On the Locality and Reality of Einstein-Podolsky-Rosen Correlations

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Abstract. The nonlocal aspects of Einstein-Podolsky-Rosen correlations are commonly regarded as a strong indication that quantum mechanics is a nonlocal theory. We disagree with this conclusion and argue in favor of the view that these correlations can be understood in terms of local physics. In particular, all features of quantum erasure — a procedure that exploits the nonlocal quantum correlations extensively — result from local actions. The classical notion of locality remains valid, but the concept of reality needs to be refined. Only those properties of quantum objects that can be verified in control experiments should be regarded as real. Further, everything can be viewed from the perspective of the time-symmetric two-state formalism, which is Lorentz covariant by construction and has no room for nonlocal elements.

INTRODUCTION

"Quantum theory is nonlocal." So reads the opening line of a recent report [1] on a new demonstration that Bell's inequality [2] is violated by entangled photon pairs. We tend to disagree and wish to offer a number of remarks in support of our view that local physics suffices to understand the correlations in question.

It must be noted at the outset that the term "nonlocality" is employed with somewhat different meaning in a variety of contexts. For example, the interference pattern observed on the screen in Young's double-slit experiment originates in the superposition of two amplitudes referring to the alternatives "through this slit" and "through that slit." It is a tenable notion to regard this interference as a nonlocal consequence of the superposition principle. This is, however, not the subject matter of the present paper, nor are we addressing the nonlocal aspects of the AB effect [3].

Rather, we are here solely concerned with what that opening line refers to, viz. the nonlocal correlations that are inconsistent with Bell's inequality and the locality assumption underlying it. We'd like to have the reader judge for himself and begin

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therefore with an exposition of some of the standard nonlocality arguments (Sec. I). This is followed by counter arguments in Sec. II. We close with a summary.

Let us note that these issues are as subtle as they are fascinating and, undoubtedly, the last word is yet to be uttered.

I ARGUMENTS IN SUPPORT OF A NONLOCAL INTERPRETATION OF EPR CORRELATIONS

In this section we shall review the most familiar arguments that are routinely used in the nonlocality claim. First we recall how the incompleteness argument of Einstein, Podolsky, and Rosen is turned around for this purpose. Then we remark on the Bell-Kochen-Specker theorem and illustrate it with the example of a Greenberger-Horne-Zeilinger state. After a brief look at joint probabilities for a spin-$\frac{1}{2}$ system, we finally note that supporters of Bohmian mechanics are bound to assert that quantum systems participate in nonlocal interactions at-a-distance.

A Einstein-Podolsky-Rosen correlations

In Bohm's version [4] of the Einstein-Podolsky-Rosen (EPR) argument [5] one considers two spin-$\frac{1}{2}$ atoms and emphasizes the entanglement of their spin degrees of freedom in the singlet state[1]

$$|\Psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)\,, \tag{1}$$

where $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are common eigenkets of the z-components of the two spin vector operators, the eigenvalues being $\sigma_z^{(1)} = +1$, $\sigma_z^{(2)} = -1$ and $\sigma_z^{(1)} = -1$, $\sigma_z^{(2)} = +1$, respectively. As a consequence, $|\Psi_{\mathrm{EPR}}\rangle$ is an eigenket of the two-particle observable $\sigma_z^{(1)}\sigma_z^{(2)}$ with eigenvalue -1, although it is neither an eigenket of $\sigma_z^{(1)}$ nor of $\sigma_z^{(2)}$. And the same statements hold for the x and the y components, as summarized in the following set of equations:[2]

$$\sigma_x^{(1)}\sigma_x^{(2)}|\Psi_{\mathrm{EPR}}\rangle = -|\Psi_{\mathrm{EPR}}\rangle\,, \tag{2a}$$
$$\sigma_y^{(1)}\sigma_y^{(2)}|\Psi_{\mathrm{EPR}}\rangle = -|\Psi_{\mathrm{EPR}}\rangle\,, \tag{2b}$$
$$\sigma_z^{(1)}\sigma_z^{(2)}|\Psi_{\mathrm{EPR}}\rangle = -|\Psi_{\mathrm{EPR}}\rangle\,; \tag{2c}$$

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1) "Atom" and "spin" need not be understood in their limited literal sense. One could just as well consider two photons and their polarization states, or any other physical system with two internal states of relevance.

2) In view of $\sigma_x^{(1)}\sigma_x^{(2)} = \frac{1}{2}(\sigma_x^{(1)} + \sigma_x^{(2)})^2 - 1$ etc. one could use sums rather than products and thus emphasize the zero-total-angular-momentum aspect. But we opt for products to establish the close analogy with the GHZ state of section I B.

since identities such as $(\sigma_x^{(1)}\sigma_x^{(2)})\,(\sigma_y^{(1)}\sigma_y^{(2)}) = -\sigma_z^{(1)}\sigma_z^{(2)}$ hold, two of them imply the third. More generally, $|\Psi_{\mathrm{EPR}}\rangle$ of (1) is an eigenket of $\vec{e}\cdot\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}\cdot\vec{e}$ with eigenvalue -1 for any unit vector $\vec{e}$.

The correlations between the results of measurements performed independently on the two atoms are of central interest in the EPR argument. Because any measurement occupies a certain spatial volume and has a finite duration, the independence is ensured by separating the two atoms by a distance so large that there is no causal relation between the two relevant space-time regions. These matters are illustrated in Fig. 1.

Accordingly, the operational meaning of Eqs. (2) is as follows. If one measures the two z components, the individual results are unpredictable — in many repetitions, the outcomes $\sigma_z^{(1)} = \pm 1$ are equally frequent and so are the outcomes $\sigma_z^{(2)} = \pm 1$ — but their product is always -1. Knowing $\sigma_z^{(1)} = +1$, say, is perfectly equivalent to knowing $\sigma_z^{(2)} = -1$. Either one of the σ_z measurements can be regarded as a control measurement for the other.

So far, it is much like the purely classical situation in which single gloves are sealed in boxes to be opened in spatially separated regions. Finding the left-hand glove of a pair in a box, implies that the partner box contains the right-hand one, and vice versa.

But there is, of course, a real and very significant difference between the two-atom and the paired-gloves experiment. We could also determine the x components of the atoms and confirm that they exhibit the same features as the z measurements: Individual results are random, but the product is always the same. And there is no paired-gloves analog to σ_x measurements.

