} \rangle$. This is a measurement of $\langle P_{V_n} \rangle$ with a precision equal to Δq , the width of the initial pointer variable distribution. (Note that Δq can not be infinitesimally small because Δp is bounded.)

Obviously, this procedure will also work if we couple the electron to many such measuring devices, thus measuring the projection operators in different regions, namely,

$$H = \sum g(t)p_n P_{V_n}. \tag{5}$$

Performing these measurements simultaneously will require a longer measurement period but, in principle, we can measure the value of Ψ of a single particle everywhere in space.

Protection due to energy conservation also suffices for measurement of stationary nondegenerate eigenstates with nonzero current. These are, for example, the eigenstates of particles described by a Hamiltonian with a vector potential, such as electrons confined to a toroid through which there is a flux of magnetic field (the Aharonov-Bohm effect). In this case the state cannot be chosen real and we have to find its local phase. (The overall phase is unmeasurable and has no physical meaning.) To this end we will measure, in a similar adiabatic way, the following operators:

$$B_n = \frac{-i}{2}(P_{V_n} \nabla + \nabla P_{V_n}). \tag{6}$$

The expectation values of these operators are proportional to the current density at these regions, and using also the values of the density itself we may find the gradient of the phase. Indeed, if we represent the Schrödinger wave as $\Psi(\mathbf{x}) = r(\mathbf{x})e^{i\theta(\mathbf{x})}$ then

$$\frac{\langle B_n \rangle}{\langle P_{V_n} \rangle} = \nabla\theta. \quad (7)$$

The phase can be found by integration of its gradient up to an (unobservable) constant. Note, that if there are regions which are disconnected one from the other, then, by this method, the relative phase cannot be defined.

The density $|\Psi|^2$ multiplied by the charge of the electron yields the effective charge density. An adiabatic measurement of the Gauss flux out of a certain region must yield the expectation value of the charge inside this region. Similarly, in the case of a stationary state with non-zero current, an adiabatic measurement of the Ampere contour integral around a small loop yields the current density times the charge of the electron.

3. THE OUTCOMES OF THE ADIABATIC MEASUREMENTS ARE SINGLE-TIME PROPERTIES OF A QUANTUM SYSTEM

We have shown the observability of a quantum stationary state. This is our main argument for associating physical reality with a quantum state. Since our measurement lasts a long period of time we do not have a method for measuring the Schrödinger wave at a given time. Thus, we have a direct argument for associating physical reality with stationary Schrödinger waves only over a *period* of time. The reader may suspect that what we have measured represents time-averaged physical properties of the system. Let us present a few arguments explaining why we should associate the outcomes of our measurement with single-time properties of our quantum system.

The essential feature of our adiabatic measurement is that the state $|\Psi\rangle$ does not change throughout the experiment. Since in the standard approach to quantum theory the Schrödinger wave yields the complete description of the system, we conclude that the action of the system on the measuring device is the same at any moment during the measurement.

However, perhaps there is a description of a system beyond its quantum state which does change during the measurement process, and the Schrödinger wave we measure does not have inherent meaning at a given time but represents only a time average over the period of the measurement. Indeed, it is possible to construct a simple classical model in which the outcomes of our measurements follow our predictions, but no physical meaning can be associated with the wave at a given time. Consider a model of an atom in which the electron performs very fast ergodic motion in the region corresponding to the quantum cloud. The charge density might be either zero (if the electron is not there) or singular (if the electron is inside the infinitesimally small region including the space point in question). In spite of this fact, the measurement we have described will yield outcomes corresponding to a nonsingular charge density cloud. What is measured

here is the time average of the density, or how long time the electron spent in a given place.

