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ABOUT POSITION MEASUREMENTS WHICH DO NOT SHOW THE BOHMIAN PARTICLE POSITION

This work is inspired by our discussions with David Bohm in different periods in Tel-Aviv, London, and South Carolina. We were excited by the results we obtained in the framework of two-state vector formalism about weak measurements (Aharonov and Vaidman 1990) and we were trying to understand these results using Bohm's causal interpretation (Bohm 1952a,b). Both Bohm's theory and the two-state vector formalism yield the same predictions for the results of experiments as the standard quantum theory. Therefore, clearly, there cannot be a technical contradiction between these two approaches. However, it is a legitimate question to ask how close or how different are the concepts of the two theories. Friction between the basic concepts might lead to a direction for the modification of quantum theory; the search for a useful modification was the goal of David Bohm.

I (L.V.) have a vivid memory of an excursion the day after the conference in South Carolina honoring 30 years of the Aharonov–Bohm effect when I, David and Saral Bohm were riding in a carriage in the old streets of Charleston. I remember discussing with David my passion for the many-worlds interpretation (Everett 1957), and then, what I find most attractive in the Bohm theory: that it is unambiguous, deterministic and complete. I see it as the best candidate for the final theory of the world for a physicist (such as Y.A.) who does not want to accept the existence of many worlds. David said to me that what I liked in his theory did not have much value for him. He had a strong belief that a man cannot find the final theory of the world. All that we can do is to look for a better and better approximation to the correct theory which is intrinsically unattainable to us. His vision was that the causal interpretation should suggest a way for generalization to the next-level theory which also, by no means, will be the final theory of the world.

We believe that the observations we make in this work are somewhat disturbing for a physicist who wants to see in the causal interpretation the final word about the world, but they are not a real threat for the theory from the perspective of Bohm. Apart from discussing weak measurements, we will consider recently proposed "delayed observation" measurements (Englert *et al.* 1992) which exhibit similar features. Note, however, that the difficulties we see follow mostly form a particular approach to the Bohm theory we adopt, in which only the Bohmian particle corresponds to the "reality" which we experience, while the wave function is just a pilot wave which governs the motion of the particle. By the position of the "Bohmian particle" we mean what is frequently called actual position of the particle in the Bohm theory. For a composite system consisting of many

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particles we shall, somewhat abusing the language, refer to the point in configuration space formed from the coordinates of all the Bohmian particles as the "Bohmian particle."

We consider the following two principles desirable for a causal interpretation. In Bohmian mechanics in most cases they are indeed valid. However, as we will show, there are situations in which they are not.

(I) *A procedure which we usually consider as a good measurement of position should yield the position of the Bohmian particle.*

(II) *An empty wave (the one without the Bohmian particle inside) should not yield observable effects on other particles.*

The motivation for the first principle is clear. The second principle we find desirable because otherwise the Bohmian picture becomes very complex. In the Bohm theory there is no collapse of the wave function, so the total wave incorporates all the complexity of the many worlds of the Everett interpretation. We hoped that the Bohm theory could avoid it. The Bohm theory, as we understand it, says that when we observe a person whose Schrödinger wave is split into a superposition of two macroscopically different waves, we actually see only the component of the superposition which has the Bohmian particle inside. We thought that this is true also when we "observe" a single particle – by means of any type of interaction that we might speak of as detecting it.

Let us start with the following example. The wave function of the particle consists of two identical wave packets running in opposite directions (see Figure 1). For simplicity we will consider a rectangular shape and assume

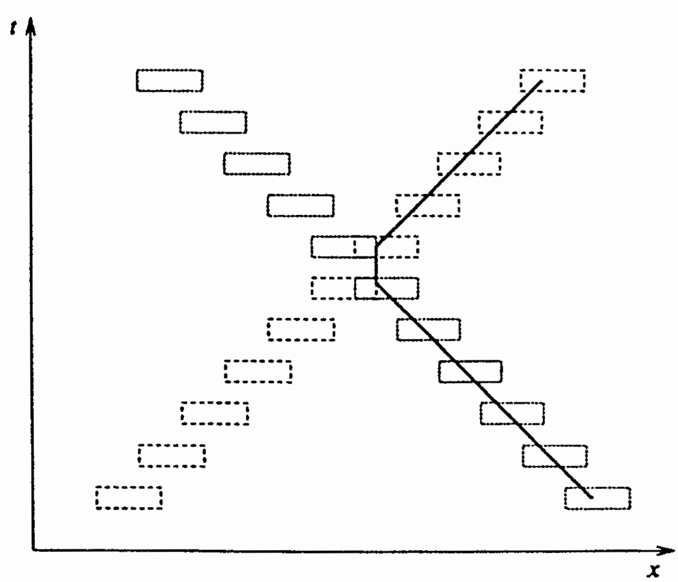

Fig. 1. Space-time diagram for the wave and the Bohm coordinate of the particle. The initial state is the superposition of two identical wave packets moving one towards the other. The Bohmian particle coordinate located initially in the right wave packet. Contrary to the naive expectation the Bohmian particle makes a turn in the region where no (Hamiltonian type) interaction takes place.

that the spread of the wave packets during the process can be neglected. In the beginning the Bohmian particle is inside the right wave packet. The Bohm theory yields an extremely simple prescription for finding the evolution of the system. Until the overlap, the Bohmian particle runs together with the right wave packet. At the moment it reaches the area of the overlap, it stops (the currents of the two wave packets cancel each other in the calculation of the Bohmian particle velocity). Since at that moment the left wave packet has its whole length to go over the particle, while the right wave packet has only a part of its length, the particle will end up inside the left wave packet and will run with it to the right.