Now, suppose a measurement on the 1$^{\mathrm{st}}$ atom has determined the value of $\sigma_z^{(1)}$. Then there is nothing to be learned from measuring also the z component of the 2$^{\mathrm{nd}}$ atom's spin vector. A measurement of $\sigma_x^{(2)}$ is much more interesting, because it reveals the value of the 1$^{\mathrm{st}}$ atom's x component as well. Thus, having determined the actual values of $\sigma_z^{(1)}$ and $\sigma_x^{(2)}$, we can infer the values of $\sigma_x^{(1)}$ and $\sigma_z^{(2)}$ as well, so that each atom has known values of two orthogonal components of its spin vector. Inasmuch as the formalism of quantum mechanics is incapable of describing an atom with well-defined — actually measured! — values of σ_x and σ_z, the EPR trio concluded that the formalism, and therefore quantum mechanics itself, is incomplete.

They did not regard their chain of arguments as proof of a nonlocal character of quantum mechanics. Rather, from its assumed locality (and a second assumption concerning reality to which we will return in Sec. II B) they inferred its incompleteness.

Today, claims that quantum mechanics is incomplete are no longer heard very frequently. There is a general agreement that quantum mechanics is complete in the sense that one cannot add further elements to it without getting into trouble with well-established experimental facts. But, reversing the EPR argument to some extent, doesn't it then follow that the EPR correlations are of an intrinsically

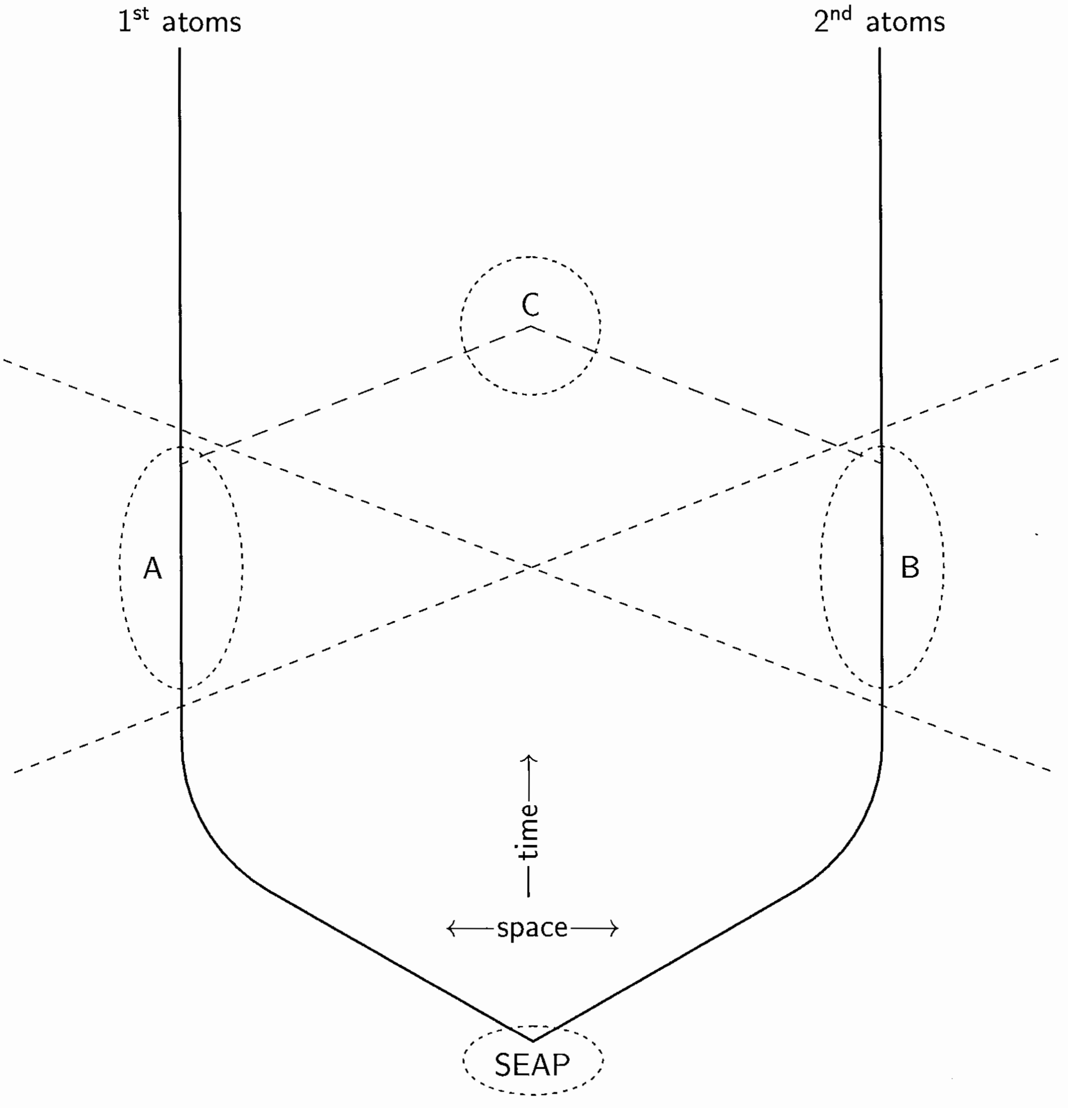

FIGURE 1. Space-time diagram of the EPR argument. After being emitted by the source of entangled atom pairs SEAP, the atoms separate so that the experimenters A and B can perform independent measurements in the space-time regions indicated. The light cone associated with an event half-way in between confirms that there is no causal relation between the two regions. The measurement results are reported, at the speed of light, to theorist C who analyzes the data even faster.

nonlocal nature?

A typical way of justifying the answer "yes" runs along the following lines; see Fig. 1. Experimenter A performs measurements on the 1st atoms of each pair, experimenter B on the 2nd ones. Both can determine either the x or the z component, but which one will be measured for any given pair is not agreed upon beforehand. Suitable random-choice devices select the components independently for A and B. It is here where the spacelike separation is essential because it ensures that the random selections are indeed independent.[3] A third experimenter C, located half-way between A and B and at rest relative to both, collects the data from A and B for analysis. For 50% of all atom pairs, the settings of A and B were different, and C observes no correlations whatsoever. However, for the other 50% the same spin component is measured on both atoms, and C confirms that the data exhibit the EPR correlations: There is always one outcome $+1$ and one outcome -1, whereas the individual sequences of $+1$'s and -1's recorded by A and B are completely random. It should be clear that C observes the EPR correlations at a time when A and B cannot yet know what the other one has found.

Although it is impossible to predict the random strings of $+1$'s and -1's because we do not know "how Nature decides about the actual measurement result" it is seemingly natural to assume that some unknown mechanism is at work. Equally natural convictions about locality and reality (to which we shall return in Sec. II B) then dictate that the hypothetical mechanism should function locally. Put differently, the outcome of A's measurement of $\sigma_z^{(1)}$, say, is not influenced by the setting of B's equipment because that is chosen at random far away. How could it possibly matter whether B measures also the z component or happens to determine $\sigma_x^{(2)}$? If everything were truly local it couldn't matter, but the observed EPR correlations indicate that it does, and therefore one must conclude, so it seems, that somehow the imagined mechanism in A's space-time vicinity is aware of what is going on at B's end in a nonlocal fashion.

