

## PROTECTIVE MEASUREMENTS OF TWO-STATE VECTORS

## INTRODUCTORY ABSTRACT

A recent result about measurability of a quantum state of a single quantum system is generalized to the case of a single pre- and post-selected quantum system, described by a two-state vector. The protection required for such measurement is achieved by coupling the quantum system to a pre- and post-selected protected device yielding a nonhermitian effective Hamiltonian.

We present here a point of contact between two approaches which have been main directions of our research in the recent years. Our numerous discussions of these subjects with Abner Shimony, whom we thank for his crystal-clear thinking, made it possible for us to see these issues as they are presented here.

Recently it has been shown that *protective measurements*<sup>1,2</sup> can be used for “observing” the quantum state of a single system. Also, in recent years an approach has been developed in which a quantum system is described, at a given time, by two (instead of one) quantum states: the usual one evolving toward the future and the second evolving backwards in time from a future measurement.<sup>3-7</sup> In this approach, the vector describing a quantum system at a given time consists of two states. The protective measurements<sup>1,2</sup> are not suitable for observing two-state vectors. Here we will present a method for measuring two-state vectors of a single (pre- and post-selected) system. We shall precede the explanation of our method by brief reviews of the method of protective measurements of a single quantum state and of the two-state vector formalism.

The basic protection procedure introduces a protective potential such that the quantum state of the system will be a nondegenerate eigenstate of the Hamiltonian. Let us consider a particle in a discrete nondegenerate energy eigenstate  $\Psi(x)$ . The standard von Neumann procedure for measuring the value of an observable  $A$  involves an interaction Hamiltonian:

$$H = g(t)PA \quad (1)$$

where  $P$  is the conjugate momentum of pointer variable  $Q$ , and the coupling parameter  $g(t)$  is normalized to  $\int g(t)dt = 1$ . The initial state of the pointer is taken to be a Gaussian centered around zero. In standard impulsive measurements,  $g(t) \neq 0$  for only a very short time interval. Thus, the interaction term dominates the rest of the Hamiltonian, and the time evolution  $\exp(-iPA)$  leads to a correlated state: eigenstates of  $A$  with eigenvalues  $a_n$  are correlated to measuring device state in which the pointer is shifted by these values  $a_n$ . (Here and below we use units such that  $\hbar = 1$ .) By contrast, the protective measurements of interest here utilize the opposite limit of extremely slow measurement. We take  $g(t) = 1/T$  for most of the time  $T$  and assume that  $g(t)$  goes to zero gradually before and after the period  $T$ . We choose the initial state of the measuring device such that the momentum  $P$  is

bounded. We also assume that  $P$  is a constant of motion not only of the interaction Hamiltonian (1), but of the whole Hamiltonian. For  $g(t)$  smooth enough we obtain an adiabatic process in which the particle cannot make a transition from one energy eigenstate to another, and, in the limit  $T \rightarrow \infty$ , the interaction Hamiltonian does not change the energy eigenstate. For any given value of  $P$ , the energy of the eigenstate shifts by an infinitesimal amount given by the first order perturbation theory:  $\delta E = \langle H_{int} \rangle = \langle A \rangle P/T$ . The corresponding time evolution  $\exp(-iP\langle A \rangle)$  shifts the pointer by the average value  $\langle A \rangle$ . By measuring the averages of a sufficiently large number of variables  $A_n$ , the full Schrödinger wave  $\Psi(x)$  can be reconstructed to any desired precision.