One of us (L.V.) used to view the Bohm interpretation as the most elegant way of pointing out one of the many worlds of the Everett interpretation as "real". This example shows that the Bohmian world might be different from any of the Everett worlds. Indeed, no turns take place in Everett worlds in this example.

Now let us add a robust position measurement on the left side of the wave packet (the empty wave; see Figure 2). By a robust measurement we mean a von Neumann-type interaction between the particle and the pointer variable of the measuring device. We can model it by the interaction Hamiltonian

$$(1) \qquad H = g(t)P\Pi_V,$$

where P is the conjugate momentum to the pointer variable Q which is, say, the spatial coordinate of the pointer. Π_V is a projection operator on the volume V. The left wave packet of the particle passes the volume V during a certain time t. The coupling constant g is chosen in such a way that during that time the wave function of the pointer is shifted by a distance much larger than its spread. We consider the initial state of the particle and the position of its Bohm coordinate to be the same as in the previous case. Of course, initially the Bohmian particle of the pointer is somewhere inside its initial wave packet. It is very easy to see the evolution, according to the causal interpretation, also in this situation. The wave function of the particle and the system become entangled; the right wave packet is entangled with no shift of the pointer while the left wave packet is entangled with the shifted wave packet of the pointer. The Bohmian particle which started inside the right wave packet runs with it all the way to the left, and the Bohmian particle of the pointer does not move. Indeed, when the wave packets of the particle overlap, there is no overlap for the wave function of the composite system (particle and the pointer) and therefore the left wave packet does not change the motion of the Bohmian particle moving inside the right wave packet (see Figure 2(a)). (If, instead, initially the Bohmian particle was inside the left wave packet, then it will run with this wave packet all the way to the right, while the Bohmian particle of the pointer will move inside the shifted wave packet of the pointer outside its initial location; see Figure 2(b).)