Apparently there is only one way how we can avoid this conclusion, namely by assuming that for each pair the outcomes of all spin measurements are already assigned at the time when the pair is created — just like it is decided, at the instant when the boxes are sealed, which box contains which glove of the pair. But then the two-atom data must be consistent with Bell's inequality [2], and they are not. As reasonable as the said assumption may sound, "The reasonable thing just doesn't work." (Bell as quoted by Bernstein [7]).

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3) A practical way of ensuring the independence could rely on counting for a certain time the photons that arrive from distant galaxies at opposite sides of the universe [6]. The x component is measured if the count is even, say, otherwise it's the z component. — There is no absolute certainty about the independence, however, because it is thinkable that the galaxies have a common past and that there are resulting correlations between the photons they emit. The possibility of a conspiracy of such a ludicrous kind cannot be excluded by logical arguments alone. But an appeal to common sense should do.

B The Bell-Kochen-Specker theorem

The EPR trio's diagnosis that quantum mechanics is incomplete relies on the fact that the x and z components of an atom's spin vector do not commute and the formalism does not accommodate atoms with definite values of both σ_x and σ_z. Perhaps the formalism has been interpreted incorrectly? Could it be that σ_x, σ_z, and all other observables always possess definite values, only that most of them remain unknown to the experimenter as a matter of principle?

Kochen and Specker [8], and Bell independently [9], have given an ingenious argument to the contrary. It is not possible to assign consistent values to all observables of the two-atom system under consideration; a charmingly simple proof of this Bell-Kochen-Specker (BKS) theorem has been found by Mermin [10]. We shall not reproduce it here.

Instead we illustrate the essence of the BKS theorem with the aid of Mermin's [11] three-atom state

$$|\Psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\Big) \tag{3}$$

of the Greenberger-Horne-Zeilinger (GHZ) kind [12,13]. The notation is an immediate generalization of the one introduced in (1). The three-particle correlations built into the GHZ state are summarized in

$$\begin{aligned}
\sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}|\Psi_{\mathrm{GHZ}}\rangle &= +|\Psi_{\mathrm{GHZ}}\rangle\,, & \text{(4a)}\\
\sigma_y^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}|\Psi_{\mathrm{GHZ}}\rangle &= +|\Psi_{\mathrm{GHZ}}\rangle\,, & \text{(4b)}\\
\sigma_y^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}|\Psi_{\mathrm{GHZ}}\rangle &= +|\Psi_{\mathrm{GHZ}}\rangle\,, & \text{(4c)}\\
\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}|\Psi_{\mathrm{GHZ}}\rangle &= -|\Psi_{\mathrm{GHZ}}\rangle\,, & \text{(4d)}
\end{aligned}$$

which are the analogs of Eqs. (2) for the EPR state (1). Here, too, the last equation is implied by the other ones since

$$\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)} = -\left[\sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}\right]\left[\sigma_y^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}\right]\left[\sigma_y^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}\right]. \tag{5}$$

The four three-atom operators on the left-hand sides of Eqs. (4) commute with each other, but local measurements on the three atoms can determine only one of them. For example, we can infer the value $m_x^{(3)}$ of $\sigma_x^{(3)}$ by measuring the values $m_y^{(1)}$ and $m_y^{(2)}$ of $\sigma_y^{(1)}$ and $\sigma_y^{(2)}$, or by measuring the values $m_x^{(1)}$ of $\sigma_x^{(1)}$ and $m_x^{(2)}$ of $\sigma_x^{(2)}$. We get

$$m_x^{(3)} = m_y^{(1)}m_y^{(2)} \qquad \text{or} \qquad m_x^{(3)} = -m_x^{(1)}m_x^{(2)} \tag{6}$$

as consequences of (4c) and (4d), respectively. Similarly, we can establish the value $m_y^{(3)}$ of $\sigma_y^{(3)}$ by measuring either $\sigma_x^{(1)}$ and $\sigma_y^{(2)}$ [cf. (4a)] or $\sigma_y^{(1)}$ and $\sigma_x^{(2)}$ [cf. (4b)].

If the measurements on the three atoms are done in spacelike separated regions, the settings of the instruments for the 1st and the 2nd atom cannot have any influence on the $m_x^{(3)}$ value of the 3rd atom. The actual value of $\sigma_x^{(3)}$ is a local property of the 3rd atom. By the same token $\sigma_y^{(3)}$ has a value $m_y^{(3)}$ determined solely by what goes on in the space-time vicinity of the 3rd atom.

Likewise, there are definite values for the x and y spin components of the 1st and the 2nd atom. It is true that one cannot measure all of them, but couldn't that just be a limitation on what one can find out about them, with no bearing on the actuality of the unmeasured values?

No, because these hypothetical values have to obey

$$m_x^{(1)} m_y^{(2)} m_y^{(3)} = +1\,, \tag{7a}$$

$$m_y^{(1)} m_x^{(2)} m_y^{(3)} = +1\,, \tag{7b}$$

$$m_y^{(1)} m_y^{(2)} m_x^{(3)} = +1\,, \tag{7c}$$

$$m_x^{(1)} m_x^{(2)} m_x^{(3)} = -1\,, \tag{7d}$$

to be consistent with Eqs. (4), and that is impossible. Irrespective of the values assigned, the product of the four left-hand sides equals +1 since it involves each m value twice, but on the right we get −1. A different way of looking at it uses Eq. (5) whose numerical analog reads

$$\begin{aligned} m_x^{(1)} m_x^{(2)} m_x^{(3)} &= -\left[m_x^{(1)} m_y^{(2)} m_y^{(3)}\right]\left[m_y^{(1)} m_x^{(2)} m_y^{(3)}\right]\left[m_y^{(1)} m_y^{(2)} m_x^{(3)}\right] \\ &= -m_x^{(1)} m_x^{(2)} m_x^{(3)} \underbrace{\left[m_y^{(1)}\right]^2 \left[m_y^{(2)}\right]^2 \left[m_y^{(3)}\right]^2}_{=+1}\,, \end{aligned} \tag{8}$$

which is an obvious contradiction.

The locality assumption, so it seems again, has led to a contradiction, and the conclusion that there is a nonlocal character to quantum mechanics appears to be inevitable — a conclusion that we shall take issue with in Sec. II.