Let us turn to the review of the two-state vector formalism originated by Aharonov, Bergmann and Lebowitz,<sup>3</sup> who considered measurements performed on a quantum system between two other measurements, results of which were given. The quantum system between two other measurements is described by two states: the usual one, evolving towards the future from the time of the first measurement, and a second state evolving backwards in time, from the time of the second measurement. If a system has been prepared at time  $t_1$  in a state  $|\Psi_1\rangle$ , and is found at time  $t_2$  in a state  $|\Psi_2\rangle$ , then at time  $t$ ,  $t_1 < t < t_2$ , the system is described by  $\langle \Psi_2 | \exp(i \int_{t_2}^t H dt) \exp(-i \int_{t_1}^t H dt) | \Psi_1 \rangle$ . For simplicity, we shall consider the free Hamiltonian to be zero; then, the system at time  $t$  is described by the two states  $\langle \Psi_2 |$  and  $|\Psi_1 \rangle$ . In order to obtain such a system, we prepare an ensemble of systems in the state  $|\Psi_1\rangle$ , perform a measurement of the desired variable using separate measuring devices for each system in the ensemble, and perform the post-selection measurement. If the outcome of the post-selection was not the desired result, we discard the system and the corresponding measuring device. We look only at measuring devices corresponding to the systems post-selected in the state  $\langle \Psi_2 |$ .

The basic concept of the two-state approach, the weak value of a physical variable  $A$  in the time interval between pre-selection of the state  $|\Psi_1\rangle$  and post-selection of the state  $|\Psi_2\rangle$  is given by:<sup>5</sup>

$$A_w \equiv \frac{\langle \Psi_2 | A | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle} \quad (2)$$

Weak values emerge from a measuring procedure with a sufficiently weak interaction. When the strength of the coupling to the measuring device goes to zero, the outcomes of the measurement invariably yield the weak value. To be more precise, such a measurement yields the real part of the weak value. Indeed, the weak value is, in general, a complex number, but its imaginary part will contribute only a phase to the wave function of the measuring device in the position representation of the pointer. Therefore, the imaginary part will not affect the probability distribution of the pointer position, which is what we see in a usual measurement. However, the imaginary part of the weak value also has physical meaning. It expresses itself as a change in the conjugate momentum of the pointer variable.

We are familiar with weak measurements performed on a single system. In fact, the first work on weak measurements<sup>4</sup> considered such a case. There, a single

measurement of the spin component of a spin- $N$  system yielded the “forbidden” value  $\sqrt{2N}$  with the uncertainty  $\sqrt{N}$ . This is the weak value of  $S_x$  for the two-state vector  $\langle S_y = N | S_x = N \rangle$ . Another such example is the measurement of the kinetic energy of a tunneling particle.<sup>8</sup> We have shown for any precision of the measurement that we can ensure a negative value reading of the measuring device by an appropriate choice of the post-selection state.

However, these examples do not represent a measurement of the two-state vector itself. If our measuring device for the spin measurement shows  $\sqrt{2N}$ , we cannot deduce that our two-state vector is  $\langle S_y = N | S_x = N \rangle$ . Indeed, there are many other two-state vectors that yield the same weak value for the spin component, but we cannot even claim that we have one of these vectors. The probability for the result of the post-selection measurement corresponding to any of these vectors is extremely small, so it is much more likely to obtain the “forbidden” outcome  $S_x = \sqrt{2N}$  as a statistical error of the measuring device. The same applies to the measurement of kinetic energy of a tunneling particle. The negative value shown by the measuring device usually is due to a statistical error, and only in very rare cases does it correspond to a particle “caught” in the tunneling process.

We could try to use several weak measurements on a single pre- and post-selected system in order to specify the two-state vector. But in that case, these measurements will change the two-state vector. Therefore, as in the case of the measurement of the forward evolving single-state vector of a single system, we need a protection procedure. At first sight, it seems that protection of a two-state vector is impossible. Indeed, if we add a potential that makes one state a nondegenerate eigenstate, then the other state, if it is different, cannot be an energy eigenstate as well. (The states of the two-state vector cannot be orthogonal.) But, nevertheless, protection of the two-state vector is impossible, as we now show.

The procedure for protection of a two-state vector of a given system is accomplished by coupling the system to another pre- and post-selected system. The protection procedure takes advantage of the fact that weak values might acquire complex values. Thus, the effective Hamiltonian of the protection might not be hermitian. Nonhermitian Hamiltonians act in different ways on quantum states evolving forward and backwards in time. This allows simultaneous protection of two different states (evolving in opposite time directions).