In this example we can see the two principles at work. The position

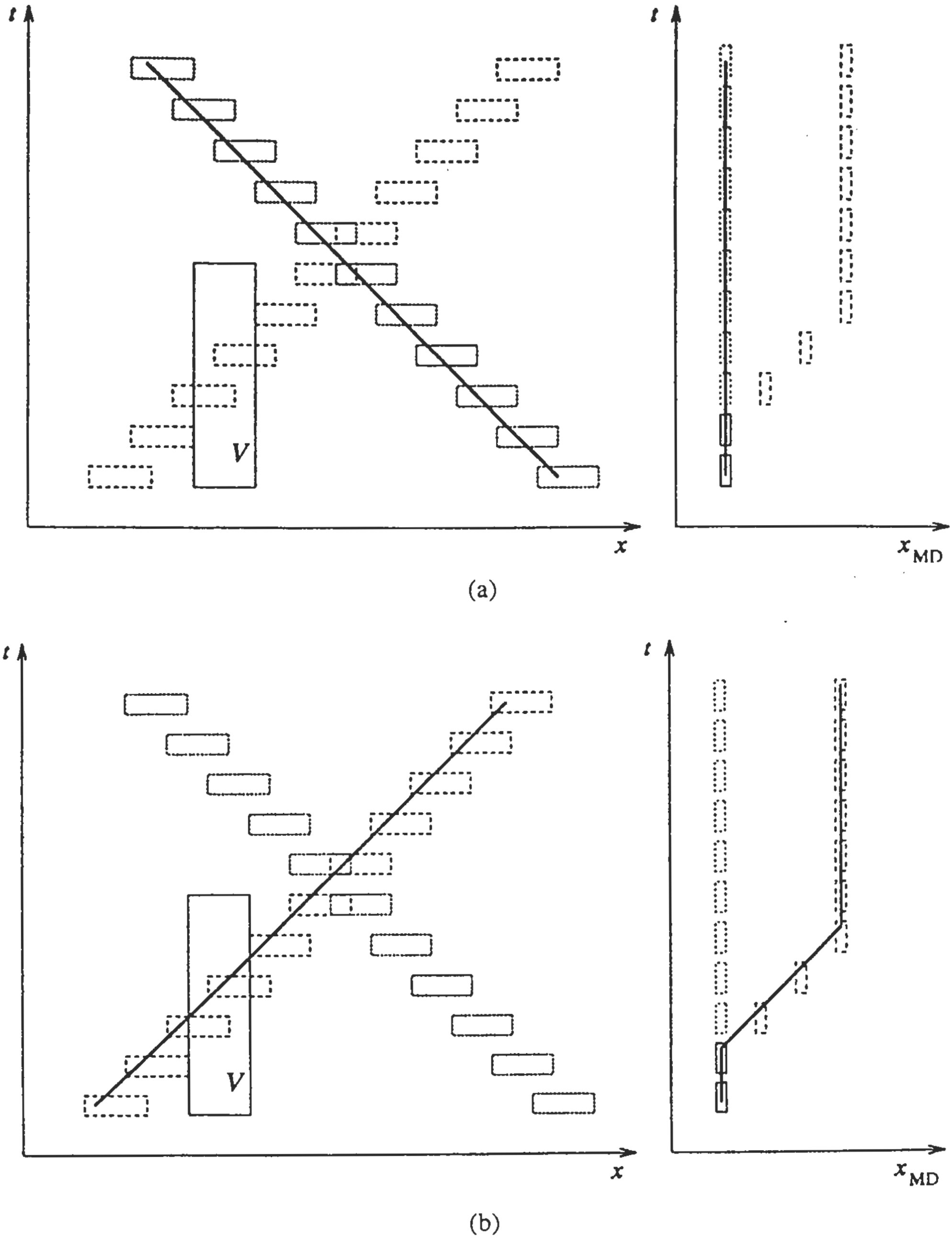

Fig. 2. Space-time diagram for the wave and the Bohm coordinate of the particle and the wave and the Bohm coordinate of the pointer of the measuring device. (a) The Bohmian particle starts on the right and does not enter the interaction region V. No shift in the Bohm coordinate of the pointer takes place. (b) The Bohmian particle starts on the left, passes through the interaction region V and the Bohm coordinate of the pointer moves, thus showing that the particle was on the left side.

measurement shows that the article is on the left if and only if the Bohmian particle was on the left. And, also, observable action on the measuring device occurs only when the Bohmian particle is at the location of the measuring device.

Let us turn now to an example in which there is some difficulty with the causal interpretation. We will consider again the same situation but, instead of a robust measurement, we will discuss "weak measurement". This is a standard measurement with a weakened coupling. The difficulty appears when we consider the pre- and post-selected ensemble. The particles are all pre-selected in the initial state which is the superposition of the two wave packets as described above. At the end we observe the location of the particle and consider only the cases in which the particle is found in the right side. Contrary to the example above, no *a priori* assumption is made about the initial Bohm coordinates. Since the final measurement is considered to be robust, we assume that the final Bohm coordinate is in the right side too.

For simplicity, instead of taking the usual "weak measurement", we will consider now a simple model which, in this case, exhibits similar features. We will assume that the wave function of the pointer has a rectangular form and that its spread during the process can be neglected. The coupling constant $g(t)$ is taken to be very small such that the shift caused by the particle passing through the volume V is equal to 10% of the width of the wave packet (see Figure 3). In this situation one measurement yields usually (in 90% of the cases) no information, but the outcomes of a number of results obtained on a large ensemble of identical pre- and post-selected systems will yield a clear answer: the particle is on the left. Indeed, the statistical outcomes are identical to those obtained from measurements on a pre-selected ensemble with a particle placed on the left side. In order to fulfill the first principle we need that in such a situation we can claim with high probability that the Bohmian particle was also on the left. This is, however, not so.

Assuming uniform distribution of the Bohmian particles we can easily see that in 90% of the cases, in spite of the results of the measurements, the Bohmian particle was not there. Indeed, if the Bohmian particle of the pointer was anywhere in the last 90% of its wave packet (i.e., not close to the beginning edge) and the Bohm coordinate of the particle was on the right, then it will behave in the same way as in the first example: the Bohmian particle will move together with the right wave packet, stops when it enters the region of the overlap of the two wave packets until it is taken to the right by the left wave packet (see Figure 3(a)). For Bohmian particle starting on the left side the situation is similar. In 90% cases, when the pointer Bohmian particle is in the beginning part of the wave function, the Bohmian particle makes the turn and, only when the Bohm's pointer coordinate is in the last 10% percent of the wave function, does it go straight to the right (see Figure 3(b)). Thus, the 90% of the Bohmian particles which ended at the right side, started from the right, and only 10% started from the left. The apparent difficulty is that the outcomes of the measurements, if interpreted in a natural way, correspond to all particles being on the left side, while only 10% of the Bohmian particles were there.