C A digression: Joint probabilities for a single spin-$\frac{1}{2}$ atom

The BKS theorem does not apply to a single spin-$\frac{1}{2}$ atom because its sets of mutually commuting observables are too small. Accordingly, one can try to get away with assigning values both to σ_x and to σ_z. To put this effort into an operational perspective, let us introduce four numbers $p_{\pm\pm}$ with the intended significance that, for example, p_{+-} is the probability that $\sigma_x = +1$ and $\sigma_z = -1$ simultaneously. The operational aspect is the requirement that the marginal sums over the $p_{\pm\pm}$'s result in the actual probabilities for σ_x and σ_z measurements [14,15]:

$$\begin{aligned}
\text{prob}\,(\sigma_x = +1) &= \frac{1}{2} + \frac{1}{2}\langle\sigma_x\rangle = p_{++} + p_{+-}\,, && \text{(9a)}\\
\text{prob}\,(\sigma_x = -1) &= \frac{1}{2} - \frac{1}{2}\langle\sigma_x\rangle = p_{-+} + p_{--}\,, && \text{(9b)}\\
\text{prob}\,(\sigma_z = +1) &= \frac{1}{2} + \frac{1}{2}\langle\sigma_z\rangle = p_{++} + p_{-+}\,, && \text{(9c)}\\
\text{prob}\,(\sigma_z = -1) &= \frac{1}{2} - \frac{1}{2}\langle\sigma_z\rangle = p_{+-} + p_{--}\,. && \text{(9d)}
\end{aligned}$$

The sum of all four $p_{\pm\pm}$'s must equal unity, and Eqs. (9) determine the values of the differences $p_{++} - p_{--}$ and $p_{+-} - p_{-+}$. So there is one free parameter, y, in the solution

$$\begin{aligned}
p_{++} &= \frac{1}{4}\left(1 + \langle\sigma_x\rangle + y + \langle\sigma_z\rangle\right)\,, && \text{(10a)}\\
p_{--} &= \frac{1}{4}\left(1 - \langle\sigma_x\rangle + y - \langle\sigma_z\rangle\right)\,, && \text{(10b)}\\
p_{+-} &= \frac{1}{4}\left(1 + \langle\sigma_x\rangle - y - \langle\sigma_z\rangle\right)\,, && \text{(10c)}\\
p_{-+} &= \frac{1}{4}\left(1 - \langle\sigma_x\rangle - y + \langle\sigma_z\rangle\right)\,. && \text{(10d)}
\end{aligned}$$

The freedom to choose y arbitrarily can be used to ensure the positivity of the $p_{\pm\pm}$'s. Any y that obeys the inequalities

$$\left|\,\langle\sigma_x\rangle + \langle\sigma_z\rangle\,\right| - 1 \le y \le 1 - \left|\,\langle\sigma_x\rangle - \langle\sigma_z\rangle\,\right| \tag{11}$$

is good enough for this purpose; in particular, the arithmetic mean of the two bounds will do. Another possibility is $y = \langle\sigma_x\rangle\,\langle\sigma_z\rangle$, for which

$$\begin{aligned}
p_{\pm\pm} &= \frac{1}{4}\left(1 \pm \langle\sigma_x\rangle\right)\left(1 \pm \langle\sigma_z\rangle\right)\\
&= \text{prob}\,(\sigma_x = \pm 1)\,\text{prob}\,(\sigma_z = \pm 1) && \text{(12)}
\end{aligned}$$

obtains, where the two $\pm$ in each expression correspond to one another in accordance with their order.

With any y value from the interval in (11), the interpretation of the $p_{\pm\pm}$'s as probabilities is permissible. In this sense, the challenge to assign values both to σ_x and to σ_z has been met.

This does not invalidate the EPR argument about the incompleteness of quantum mechanics, however, because there is still no room for simultaneous sharp values of σ_x and σ_z, as represented by $p_{++} = 1$ and $p_{--} = p_{+-} = p_{-+} = 0$. Not all positive $p_{\pm\pm}$'s with unit sum are allowed by quantum mechanics. In particular, the restriction $\langle\sigma_x\rangle^2 + \langle\sigma_z\rangle^2 \le 1$ translates into

$$\left(p_{++} - p_{--}\right)^2 + \left(p_{+-} - p_{-+}\right)^2 \le \frac{1}{2}\,, \tag{13}$$

which is obeyed for any value of y in (10). Therefore one cannot have three $p_{\pm\pm}$ values that vanish while the fourth equals unity.

At the price of negative $p_{\pm\pm}$ values — and after first contemplating $y = \pm i \langle\sigma_y\rangle$ — the simplest choice $y = 0$ was effectively made in Ref. [15], where Young's double-slit interferometer was reconsidered from the perspective of such "negative probabilities" (whatever this self-contradictory term may mean). Further, the extension to joint negative probabilities of two non-orthogonal spin components offered a fresh look at EPR correlations; for details consult [15].

Earlier, Feynman had exploited the arbitrariness of y for a different purpose [16]. In view of their unit sum, the four $p_{\pm\pm}$'s represents three independent numbers. Since the atom's spin state is characterized by the three components of the Bloch vector $\langle\vec{\sigma}\rangle$, a one-to-one correspondence can be achieved if y is related to $\langle\sigma_y\rangle$. The requirements

$$\text{prob}\,(\sigma_y = +1) = \frac{1}{2} + \frac{1}{2}\langle\sigma_y\rangle = p_{++} + p_{--}\,, \tag{14a}$$

$$\text{prob}\,(\sigma_y = -1) = \frac{1}{2} - \frac{1}{2}\langle\sigma_y\rangle = p_{+-} + p_{-+}\,, \tag{14b}$$

suggest themselves inasmuch as Eqs. (14) supplement Eqs. (9) naturally, and then $y = \langle\sigma_y\rangle$ is implied.[4] As Feynman notes, the resulting $p_{\pm\pm}$'s are a spin-$\frac{1}{2}$ analog of what the Wigner function is for a continuous x, p degree of freedom, and that is an additional argument in favor of $y = \langle\sigma_y\rangle$. It is then no surprise that his $p_{\pm\pm}$'s can be negative. Physical probabilities are given by the six different sums of two $p_{\pm\pm}$'s — the *marginals* in the general jargon that goes also with the Wigner function and the like — and their positivity is ensured.

Is it possible to choose y from the interval of (11) and also relate it to $\langle\sigma_y\rangle$? Yes, but if this relation should be applicable to any spin state, then y must involve $\langle\vec{\sigma}\rangle$ nonlinearly. An example is

$$\begin{aligned} y = \langle\sigma_y\rangle &+ \frac{1}{2}(1 - \langle\sigma_y\rangle)\,|\,\langle\sigma_x\rangle + \langle\sigma_z\rangle\,| \\ &- \frac{1}{2}(1 + \langle\sigma_y\rangle)\,|\,\langle\sigma_x\rangle - \langle\sigma_z\rangle\,|\,. \end{aligned} \tag{15}$$

As a consequence of the nonlinearity, the resulting $p_{\pm\pm}$'s are not expectation values of some observables, as they are for the choices $y = 0$ of [15] and $y = \langle\sigma_y\rangle$ of [16]. Therefore, some of the marginals will not equal physical probabilities although they are surely positive.