Let us start with an example.<sup>9</sup> We consider the protection of a two-state vector of a spin-1/2 particle,  $\langle \uparrow_y | | \uparrow_x \rangle$ . The protection procedure uses an external pre- and post-selected system  $S$  of a large spin  $N$  that is coupled to our spin via the interaction:

$$H_{prot} = -\lambda S \cdot \sigma \quad (3)$$

The external system is pre-selected in the state  $|S_x = N\rangle$  and post-selected in the state  $\langle S_y = N |$ , that is, it is described by the two-state vector  $\langle S_y = N | S_x = N \rangle$ . The coupling constant  $\lambda$  is chosen in such a way that the interaction with our spin-1/2 particle cannot change significantly the two-state vector of the protective system  $S$ ,

and the spin-1/2 particle “feels” the effective Hamiltonian in which  $S$  is replaced by its weak value:

$$S_w = \frac{\langle S_y = N | (S_x, S_y, S_z) | S_x = N \rangle}{\langle S_y = N | S_x = N \rangle} = (N, N, iN) \quad (4)$$

Thus, the effective protective Hamiltonian is:

$$H_{eff} = -\lambda N(\sigma_x + \sigma_y + i\sigma_z) \quad (5)$$

The state  $|\uparrow_x\rangle$  is an eigenstate of this (nonhermitian) Hamiltonian (with eigenvalue  $-\lambda N$ ). For backward evolving states the effective Hamiltonian is the hermitian conjugate of (5) and it has different (nondegenerate) eigenstate with this eigenvalue; the eigenstate is  $\langle\uparrow_y|$ . The forward evolving state  $|\uparrow_x\rangle$  and the backward evolving state  $\langle\uparrow_y|$  are also the eigenstates of the exact Hamiltonian (3) (when the large spin is pre- and post-selected as described above).

In order to prove that the Hamiltonian (3) indeed provides the protection, we have to show that the two-state vector  $\langle\uparrow_y|\uparrow_x\rangle$  will remain essentially unchanged during the measurement. We consider measurement which is performed during the period of time between pre- and post-selection which we choose to be equal to one. The Hamiltonian:

$$H = -\lambda S \cdot \sigma + P\sigma_\xi \quad (6)$$

can be replaced by the effective Hamiltonian:

$$H_{eff} = -\lambda N(\sigma_x + \sigma_y + i\sigma_z) + P\sigma_\xi \quad (7)$$

Indeed, the system with the spin  $S$  can be considered as  $N$  spin-1/2 particles all pre-selected in  $|\uparrow_x\rangle$  state and post-selected in  $|\uparrow_y\rangle$  state. The strength of the coupling to each spin-1/2 particle is  $\lambda \ll 1$ ; therefore, during the time of the measurement their states cannot be changed significantly. Thus, the forward evolving state  $|S_x = N\rangle$  and the backward evolving state  $\langle S_y = N|$  do not change significantly during the measuring process. The effective coupling to such system is the coupling to its weak values.

Good precision of the measurement of the spin component requires large uncertainty in  $P$ , but we can arrange the experiment in such a way that  $P \ll N$ . Then the second term in the Hamiltonian (6) will not change significantly the eigenvectors. The two-state vector  $\langle\uparrow_y|\uparrow_x\rangle$  will remain essentially unchanged during the measurement and, therefore, the measuring device on this single particle will yield  $(\sigma_\xi)_w = \langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle/\langle\uparrow_y|\uparrow_x\rangle$ . We can perform several measurements of different spin component on the same single system since the measurements do not disturb significantly the two-state vector. Thus, the results  $(\sigma_x)_w = 1$ ,  $(\sigma_y)_w = 1$ , and  $(\sigma_z)_w = i$  will uniquely define the two-state vector.

The Hamiltonian (3), with an external system described by the two-state vector  $\langle S_y = N | S_x = N \rangle$ , provides protection for the two-state vector  $\langle\uparrow_y|\uparrow_x\rangle$ . It is not difficult to demonstrate that any two-state vector obtained by pre- and post-selection of the spin-1/2 particle can be protected by the Hamiltonian (3). A general form of the two-state vector is  $\langle\uparrow_\beta|\uparrow_\alpha\rangle$  where  $\hat{\alpha}$  and  $\hat{\beta}$  denote some directions. It

can be verified by a straightforward calculation that the two-state vector  $\langle \uparrow_\beta | | \uparrow_\alpha \rangle$  is protected when the two-state vector of the protective device is  $\langle S_\beta = N | | S_\alpha = N \rangle$ .