In order to see that this picture is inappropriate for the quantum situation let us consider another example: a particle in a one dimensional box, say in the first excited state. The spatial part of the state is $\sqrt{2/L} \sin(2\pi x/L)$. The adiabatic measuring procedure described above will yield the Schrödinger wave density $(2/L) \sin^2(2\pi x/L)$. In particular, it equals zero at the center of the box. If there is some kind of underlying position of the electron which changes in time such that the result of the density measurement is proportional to the amount of time the electron "spends" there, then half of the time it must be in the left half of the box and half of the time in the right half of the box. But it can spend no time at the center of the box, i.e., it must move at infinite velocity at the center. It is absolutely unclear what this "position" of an electron might be. There is a theory [4] which introduces a "position" for a particle in addition to its Schrödinger wave; but according to this theory, the "velocity" of the particle in this energy eigenstate vanishes: it does not move at all.

The mathematical formalism yields an additional argument: in our measurement for any, even very short, period of time the measuring device shifts by an amount proportional to $\langle A \rangle$, the expectation value of the measured variable, rather than to one of the eigenvalues. Thus, the expectation values, which are the mathematical characteristics of Schrödinger waves, can be associated with very short periods of time. In the limit the expectation values and, therefore, the quantum state itself become single-time properties of a quantum system.

In order to explain this behavior of the measuring device we note that its wave function is a superposition of states shifted by the eigenvalues of the measured operator. These shifted waves are not orthogonal and we obtain an interference effect. The interference miraculously yields a shift proportional to the expectation value. The shift, however, is also proportional to the time interval, and for a short time this shift is much smaller than the uncertainty, so we cannot see it from a measurement performed on a single particle during a short time. Only the total shift accumulated during the whole period of measurement is much larger than the width of the initial distribution, and therefore, it is observable on a single particle.

One more argument showing that our measurements reflect values which are not of statistical character follows from consideration of measurements performed on an ensemble of identical systems. Simple calculations show that in the limit of a very large ensemble and correspondingly very weak and slow measurement we obtain knowledge of the quantum state of these systems without exciting even one system in the ensemble!

4. MEASUREMENTS WITH ARTIFICIAL PROTECTION

The next question is how to measure the Schrödinger wave when it is not protected by energy conservation, i.e., when it is not a discrete nondegenerate energy eigenstate. In fact, the measurement remains the same, but we must add a "protection" procedure.

If we have a degenerate eigenstate, then one of the simplest ways is to remove the degeneracy by changing the energies of the other states, such that the state to be

measured remains unchanged, but is now protected by energy conservation.

In case the state is a superposition of different energy eigenstates, then the simplest way to protect the time-dependent Schrödinger wave is by dense state-verification measurements which test and protect the time evolution of the quantum state. This is a Zeno type protection. If we are interested in all details of this time dependent state we cannot use measurements which are too slow. Thus we need stronger protection. For measurement of any desired accuracy of the Schrödinger wave, there is a density of the projective measurements which will protect the state from being changed due to the measurement interaction. The time scale of intervals between consecutive protections must be much smaller than the time scale of changing the Schrödinger wave. (It is better, however, not to make the protective measurements too dense, otherwise they might force a time evolution which has no connection to the undisturbed evolution of the quantum system.)

The conceptual disadvantage of measurement with artificial protection is that we have to know the state in order to arrange a proper protection. One might object, therefore, that we obtain no new information from our measurement. However, we can separate the protection procedure and the measuring procedure: one experimentalist provides protection and the other measures the Schrödinger wave itself. Then the second experimentalist does obtain new information. Even for dense projective measurements, most of the time the system evolves according to its free Hamiltonian, so we are allowed to say that what we measure is the property of the system and not of the protection procedure. But the most important point is that we actually “see” the Schrödinger wave of a single particle using a standard measuring procedure.

5. MEASUREMENTS OF THE SCHRÖDINGER WAVE AND SUPERLUMINAL COMMUNICATION

One of the important features of non-relativistic quantum theory is that despite the non-relativistic character of the “collapse” of the Schrödinger wave, one cannot use the collapse for sending signals faster than light. The possibility of measuring the value of a quantum state at a given location at first seems to allow such superluminal communication [3]. Let us show that our measurements of Schrödinger waves do not violate causality.