In summary, one can indeed find four positive numbers $p_{\pm\pm}$ that can be regarded as joint probabilities for definite values of σ_x and σ_z with some justification. These joint probabilities are, however, nonlinear functions of the statistical operator of the spin-$\frac{1}{2}$ degree of freedom, and σ_y is not treated on equal footing with σ_x and σ_z. If

4) Here, the inequality $p_{++}^2 + p_{--}^2 + p_{+-}^2 + p_{-+}^2 \leq \frac{1}{2}$ must hold, which is more stringent than (13).

one insists on a fair treatment of σ_y or requires that the $p_{\pm\pm}$ are expectation values, then their positivity cannot be guaranteed. So, a fully satisfactory assignment of joint probabilities is not achievable although the BKS theorem does not apply. And as noted above, none of these observations has any bearing on the incompleteness argument of the EPR trio.

D Bohm trajectories

In section I A we have said that quantum mechanics is complete because nothing can be added to the formalism without getting into trouble. Is this really true, or shouldn't one rather admit that Bohm [17] has been successful in supplementing quantum mechanics with particle trajectories without encountering any inconsistencies? Yes, he has, but these trajectories are of no phenomenological consequence — and additions of this "cheap" (Einstein in a letter to Born of May 12, 1952 [18]) kind shouldn't count.

Moreover, there are simple yet instructive examples [19–21] which show that, quite contrary to what Bohm had in mind, the trajectories of Bohmian mechanics do not reveal the actual positions of the particles to which they belong. For example, it is possible to have a well localized electromagnetic interaction with an atom whose trajectory never comes near the interaction region.

Convinced adherents of Bohmian mechanics, however, are not bothered by this observation. They are ready to turn the argument around and profess that, by definition, a particle is always located on its Bohm trajectory. Then, the particle's capability of participating in well-localized interactions far away is just another proof of the nonlocal nature of quantum mechanics.

II ARGUMENTS IN FAVOR OF "EPR CORRELATIONS ARE LOCAL"

In this section we counter the arguments in support of nonlocality and try to make a case for locality. We can immediately dispose of the nonlocality claim based on the unavoidable interactions at-a-distance in Bohmian mechanics, because that is nothing more than an artifact of the Bohm trajectories. At best, it demonstrates a fundamental lack of plausibility of the picture drawn by Bohmian mechanics.

More substantial is the discussion of correlations at-a-distance without effects at-a-distance in Sec. II A. The so-called stronger locality principle, on which Bell's inequality is based, is the subject of Sec. II B where we exhibit the underlying concept of reality and put the notion of real factual situations into perspective. Then, in Sec. II C, we offer some remarks to the extent that the EPR correlations can be understood as an implication of more basic phenomena. Quantum erasure, which exploits EPR correlations in a particular way, is the topic addressed in Sec. II D where we emphasize that there is nothing nonlocal about it. The point of view entertained by the two-state formulation of quantum mechanics is reported in Sec. II E.

A Action, correlations, effects at-a-distance

In the first place, the EPR correlations that are built into $|\Psi_{\rm EPR}\rangle$ of (1) refer solely to the spin degrees of freedom of the two atoms and have nothing to do with their center-of-mass motion. The question of locality enters only because of the spacelike separation between the atoms when (randomly) chosen spin components are measured. As noted in Sec. I A, this separation is a technical means to ensure the independence of the two spin measurements. The EPR correlations are a property of the spin-singlet state (1) even if the two atoms are in the same place.[5] In this sense, EPR correlations are neither local nor nonlocal — the category of locality simply doesn't apply. Said differently, the EPR correlations are the same at the moment of initial dissociation as they are when the atoms are separated by "galactic" distances.

The spacelike separation is not only a pedagogical tool for illustrating and emphasizing that one can make independent choices of the spin components that are measured, it is also the manner in which the independence is achieved in experiments [22,23,1]. The causal independence of macroscopic events at spacelike separations is beyond reasonable doubt (unless, of course, the events have a common cause, as they would have in the conspiracy scenario of footnote 3), and since the settings of the measurement devices for observers A and B in Fig. 1 involve *macroscopic* events, the causal independence of the two spin measurements is implied although they explore *microscopic* properties of the entangled two-atom state $|\Psi_{\rm EPR}\rangle$.

It is not necessary to invoke lessons of relativistic quantum field theory to arrive at these conclusions. The logic is rather the other way round: All attempts at formulating a relativistic quantum theory must incorporate the independence of observations at spacelike separations. The recent monograph by Haag [24] gives a lucid account of this program.

A fundamental postulate of relativistic quantum theory is, therefore, that an observable that is local to a certain space-time region must commute with all other observables that are local to other regions at spacelike separations. As self-evident as this property may seem, it is not at all obvious that it does not lead to contradictions.[6] Indeed it doesn't and, accordingly, relativistic quantum field theory is *local*.

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5) In the $J = 0$ ground state of hydrogen, the spin state is the EPR state of (1). If one could measure the electron spin and the proton spin independently, EPR correlations would surely be observed.

6) The typical pitfalls that must be recognized and then avoided are well illustrated by the consistent treatment [25,26] of Fermi's problem [27] in which a photon emitted by one atom is absorbed by a distant second atom.

Owing to this locality property of quantum fields, it is impossible to send faster-than-light signals by exploiting EPR correlations. There really is no "spooky action at-a-distance" (Einstein in a letter to Born of March 3, 1947 [18]), at best one could speak of *correlations at-a-distance*,[7] and we leave it as a moot point if they deserve to be called spooky. In a different terminology [28], this distinction appears as the *stronger locality principle* (used in the derivation of Bell's inequality, for example, and discussed below in Sec. II B) and the *weaker locality principle* (obeyed by quantum fields, see above).

The weaker locality principle ensures the absence of *effects at-a-distance*. In the operational context of Fig. 1, this means that B cannot verify whether A is actually doing what has been agreed upon. There is no physical effect at B's end which would tell him what is happening at A's end. Suppose, for example, that A's random device is broken and A determines always the z component or always the x component rather than a random sequence of both. Does B's data exhibit any signs thereof? No, it doesn't, and even if B were told that A is measuring the same component all the time, he wouldn't be able to find out which one.