At least formally, we can generalize this method to make a protective measurement of an arbitrary two-state vector  $\langle \Psi_2 | | \Psi_1 \rangle$  of an arbitrary system. Let us decompose the post-selected state  $|\Psi_2\rangle = a|\Psi_1\rangle + b|\Psi_\perp\rangle$ . Now we can define ‘‘model spin’’ states:  $|\Psi_1\rangle \equiv |\uparrow_z\rangle$  and  $|\Psi_\perp\rangle \equiv |\downarrow_z\rangle$ . On the basis of the two orthogonal states, we can obtain all other ‘‘model spin’’ states. For example,  $|\uparrow_x\rangle = 1/\sqrt{2}(|\uparrow_z\rangle + |\downarrow_z\rangle)$ , and then we can define the ‘‘spin model’’ operator  $\tilde{\sigma}$ . Now, the protection Hamiltonian in complete analogy with the spin-1/2 particle case is:

$$H_{prot} = -\lambda S \cdot \tilde{\sigma} \quad (8)$$

In order to protect the state  $\langle \Psi_2 | | \Psi_1 \rangle$ , the pre-selected state of the external system has to be  $\langle S_z = N \rangle$  and the post-selected state has to be  $\langle S_x = N \rangle$  where the direction  $\hat{\chi}$  is defined by the ‘‘spin model’’ representation of the state  $|\Psi_2\rangle$ :

$$|\tilde{\uparrow}_\chi\rangle \equiv |\Psi_2\rangle = \langle \Psi_1 | \Psi_2 \rangle |\uparrow_z\rangle + \langle \Psi_\perp | \Psi_2 \rangle |\downarrow_z\rangle \quad (9)$$

Let us come back to our first example. The Hamiltonian (5) has more interesting features than just protecting the two-state vector  $\langle \uparrow_y | | \uparrow_x \rangle$ . First, there is another two-state vector which is protected: the two state  $\langle \downarrow_x | | \downarrow_y \rangle$  with corresponding eigenvalue  $\lambda N$ . There is, however, a certain difference: while  $\langle \uparrow_y |$  and  $|\uparrow_x\rangle$  are exact eigenstates also of the Hamiltonian (3) (with the chosen pre- and post-selection of the spin  $S$ ), the states  $\langle \downarrow_x |$ ,  $|\downarrow_y\rangle$  are not. An easy calculation shows that the probability to find  $\sigma_y = 1$  at an intermediate time, given the initial state  $|\downarrow_y\rangle$ , does not vanish, but it is small: the probability is of order  $1/N^2$ . Straightforward (but lengthy) calculations show that the (not too strong) measurement coupling,  $P\sigma_\xi$ , adds to the probability of finding  $\sigma_y = 1$  corrections proportional to  $P^2/N^2$ ,  $P^2/\lambda^2 N^2$ , and  $P^4/\lambda^2 N^2$  which are also small for large  $N$ .

The calculations show that  $\lambda$  need not be small for the protection measurement. In fact, larger  $\lambda$  yields better protection. We required small  $\lambda$  to ensure that the coupling (3) will not cause significant change of the two-state of the large spin  $S$  system, irrespectively of the evolution of the spin-1/2 particle. But when the additional coupling  $P\sigma_\xi$  is small compared to the protection Hamiltonian (3), the spin-1/2 particle evolves in such a way that the two-state vector  $\langle S_y = N | | S_x = N \rangle$  remains essentially unchanged even when  $\lambda$  is large.