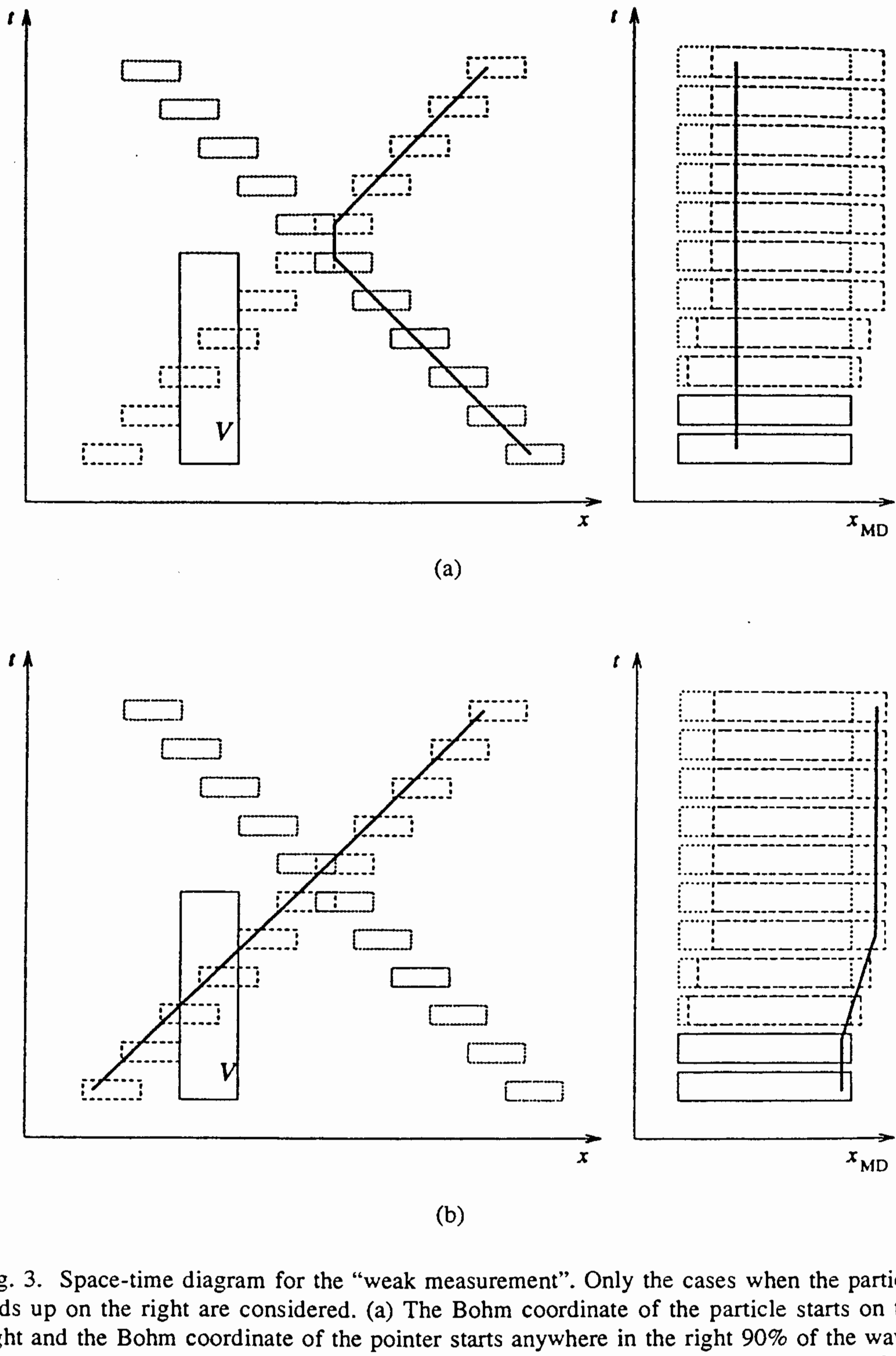

Fig. 3. Space-time diagram for the "weak measurement". Only the cases when the particle ends up on the right are considered. (a) The Bohm coordinate of the particle starts on the right and the Bohm coordinate of the pointer starts anywhere in the right 90% of the wave. (b) The Bohm coordinate of the particle starts on the left and the Bohm coordinate of the pointer starts anywhere in the right 10% of the wave. The final distribution of the Bohm coordinates of the measuring device cooresponds (according to the usual, pre-select only, situation) to the particles which were initially on the left. Indeed, the Bohm coordinate of the pointer cannot be found at the end in the first (left) 10% of the initial wave. Nevertheless, 90% of the Bohmian particles started on the right.

It is important to emphasize that we have difficulty only with the first principle. The change in the measuring device in all cases can be seen as a direct influence of the nonempty waves. Although we had just 10% of particles on the left side, for all of them the Bohmian particle of the pointer was at the far end, such that after the shift due to the interaction, they fill the missing part of the spatial distribution of the pointer relative to the distribution corresponding to no particle on the left.

The difficulty which we have in this example can be seen only on an ensemble of pre- and post-selected systems. We may try to consider the whole ensemble of N particles as a single system, but then the difficulty will not appear. Indeed, in this case we have to consider a single measuring device. If we take the same measuring device which is coupled, one after the other, to all particles, then the picture becomes different. Straightforward analysis shows that for $N \gg 10$ most of Bohmian particles start from the left and pass the volume V.