Since A and B have access to their own data sets only but not to the other's, they remain unaware of the correlations at-a-distance, which are a fact to C who has both data sets at his disposal. In general terms, if A determines the value m_A of the spin component $\vec{e}^{(1)} \cdot \vec{\sigma}^{(1)}$ and B the value m_B of $\vec{e}^{(2)} \cdot \vec{\sigma}^{(2)}$, then C notes that the four different pairs of outcomes $(m_\mathsf{A}, m_\mathsf{B}) = (\pm 1, \pm 1)$ occur with the relative frequencies

$$\frac{1}{4}\left(1 - m_\mathsf{A} m_\mathsf{B}\, \vec{e}^{(1)} \cdot \vec{e}^{(2)}\right) = \begin{cases} \dfrac{1}{2}\left(\cos\dfrac{\theta}{2}\right)^2 & \text{if} \quad m_\mathsf{A} m_\mathsf{B} = -1\,, \\ \dfrac{1}{2}\left(\sin\dfrac{\theta}{2}\right)^2 & \text{if} \quad m_\mathsf{A} m_\mathsf{B} = +1\,, \end{cases} \tag{16}$$

where θ is the angle between the two unit vectors $\vec{e}^{(1)}$ and $\vec{e}^{(2)}$. In particular, C observes that there are no correlations for $\vec{e}^{(1)} \perp \vec{e}^{(2)}$ and perfect anticorrelations for $\vec{e}^{(1)} = \vec{e}^{(2)}$. These regularities are a property of the $(m_\mathsf{A}, m_\mathsf{B})$ pairs; the m_A values by themselves constitute a sequence of $+1$'s and -1's that is random in all respects, and so do the m_B values.

B Locality and reality

The EPR correlations (16) in the spin singlet state (1) do not obey Bell's inequality, indeed, and therefore one can conclude that the stronger locality principle (SLP) is violated.[8] As we shall see shortly, however, the validity of the SLP is highly questionable.

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7) We owe this term to G. Süssmann.

8) Bell's original argument made use of a couple of other assumptions (local determinism is one of them) in addition to the SLP. An analysis seems to indicate that only the SLP is mandatory [28].

The SLP is intimately related to certain convictions about *reality.* They are most eloquently phrased in the following oft-quoted sentences.

> The EPR trio in 1935 [5]:
> If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. (17a)

> Einstein in 1949 [29]:
> [T]he real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former.[9] (17b)

Let us discuss these matters in the context of Fig. 1.

As soon as A has determined the value of $\sigma_z^{(1)}$ he can predict the outcome of B's $\sigma_z^{(2)}$ measurement on the partner atom. According to (17a), an "element of physical reality" (EoPR) must thus be associated with $\sigma_z^{(2)}$, which is to say that the 2nd atom possesses a definite σ_z value. And since A's determination of $\sigma_z^{(1)}$ does not "disturb the 2nd atom in any way," the conclusion is inevitable that the definite $\sigma_z^{(2)}$ value exists even if A doesn't measure $\sigma_z^{(1)}$. The same chain of arguments applies to the other components of $\vec{\sigma}^{(2)}$ and so the whole spin vector of the 2nd atom is an EoPR and, by symmetry, that of the 1st atom is also one.

The actual situation is quite different, however. Suppose A has determined the value +1 for $\sigma_z^{(1)}$, and B got +1 for $\sigma_x^{(2)}$. Then C can state the following.

> Given that A has determined +1 for the z component of the 1st atom, it is clear that B would have found −1 had he measured the z component for the 2nd atom. (18a)

> Given that B has determined +1 for the x component of the 2nd atom, it is clear that A would have found −1 had he measured the x component for the 1st atom. (18b)

Although both statements are about measurements that could have been made, but haven't been made and can no longer be made, they are nevertheless correct and fully justified by the EPR correlations of (16) that C has previously observed for the pairs emitted by the SEAP. Of course, one cannot verify (18a) or (18b) by control experiments on the same pair of atoms; verifiable predictions can only concern another $\sigma_z^{(1)}$ measurement by A or another $\sigma_x^{(2)}$ measurement by B. The

9) This is P. A. Schilpp's translation of the German original which reads: Der reale Sachverhalt (Zustand) des Systems S_2 ist unabhängig davon, was mit dem von ihm räumlich getrennten System S_1 vorgenommen wird.

outcomes of the complementary control measurements of $\sigma_x^{(1)}$ and $\sigma_z^{(2)}$ are utterly unpredictable.

As a consequence of (17a) one could conclude that the statement

> If A had measured $\sigma_x^{(1)}$ and B had measured $\sigma_z^{(2)}$, then both would have found -1. (18c)

is equally well justified, because it is just a logical implication of (18a) and (18b) as soon as the status of an EoPR has been granted to both $\sigma_x^{(1)}$ and $\sigma_z^{(2)}$. In view of what is said at the end of the preceding paragraph, however, they do not possess this status since the control experiments will not necessarily yield the assigned values of -1.

In other words, the inference of (18c) from (18a) and (18b) makes implicit use of the (incorrect) assumption that the various spin components have definite values prior to any measurement by A and B, so that these values are merely recognized by the observers, thereby changing their status from definite-but-unknown to definite-and-known. But, as a matter of fact, no component of the spin vectors $\vec{\sigma}^{(1)}$ and $\vec{\sigma}^{(2)}$ has a definite value prior to a measurement. Only the products of (2) have values, their factors don't; the products are EoPRs, the factors are not.[10] In the entangled two-atom state $|\Psi_{\text{EPR}}\rangle$ of (1), the value of $\sigma_x^{(1)}$, say, is not just unknown — it is unknowable, it simply doesn't exist.

Having thus seen that claims of nonlocality based on (17a) are rather dubious, let us now turn to (17b). In the context of Fig. 1 the question arises: What is the *real factual situation* (RFS) in A's space-time region, and what is it in B's? The ensemble of 1st atoms investigated by A has a statistical operator ρ_1 that is obtained by tracing $|\Psi_{\text{EPR}}\rangle\langle\Psi_{\text{EPR}}| = \frac{1}{4}(1 - \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)})$ over the 2nd atom,

$$\begin{aligned}
\rho_1 &= \text{tr}_2\left\{|\Psi_{\text{EPR}}\rangle\langle\Psi_{\text{EPR}}|\right\} \\
&= \frac{1}{2}\left[\frac{1}{2}(1+\sigma_z^{(1)}) + \frac{1}{2}(1-\sigma_z^{(1)})\right] && \text{(19a)} \\
&= \frac{1}{2}\left[\frac{1}{2}(1+\sigma_x^{(1)}) + \frac{1}{2}(1-\sigma_x^{(1)})\right] && \text{(19b)} \\
&= \frac{1}{4}\left[\frac{1}{2}(1+\sigma_z^{(1)}) + \frac{1}{2}(1-\sigma_z^{(1)}) + \frac{1}{2}(1+\sigma_x^{(1)}) + \frac{1}{2}(1-\sigma_x^{(1)})\right] && \text{(19c)} \\
&= \frac{1}{2}\,.
\end{aligned}$$