Another important point is that the bound on  $P$ , and thus the bound on the precision of the measurement, can be reduced by increasing the period of time  $T$  of the measurement with the appropriate reduction of the strength of the coupling term,  $P\sigma_\xi/T$ . For this regime we can give another proof that our intermediate measurements yield the weak values.<sup>10</sup>

In general, a nondegenerate nonhermitian Hamiltonian can be written in the following form:

$$H = \sum_i \omega_i \frac{|\Phi_i\rangle \langle \Psi_i|}{\langle \Psi_i | \Phi_i \rangle} \quad (10)$$

where  $\langle \Psi_i |$  are the ‘‘eigen-bras’’ of  $H$ , and  $|\Phi_i\rangle$  are the ‘‘eigen-kets’’ of  $H$ . The  $\langle \Psi_i |$  form a complete but, in general, non-orthogonal set, and so do the  $|\Phi_i\rangle$ . They obey the mutual orthogonality condition:  $\langle \Psi_i | \Phi_j \rangle = \delta_{ij} \langle \Psi_i | \Phi_i \rangle$ .

The Hamiltonian of our example gets the form:

$$H_{eff} = -\lambda N \frac{|\uparrow_x\rangle\langle\uparrow_y|}{\langle\uparrow_y|\uparrow_x\rangle} + \lambda N \frac{|\downarrow_y\rangle\langle\downarrow_x|}{\langle\downarrow_x|\downarrow_y\rangle} + \frac{P}{T} \sigma_\xi \quad (11)$$

Diagonalization of the Hamiltonian yields the modified energy eigenstates:

$$\omega_1 = -\lambda N + \frac{P \langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle}{T \langle\uparrow_y|\uparrow_x\rangle}, \quad \omega_2 = \lambda N + \frac{P \langle\downarrow_x|\sigma_\xi|\downarrow_y\rangle}{T \langle\downarrow_x|\downarrow_y\rangle} \quad (12)$$

This means that if the initial state of the system is  $|\uparrow_x\rangle$ , then the measuring device will record the weak value of  $\sigma_\xi$  for the two-state vector  $\langle\uparrow_y||\uparrow_x\rangle$ . This result is even stronger than what we wanted to show since we do not require the post-selection of the state  $\langle\uparrow_y|$ . The reason why other components of the backward evolving state do not contribute is because the corresponding component of the forward evolving state has zero amplitude. This feature will be clearer after the following discussion.

It is interesting to analyze the behavior of a system described by nonhermitian Hamiltonian (10) when the initial state is not one of the eigenstates. In this case, the initial state should be decomposed into a superposition of the eigenstates  $|\Psi\rangle = \sum_i \alpha_i |\Psi_i\rangle$  and its time evolution will be given by:

$$|\Psi(t)\rangle = \mathcal{N}(t) \sum_i \alpha_i \exp(-i\omega_i T) |\Psi_i\rangle \quad (13)$$

In order to keep the state normalized, we have to introduce the time dependent normalization factor  $\mathcal{N}(t)$ . This is the difference in the action of the effective Hamiltonian, and it signifies the fact that the probability for the appropriate result of the post-selection measurement (which leads to the nonhermitian effective Hamiltonian) depends on the time when it is performed.

If an adiabatic measurement of a variable  $A$  is performed, then the final state of the system and the measuring device is:

$$\mathcal{N}'(t) \sum_i \alpha_i \exp(-i\omega_i T) |\Psi_i\rangle \Phi(Q - \frac{\langle\Phi_i|A|\Psi_i\rangle}{\langle\Phi_i|\Psi_i\rangle}) \quad (14)$$

The state of the measuring device is amplified to a macroscopically distinguishable situation and, according to standard interpretation, a collapse takes place to the reading of one of the *weak values* of  $A$  with the relative probabilities given by  $|\alpha_i \exp(-i\omega_i T)|^2$ .