In order to explain our next example, which is a weak measurement performed on a single (pre- and post-selected) system, we will start with a brief review of the two-state vector formalism. In 1964 Aharonov, Bergmann and Lebowitz considered measurements performed on a quantum system between two other measurements, results of which were given. They proposed describing the quantum system between two measurements by using two states: the usual one, evolving towards the future from the time of the first measurement, and a second state evolving backwards in time, from the time of the second measurement. If a system has been prepared at time t_1 in a state $|\Psi_1\rangle$ and is found at time t_2 in a state $|\Psi_2\rangle$, then at time t, $t_1 < t < t_2$, the system is described by

$$(2) \qquad \langle\Psi_2| e^{i\int_{t_2}^{t} H dt} \qquad \text{and} \qquad e^{-i\int_{t_1}^{t} H dt} |\Psi_2\rangle.$$

For simplicity, we shall consider the free Hamiltonian to be zero; then, the system at time t is described by the two states $\langle\Psi_2|$ and $|\Psi_1\rangle$. In order to obtain such a system, we prepare an ensemble of systems in the state $|\Psi_1\rangle$, perform a measurement of the desired variable using separate measuring devices for each system in the ensemble, and perform the post-selection measurement. If the outcome of the post-selection was not the desired result, we discard the system and the corresponding measuring device. We look only at measuring devices corresponding to the systems post-selected in the state $\langle\Psi_2|$.

The basic concept of the two-state approach, the weak value of a physical variable A in the time interval between pre-selection of the state $|\Psi_1\rangle$ and post-selection of the state $|\Psi_2\rangle$, is given by (Aharonov and Vaidman 1990)

$$(3) \qquad A_w \equiv \frac{\langle\Psi_2|A|\Psi_1\rangle}{\langle\Psi_2|\Psi_1\rangle}.$$

Let us show briefly how weak values emerge from a measuring procedure with a sufficiently weak coupling. We consider a sequence of measurements: a pre-selection of $|\Psi_1\rangle$, a (weak) measurement interaction of the form of Eq. (1), and a post-selection measurement finding the state $|\Psi_2\rangle$. The state of the measuring device (which was initially in a Gaussian state) after this sequence is given (up to normalization) by

$$(4) \qquad \Phi(Q) = \langle\Psi_2|e^{-iPA}|\Psi_1\rangle e^{-Q^2/2\Delta^2}.$$

In the P-representation we can rewrite it as

$$(5) \qquad \tilde{\Phi}(P) = \langle\Psi_2|\Psi_1\rangle e^{-iA_w P}\, e^{-\Delta^2 P^2/2} + \langle\Psi_2|\Psi_1\rangle \sum_{n=2}^{\infty} \frac{(iP)^n}{n!} [(A^n)_w - (A_w)^n] e^{-\Delta^2 P^2/2}.$$

If Δ is sufficiently large, we can neglect the second term of (5) when we Fourier transform back to the Q-representation. Large Δ corresponds to weak measurement in the sense that the interaction Hamiltonian (1) is small. Thus, in the limit of weak measurement, the final state of the measuring device (in the Q-representation) is

$$(6) \qquad \Phi(Q) = e^{-(Q - A_w)^2/2\Delta^2}.$$

This state represents a measuring device pointing to the weak value, A_w.

Although we have showed this result for a specific von Neumann model of measurements, the result is completely general: any coupling of a pre- and post-selected system to a variable A, provided the coupling is sufficiently weak, results in effective coupling to A_w. Since Δ has to be large, the weak coupling between a single system and the measuring device will not, in most cases, lead to a distinguishable shift of the pointer variable, but collecting the results of measurements on an ensemble of pre- and post-selected systems will yield the weak values of a measured variable to any desired precision.

As an example, consider a spin-1/2 particle prepared with the spin state pointing in the x direction and found later with the spin in the y direction. We consider weak measurement of the spin component in the ξ direction which is the bisector of $\hat{x}$ and $\hat{y}$, i.e., $\sigma_\xi = (\sigma_x + \sigma_y)/\sqrt{2}$. Thus, $|\Psi_1\rangle = |\uparrow_x\rangle$, $|\Psi_2\rangle = |\uparrow_y\rangle$, and the weak value of σ_ξ in this case is:

$$(7) \qquad (\sigma_\xi)_w = \frac{\langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \frac{1}{\sqrt{2}} \frac{\langle\uparrow_y|(\sigma_x + \sigma_y)|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \sqrt{2}\,.$$

This value is, of course, "forbidden" in the standard interpretation where a spin component can obtain the (eigen)values ±1 only. If the angle between the spin directions of the initial and final states is close to 180°, then the weak measurement in the direction of the bisector can be even much larger (Aharonov *et al.* 1988).

Discussing spin-measurements in the framework of the Bohm theory requires additional care. The most elegant way to analyze spin in causal interpretation is not to attribute to it some kind of a Bohm pointer (e.g., Dewdney *et al.* 1988), but to consider the spin as a property of the quantum wave. Then the probability one-half in the result of a spin z measurement of a spin prepared in, say, the $\sigma_x = 1$ state corresponds to uniform distribution of the Bohmian particles inside the wave packet. If the Bohmian particle is in the upper half of the wave, the Stern–Gerlach measurement will end up with the particle going up; otherwise it will go down. This is a manifestation of *contextuality* of quantum measurement: the fact that the particle goes up is interpreted as spin "up" or spin "down" depending on the direction of the gradient of the magnetic field in the Stern–Gerlach device (see, for example, Albert 1992, 153–155).