The most naive way of sending superluminal signals is as follows. Consider a particle in a superposition $\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ of being in two boxes separated by a very large distance. The expectation value of the projection operator on the first box in the initial state is $\langle P_1 \rangle = 1/2$. This must be the outcome of the measurement performed in the first box. If, however, just prior to our adiabatic measurement in the first box, someone opens and “looks” into the second box, causing collapse into a localized state in box 1 or in box 2, then the outcome of the measurement of the projection operator in the first box will drastically change: we no longer find $\langle P_1 \rangle = 1/2$ but rather 0 or 1 (if the other observer found 1 or 0 respectively). Therefore, it seems that by strong measurement in box 2 we can send information to box 1 located arbitrarily far away.

The above argument is not valid for a very simple reason: the state $\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$

is not a discrete nondegenerate eigenstate. Since there is no overlap between the states $|1\rangle$ and $|2\rangle$, the orthogonal state $\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)$ has the same energy. Thus, there is no natural protection due to the energy conservation, and our measurement requires an artificial protection. The latter, however, includes in this situation nonlocal, explicitly nonrelativistic interactions. The artificial protection is the source of the alleged superluminal signal propagation. (This is another disadvantage of measurements with artificial protection.)

Much more subtle is the resolution of the apparent paradox for another situation. Consider a particle bound in a potential well in a ground state of energy E_0 . We perform an adiabatic measurement of the value of $\Psi^*\Psi$, which is non-zero, somewhere inside the well. The outcome of the measurement has to be this value, call it α . Even if we repeat the measurement many times we always have to obtain α . Suppose now that someone just prior to our measurement performs a strong measurement searching for the particle at a distance l far outside the well. There is an exponentially small but non-zero probability that he will find the particle there:

$$prob_1 = e^{-2l\sqrt{2m|E_0|}}, \tag{8}$$

(we take from now on $\hbar = 1$). Thus, if the measurements were performed on an ensemble of particles, a few of the outcomes of our adiabatic measurements have to be zero instead of α . (The other outcomes are affected too, but this effect is probably unobservable.) We have presented a procedure which seemingly transmits superluminal signals a distance l from the distant observer to the well: if the observer made the measurements then some of the outcomes of the Schrödinger wave measurements have to be zero, and if the observer did not perform any measurement, all outcomes have to be α .

The resolution of the paradox involves the relativistic formula: $E = mc^2$. It is necessary that the minimal time for our adiabatic measurement (without any artificial protection procedure) is larger than the time it takes light to arrive from the location of the particle, $T \geq l/c$. However, if the adiabatic measurement takes time T , there is small but non-zero probability for it to kick the particle out of its bound state:

$$prob_2 \geq e^{-2T|E_0|}. \tag{9}$$

In this case the measurement will give a mistaken result which might be zero. The requirement that the probability of error should not be smaller than the probability of finding the particle at the distance $l = Tc$ from the potential well implies

$$e^{-2T|E_0|} \geq e^{-2Tc\sqrt{2m|E_0|}}, \tag{10}$$

and therefore $|E_0| \leq 2mc^2$. Thus causality is fulfilled if Eq.(10) is valid. In other words, if the binding energy of a single particle is larger than $2mc^2$, then there is a paradox associated with causality. This limitation is easily understood in relativistic mechanics: it is the regime where pair production must be taken in account.

6. CONCLUSIONS

We have shown that expectation values of quantum variables and the quantum state itself have physical meaning, i.e., they are measurable for individual quantum systems. This is in sharp contrast to the standard approach in which expectation values and the Schrödinger wave are statistical properties of ensembles of identical systems.

Although our discussion is based on Gedanken experiments, recent experimental work with so-called “weak links” in quantum circuits shows that slow adiabatic measurements of the Schrödinger wave can be performed in the laboratory [5].

Finally, we have shown that adiabatic measurements cannot be used for superluminal transmission of signals via collapse of the Schrödinger wave.

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