In general, construction of the formal protection Hamiltonian (8) which leads to the nonhermitian Hamiltonian is a *gedankenexperiment*. It generates nonlocal interactions which can contradict relativistic causality. However, effective nonhermitian Hamiltonian can be obtained in a real laboratory in a natural way when we consider a decaying system and we post-select the cases in which it did not decay during the period of time  $T$  which is larger than its characteristic decay

time. Kaon decay is such an example.  $|K_L^0\rangle$  and  $|K_S^0\rangle$  are the eigen-kets of the effective Hamiltonian and they have corresponding eigen-bras  $\langle K_L^0|$  and  $\langle K_S^0|$  evolving backward in time. Due to the *CP* - violation, the states  $|K_L^0\rangle$  and  $|K_S^0\rangle$  are not orthogonal. However, the mixing is small:  $|\langle K_S^0|K_L^0\rangle| \ll 1$  and, therefore, the corresponding backward evolving states are almost identical to the forward evolving states:  $|\langle K_S^0|K_S^0\rangle| = |\langle K_L^0|K_L^0\rangle| = 1/\sqrt{(1 - |\langle K_S^0|K_L^0\rangle|^2)}$ . Thus, it is difficult to expect a large effect in this system and for a realistic experimental proposal one should look, probably, for another system.

We have shown in the framework of nonrelativistic quantum theory that we can measure (or, maybe a better word, “observe”) two-state vectors describing pre- and post-selected quantum systems. A number of such measurements define the two-state vector and we have a procedure to protect the two-state vector from significant change due to these measurements. In order to protect, we have to know the two-state vector. Thus, this procedure is liable to a criticism<sup>11-13</sup> leveled at our first proposal. (Our response to this can be found in Ref. 14.) Although we consider our present proposal to be a measurement performed on a single system, it should also be mentioned that in any realistic practical implementation we will need ensembles of particles, protective systems, and measuring devices. The external system of the protective device has to be not only prepared (pre-selected) in a certain state, but also post-selected in a given state. In all interesting cases the probability for an appropriate outcome of the post-selection measurement is extremely small. Still, there is a non-zero probability that our first run with a single system, a single protective device, and a single set of measuring devices will yield the desired outcomes. In this case, we have a reliable measurement performed on a single system. However, even when we use a pre-selected ensemble, we actually use only a single pre- and post-selected system. After achieving the first successful post-selection, we have completed the experiment. (For more discussion of this point, see Ref. 15.)

It is interesting to notice that our procedure cannot protect a *generalized two-state vector*<sup>7</sup> which is a superposition of two-state vectors. The system described by a generalized two-state vector is correlated to some external system. It seems that it is impossible to find any protective procedure of the generalized two-state vector that does not involve coupling to that external system. This feature hints that the generalized two-state vector, although useful as a tool, is not a basic concept. The composite system consisting of the system under study and the system correlated to it is described by the usual, basic two-state vector.

We have shown that the two-state vector is observable. Previously we have shown that the single-state vector is observable. For weak coupling interactions to an observable  $A$ , in the first approach, we obtain the effective coupling to the weak value  $\langle \Psi_2|A|\Psi_1\rangle/\langle \Psi_2|\Psi_1\rangle$ , while in the second, to the expectation value  $\langle \Psi_1|A|\Psi_1\rangle$ . Since the two values are, in general, different, we encounter an apparent paradox. The resolution of the paradox is as follows.

In order to observe a quantum state, it has to be protected. When we discussed the protective experiments of single-state vectors we did not say anything about quantum states evolving backwards in time (it was not related to the point we

wanted to make). However, the protective procedure that we proposed automatically protects the *identical* state evolving backward. Thus, what we have proposed as an observation of a single-state vector is in fact an observation of a two-state vector with identical forward and backward evolving states. For example, the protection of spin-1/2 particle state,<sup>2</sup> a strong magnetic field in a given direction, protects the two-state vector with either both states parallel or anti-parallel to this direction. This procedure is incompatible with the protection of the forward evolving state parallel to one direction and the backward evolving state parallel to another. If the particle is described by  $(\uparrow_y || \uparrow_x)$ , then the strong magnetic field in the  $\hat{x}$  direction will change the backward evolving spin-state. There exists a protection procedure for  $|\uparrow_x\rangle$  that does not change the backward evolving state as was described in the preceding. The “observation” of the state protected in such a way will not yield the pre-selected quantum state, but it will yield the picture defined by the two-state vector.

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