Bohm's picture nicely explains the peculiar result of a weak measurement of the spin component. Due to the coupling of the weak measurement, the Bohm coordinates of the pointer variable (which is the coordinate of the particle) are not shifted by the "forbidden" value $(\sigma_\xi)_w$. The interaction causes correlations such that the post-selection measurement will pick the Bohmian particle from the appropriate region. In the case of a large value of $(\sigma_\xi)_w$, the post-selected Bohmian particles are coming from the "tail" of the wave function. We will not present here the derivation of this result, but we will give a hint how it works. Assume that the initial state of the particle is a Gaussian with the spread Δ. Then, to calculate the motion of the Bohmian particles in the post-selection measurement we have to consider a mixture of two, almost identical, slightly shifted Gaussians. The particles in a certain region go towards a certain (post-selection) direction if the "weight" of one packet is significantly larger than that of the other. But this, for very small shift, happens far away in the tail: $e^{-(x-\delta)^2/\Delta^2}/e^{-x^2/\Delta^2} \gg \Delta^2/\delta$.

The example we want to consider here is less interesting for practical applications, but more striking conceptually. It is a weak measurement performed on a single system with a particular (very rare) post-selection which does lead to a distinguishable shift of the pointer. Such was the example considered in the first work on weak measurements (Aharonov *et al.* 1987). In this work a single system of a large spin N is considered. The system is pre-selected in the state $|\Psi_1\rangle = |S_x = N\rangle$ and post-selected in the state $|\Psi_2\rangle = |S_y = N\rangle$. At an intermediate time the spin component S_ξ is weakly measured and again the "forbidden" value $\sqrt{2}N$ is obtained. The uncertainty has to be only slightly larger than $\sqrt{N}$. The probability distribution of the results is centered around $\sqrt{2}N$, and for large N it lies clearly outside the range of the eigenvalues, $(-N, N)$. More generally, we can consider pre-selection and post-selection in the directions such that the angle with the bisector $\hat{\xi}$ is θ. Then, the weak value of S_ξ is $(S_\xi)_w = N/\cos\theta$ and it can be much larger than N.

Our purpose here is to consider a position measurement performed on a single system. This is, however, different from the "position" measurements we discussed above. In previous examples we measured a projection operator on a certain volume. Now we consider a measurement of the operator of position. Our example is similar to the spin-N particle experiment discussed above. We consider a massive particle which was prepared in a state $|\Psi_1\rangle$ and was found at time t_2 in a state $|\Psi_2\rangle$. The states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are the superpositions of very well localized wave packets around locations between −1 and 1. The states, up to normalization, are

$$(8) \qquad |\Psi_1\rangle = \sum_{i=0}^{N} \left| x = \frac{N-2i}{N} \right\rangle, \qquad |\Psi_2\rangle = \sum_{i=0}^{N} c_i \left| x = \frac{N-2i}{N} \right\rangle,$$

where $c_i = (-\tan^2 \theta/2)^i/i!(N-i)!$. In this case the weak measurement (the standard measurement of position with precision of the order $\sqrt{N}$) yields a number around the weak value of x which is $x_w = 1/\cos\theta$. Mathematically, this is identical to the superposition of small forces which is equivalent to a large force, as discussed by Aharonov, Anandan, Popescu and Vaidman (1990). The details can be found there.

If we choose θ and N such that $\sqrt{N} \gg 1/\cos\theta$, we may view this experiment as a good measurement of position: we can repeat the procedure several times (including the pre-selection, the position measurement and the post-selection) and we will obtain values around x_w with relatively small statistical spread. Nevertheless, the outcome of this kind of position measurement has no relation to the Bohmian particle position; the latter can have only values between −1 and 1. It is not that there is any real contradiction, or that the causal interpretation cannot explain the outcome of the measurement. If we consider the Bohmian particle position of the pointer variable, it will be around the value x_w. Again, it was not shifted by the coupling of weak measurement, but "picked" from the tail in the post-selection process, and this is due to the peculiar wave function correlation created by the whole process. The unfortunate feature is that again, the first principle is not filled in this specific case: the procedure which is very close to the usual position measurement yields consistently results which have no relation to the Bohmian particle position.

Another example where the Bohmian particle position does not help to understand the result of measurement is the *protective measurement* (Aharonov and Vaidman 1993) of the Schrödinger wave function. If the only "reality" is the Bohmian particle position and the Schrödinger wave is just a pilot wave which governs the motion of the particle, then the wave should not be observable directly. Let us briefly show how, in certain cases, we can observe the complete Schrödinger wave of a single particle, in spite of the fact that the Bohmian particle position represents almost no features of the wave.