Adopting Süssmann's terminology [32] we say that (19a–c) represent three different *blends* (German: Gemenge) of the same *mixture* (Gemisch) $\rho_1 = \frac{1}{2}$. Each blend is associated with an *as-if-reality* (AIR): (a) it is *as if* there were 50 % each of

10) This is a central point of what Mermin calls the "Ithaca interpretation" [30]. Arguably, it is an exegesis of Bohr's response [31] to the EPR paper.

atoms with $\sigma_z^{(1)} = +1$ and $\sigma_z^{(1)} = -1$; (b) it is *as if* there were 50 % each of atoms with $\sigma_x^{(1)} = +1$ and $\sigma_x^{(1)} = -1$; (c) it is *as if* there were 25 % each of atoms with $\sigma_z^{(1)} = +1$, $\sigma_z^{(1)} = -1$, $\sigma_x^{(1)} = +1$, and $\sigma_x^{(1)} = -1$. It appears that Einstein would have regarded each blend as a different RFS, namely (a) applying when B measures the z components of the partner atoms; (b) holding when the value of $\sigma_x^{(2)}$ is determined; and (c) being relevant when B measures either the x or the z component at random. It is clear, however, that this distinction has no relevance for what A is observing. The statistical properties of his data are completely and correctly described by the mixture $\rho_1 = \frac{1}{2}$. There is absolutely no way in which A could tell which one of the three AIRs is the actual one, all three (and many more) are perfectly equivalent. It really doesn't matter to A and the RFS he perceives whether B is performing any of these measurement.

We must therefore conclude that ρ_1 summarizes the RFS in A's space-time region [33]. It is then true, indeed, that A's RFS is independent of what is done by B, in full agreement with the requirement (17b). Likewise, $\rho_2 = \frac{1}{2}$ states the RFS at B's end.

Admittedly, Einstein may not have been satisfied with this solution because he might have argued that the impossibility of distinguishing between (19a–c) is just an indication of the incompleteness of quantum mechanics that the EPR trio had diagnosed [5]. Perhaps the observation that A and B could exchange instantaneous messages if the distinction were feasible [34] — in blatant violation of the weaker locality principle of quantum field theory — would have convinced Einstein that quantum mechanics is as complete as it can possibly be.

As a final remark concerning the quotes (17) we note the implicit assumption that each atom of an EPR pair qualifies as an individual "system" when the spacelike separation is achieved. In a more consistent view the entangled pair is a two-atom system, whereas the constituent atoms are not regarded as subsystems themselves, as long as their EPR correlations are of interest. This point is emphasized by Bohr in his reply to the EPR paper [31]. The holism associated with this view has a nonlocal nature of its own.

C Understanding EPR correlations

The reality of the perfect anticorrelations of (16) when A and B both determine the z component is beyond doubt. They are an undisputed fact. In view of the other fact that the individual values of $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ do not even exist before they are determined (in marked contrast to the handedness of the gloves mentioned in Sec. I A), some authors are prompted to demand an explanation of how the two atoms may "know of the measurement performed on the other one" so that the atoms "can act in unison." In other words, in addition to the result (16) produced by the quantum mechanical formalism, they request an "understanding of what is really going on." Since *understanding* and *really* are ill-defined terms, an answer to everybody's satisfaction can hardly be given. But those, who are willing to regard as an explanation a reduction to more basic phenomena that are generally accepted, may find the following remarks of some use.

The EPR correlations (16) are not an isolated fact, rather they are the consequence of two more elementary phenomena. The first is that the measurement of any spin component has only the two possible outcomes +1 and −1 and nothing in between. In operational terms: A Stern-Gerlach apparatus splits a beam of magnetic spin-$\frac{1}{2}$ atoms in two, it does not spread it out. The second is the validity of the superposition principle not just for single-particle states (as in Young's double-slit interferometer) but also for many-particle states [two particles in $|\Psi_{\rm EPR}\rangle$ of (1), three in $|\Psi_{\rm GHZ}\rangle$ of (3)]. Then the two alternatives described by $|{\uparrow\downarrow}\rangle$ (in words: 1st atom has definitely $\sigma_z = +1$, 2nd atom has definitely $\sigma_z = -1$) and $|{\downarrow\uparrow}\rangle$ are superimposed in $|\Psi_{\rm EPR}\rangle$ of (1) and a new two-atom state obtains which has properties that are quite different from $|{\uparrow\downarrow}\rangle$ and $|{\downarrow\uparrow}\rangle$.

The only-two-outcomes result of the Stern-Gerlach experiment is "a brutal, irreducible fact of life, and quantum mechanics is about learning to live with it" [35]. Irrespective of whether one agrees with this dictum, as we do, there is nothing to be disputed. Similarly, the validity of the superposition principle for two-particle states cannot be questioned in the face of, for example, the correct quantum-mechanical predictions for the He spectrum which rely heavily upon the two-electron spin-singlet state $|\Psi_{\rm EPR}\rangle$ of (1) for para-helium and the triplet states $|{\uparrow\uparrow}\rangle$, $(|{\uparrow\downarrow}\rangle + |{\downarrow\uparrow}\rangle)/\sqrt{2}$, and $|{\downarrow\downarrow}\rangle$ for ortho-helium.

D Quantum erasure

Another viewpoint emphasizing the local nature of all operational aspects is provided by quantum erasure (QE) [36]. Again we utilize the context of Fig. 1 for an illustration.

Having noted that the 1st atoms come in equal proportions of $\sigma_z^{(1)} = +1$ and $\sigma_z^{(1)} = -1$, experimenter A decides to test for coherent superpositions of $|\uparrow\rangle$ and $|\downarrow\rangle$. For this purpose he measures the spin component $\sigma_\phi \equiv \sigma_x^{(1)} \cos\phi + \sigma_y^{(1)} \sin\phi$ with varying values of the angle parameter ϕ. The looked-for coherence would manifest itself in a periodic ϕ dependence of

$$p(\phi) = \frac{\mathrm{prob}\,(\sigma_\phi = +1)}{\mathrm{prob}\,(\sigma_\phi = +1) + \mathrm{prob}\,(\sigma_\phi = -1)}\,. \tag{20}$$

Of course, since the statistical properties of A's data result from $\rho_1 = \frac{1}{2}$ of (19), A will find $p(\phi) = \frac{1}{2}$ without any ϕ dependence. In view of the AIR associated with (19a), this is as it should be: B could have measured the z component of each 2nd atom, so that C could know, in the spirit of (18a,b), if each of A's 1st atoms is of the $|\uparrow\rangle$ kind or the $|\downarrow\rangle$ kind.