As an example of a simple protective measurement, let us consider a particle in a discrete nondegenerate energy eigenstate $\Psi(x)$. The standard von Neumann procedure for measuring the value of the projection operator, Π_V, on the volume V involves an interaction Hamiltonian (1). In the protective measurements $g(t) = 1/T$ for most of the time T and it goes to zero gradually before and after the period T. For $g(t)$ smooth enough we obtain an adiabatic process in which the particle cannot make a transition from one energy eigenstate to another, and, in the limit $T \rightarrow \infty$, the interaction Hamiltonian does not change the energy eigenstate. For any given value of P, the energy of the eigenstate shifts by an infinitesimal amount given by first order perturbation theory: $\delta E = \langle H_{int} \rangle = \langle \Pi_V \rangle P/T$. The corresponding time evolution $e^{-iP\langle \Pi_V \rangle}$ shifts the pointer by the average value $\langle \Pi_V \rangle$. By measuring the averages of a sufficiently large number of projection operators of different regions, the density of the full Schrödinger wave can be reconstructed to any desired precision.

Again, we would like naively to think that the projection operator Π_V should yield 1, if the Bohmian particle is inside V and zero if it is outside V. The measurement, however yields $|\Psi(x)|^2$ irrespectively of where the Bohmian particle is. If the system is in an energy eigenstate, then the Bohmian particle does not move, so, in fact, for measurements of almost all projection operators the Bohmian particle is outside the volume during the whole period of the measurement, and for one projection operator it is always inside. Thus, the second principle is broken here: the measuring devices measuring $|\Psi(x)|^2$ in the "empty" volumes yield non-null outcomes.

More precisely, the measuring interaction changes the state of the system and the Bohmian particle starts moving. However, in the adiabatic limit its velocity is very small. Certainly, the bohmian particle does not visit every volume many times during the measurement in such a way that the average time inside the volume V is proportional to $|\Psi(x)|^2$ – a possible naive explanation based on Bohm's ontology.

The breakdown of the second principle (as well as the first one) can be seen much more clearly in an experiment suggested recently by Englert *et al.* (1992). In order to explain their idea, we consider again our first example in which the initial state of the particle is the superposition of two identical wave packets moving in opposite directions; the Bohmian particle initially is inside the right wave packet and we measure on the left side the existence of the particle. Our measurement is robust in the sense that the measuring device evolves into an orthogonal state if the particle is in the left side, but the density of the wave function of the measuring device is not changed significantly during the time of motion of the particle. Englert *et al.* considered a spin flip which is a very clean example, but, an example which fits better the spirit of the present work is a standard von Neumann interaction which makes a shift of the wave function of the pointer in the momentum representation. Then the Hamiltonian is $H = g(t)Q\Pi_V$, instead of (1). In fact, the realization of the von Neumann measurement in the laboratory is of this kind: in the Stern–Gerlach experiment the pointer is the particle itself and it gets a shift in momentum due to the interaction. Only later, the shift in momentum is translated into the shift in position. We consider the case that the shift in momentum will lead to the shift in position comparable with the width of the spatial wave function of the pointer only much later, such that at the time of the overlap of the wave packets of the particle, the two wave packets of the pointer, the one shifted in momentum which correlated to the left wave packet of the particle and the other, undisturbed, which correlated to the right wave packet, will essentially coincide. In this case, the behavior of the Bohmian particle will be as in the case that no measurement was performed at all (see Figure 4). The Bohmian particle will stop in the area of the overlap and will finally change the direction of its motion. The particle started on the right will end up on the right side and will never reach the left side. The Bohmian particle of the pointer, however, will follow the wave packet which got the shift in the momentum: it eventually will move far away from its original place. This corresponds to what we usually consider the result of the measurement saying that the particle is on the left side. Therefore, it goes against the first principle. But it also goes against the second principle.

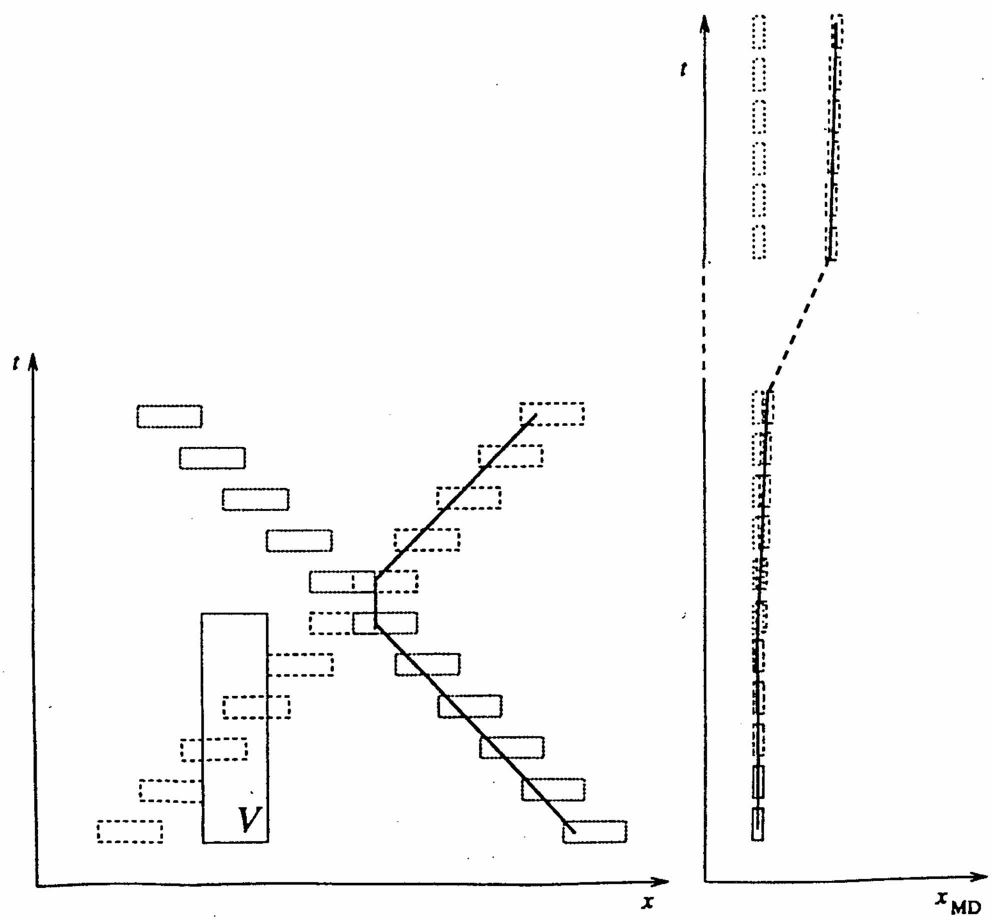

Fig. 4. Space-time diagram for the "delayed measurement". The Bohmian particle does not pass through the interaction region V, but, nevertheless, the Bohm coordinate of the measuring device moves. Note that it starts moving not immediately after the interaction in the region V, but only after the overlap of the two wave packets of the particle and that the Bohm coordinate of the pointer moves out of the location of its initial wave packet only at much later time.

The Bohmian particle was never on the left side. There was only an "empty" wave there. Nevertheless, it causes the action of the measuring device located on the left side; the Bohmian particle of the pointer moves because of the existence of the particle.

It is instructive to see how, exactly, the empty wave causes the change in the measuring device. At the time of the interaction between the particle and the measuring device, when the empty wave passes the region V, no change in the motion of the Bohmian particle of the pointer takes place. Only after the overlap of the empty and nonempty waves of the particle, the Bohmian particle of the pointer starts moving. Note, that if we "look" on the problem in configuration space, then no surprising behavior is observed. The empty wave in configuration space does not influence the motion of the Bohmian particle in configuration space until the particle is "inside" the wave and the wave ceases to be empty. The difficulty is seen only in the ordinary three dimensional space. It is even more dramatic when our particle moves inside a special bubble chamber. The bubbles created due to the passage of the particle are developed slowly enough such that during the time of the motion of the particle the density of the spatial wave function of each bubble does not change significantly. Then, what we will see as a trace of bubbles is the particle moving from the left to the right, while the Bohmian particle, in fact, will move from the right side, stop, and come back to the right. Note, however, that the picture we see via appearance of the bubbles is time delayed. In a "fast" bubble chamber, in which we see a real time motion, the Bohmian particle moves together with the trace of bubbles.

The examples considered in this work do not show that the Bohm's causal interpretation is inconsistent. It shows that Bohmian trajectories behave not as we would expect from a classical type model. Before seeing the examples discussed in this work we could think otherwise. In fact we did, and we were very reluctant to accept that the Bohmian picture is different, in some cases, from a naive picture based on the outcomes of the experiments; we worked hard, but in vain, searching for an error in our and the Englert *et al.* arguments. If we follow David Bohm, viewing his theory as an alternative formalism which should lead us to fruitful generalizations and modifications, the difficulties we have discussed can be considered on the positive side as showing a direction for constructing a better theory. However, in order to put our work in a proper perspective, we have to note that the proponents of the Bohm theory do not see the phenomena we described here as difficulties of the theory; see, for example, Dürr, Fusseder, Goldstein and Zanghi (1993). The fact that we see these difficulties follows form our particular approach to the Bohm theory in which the wave is not considered to be a "reality". Thus, our difficulties are not surprising in the light of the words of Bell (1987a, 128): "*No one can understand this theory until he is willing to think of* ψ *as a real objective field rather than just a 'probability amplitude'*." Although we are certainly sympathetic to the approach in which ψ is real (see Aharonov and Vaidman 1993), one of us (L.V.) does not see it fit well into the framework of the Bohm theory, since in his view, the main purpose of the Bohm theory is to avoid the reality of the many worlds that he believes is incorporated in the wave function of the Universe (Vaidman 1993).

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