So, those 1st atoms for which a $\sigma_z^{(2)}$ measurement has been made by B, can be sorted by C into two groups characterized by the actual value of $\sigma_z^{(2)}$. This is the which-alternative (WA) sorting,[11] and C notes ϕ independent patterns,

$$p_{\mathrm{WA}}^{(+)}(\phi) = p_{\mathrm{WA}}^{(-)}(\phi) = \frac{1}{2}\,, \tag{21}$$

for the $p(\phi)$ distribution of the WA subensembles.

In addition to (19a) there is also the equivalent AIR of (19b). The grouping of the 1[st] atoms in accordance with the $\sigma_x^{(2)}$ values found by B is the QE sorting. As a consequence of (16), C gets the patterns

$$p_{\mathrm{QE}}^{(\pm)}(\phi) = \frac{1}{2}(1 \pm \cos\phi) \tag{22}$$

here, that is: fringes and antifringes with perfect visibility. The fringeless pattern $p(\phi) = \frac{1}{2}$ observed by A comes about because A cannot keep the $\sigma_x^{(2)} = +1$ group apart from the $\sigma_x^{(2)} = -1$ group, so that he can only register $\frac{1}{2}(p_{\mathrm{QE}}^{(+)} + p_{\mathrm{QE}}^{(-)}) = \frac{1}{2}$, where the crests of $p_{\mathrm{QE}}^{(+)}$ meet the troughs of $p_{\mathrm{QE}}^{(-)}$.

In the situation in which B measures either the x or the z component of $\vec{\sigma}^{(2)}$ at random, corresponding to A's AIR of (19c), C ends up with four subensembles, two with a determined value of $\sigma_x^{(2)}$ and two with a definite $\sigma_z^{(2)}$ value. The latter are the WA groups, the former are the QE groups.

Which component is measured by B determines whether the partner atom is put into a WA group or a QE group, but despite the spacelike separation between A and B it should be clear that no nonlocal actions are involved. And, although C exploits the EPR correlations extensively in his data analysis, everything he does is local to his space-time vicinity. No actions or effects at-a-distance need to be invoked for an understanding of the kind of atom sorting we call quantum erasure.

It is remarkable though that the late Edwin Jaynes, an acclaimed information theorist, thought (erroneously) that some effect at-a-distance should be at work. He identified it as state reduction at-a-distance, interpreted as a real physical process, and found that the implications are quite bizarre. The following quote (from Ref. [37], first paragraph adapted to the present context) is telling:

> We have, then, the full EPR paradox — and more. By measuring $\sigma_z^{(2)}$ or $\sigma_x^{(2)}$ experimenter B can, at will, force A's partner atom into either: (1) a state with a known value of $\sigma_z^{(1)}$ and no possibility of an interference effect in any subsequent measurement of σ_ϕ; (2) a state with a known value of $\sigma_x^{(1)}$ in which both $|\sigma_z^{(1)} = +1\rangle$ and $|\sigma_z^{(1)} = -1\rangle$ are present with a known relative phase. Interference effects are then not only observable, but predictable. And B can decide which to do after the interaction in the SEAP is over and the atoms are far apart, so there can be no thought of any physical influence on the state of A's atom! [...]

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11) In analogous situations, in which the alternatives refer to the ways through an interferometer, one usually speaks of which-way sorting.

From this, it is pretty clear why present quantum theory not only does not use — it does not even dare to mention – the notion of a "real physical situation." Defenders of the theory say that this notion is philosophically naive, a throwback to outmoded ways of thinking, and that recognition of this constitutes deep new wisdom about the nature of human knowledge. I say that it constitutes a violent irrationality, that somewhere in this theory the distinction between reality and our knowledge of reality has become lost, and the result has more the character of medieval necromancy than of science. It has been my hope that quantum optics, with its vast new technological capability, might be able to provide the experimental clue that will show us how to resolve these contradictions.

From the above quote, it is "pretty clear" that Jaynes is upset by the notion of influencing events at a distant point in a way that he considered a "violent irrationality" and black magic. A reading of his paper [37] will reinforce this impression. He thought that simply the measurement of $\sigma_x^{(2)}$ by B (in the context of the present example) would change what A observes. This is incorrect. In fact, it is only after correlating A's data with B's data that C reveals the interference pattern. As we have emphasized already above, nothing that is done at B's end changes in any way what A perceives. It is thus "pretty clear" why QE argues strongly in favor of the (local) quantum correlation (data reduction) point of view. This is the essence of the physics. Everything happens locally, it is only that later one correlates or sorts the data so as to recognize particular subensembles, some of which exhibiting well-visible interference fringes.

E Two-state formalism

A systematic formalism that treats pre-selection and post-selection on equal footing has been developed in recent years [38–41]. It enables us to view quantum correlations from yet another angle. In the two-state formalism, the complete description of the state of the pair of atoms requires a final bra vector $\langle\Psi_{\mathrm{FIN}}|$ in addition to the initial ket vector $|\Psi_{\mathrm{EPR}}\rangle$ of (1). Consider as an example the situation of (18a–c), discussed in Sec. II B, in which A measures $\sigma_z^{(1)}$ and finds +1 while B finds +1 for $\sigma_x^{(2)}$. In this case the final bra vector is

$$\langle\Psi_{\mathrm{FIN}}| = \langle\uparrow| \otimes \frac{1}{\sqrt{2}}\Big(\langle\uparrow| + \langle\downarrow|\Big) = \frac{1}{\sqrt{2}}\Big(\langle\uparrow\uparrow| + \langle\uparrow\downarrow|\Big) . \tag{23}$$

A full account is given by the *two-state vector* (this is a bra-ket pair, not a scalar product)

$$\langle\Psi_{\mathrm{FIN}}|\ |\Psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}\Big(\langle\uparrow\uparrow| + \langle\uparrow\downarrow|\Big)\ \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big) , \tag{24}$$

which incorporates both the pre-selected ket and the post-selected bra alike. It yields probabilities for outcomes of all possible intermediate measurements of observables C according to the ABL formula [42] (see also [40]),

$$\text{prob}\,(C = c_n) = \frac{\left|\langle\Psi_{\text{FIN}}|\mathbf{P}_{C=c_i}|\Psi_{\text{EPR}}\rangle\right|^2}{\sum_i \left|\langle\Psi_{\text{FIN}}|\mathbf{P}_{C=c_i}|\Psi_{\text{EPR}}\rangle\right|^2} \tag{25}$$

where $\mathbf{P}_{C=c_i}$ is the projection operator on the subspace defined by $C = c_i$. In particular, this formula reproduces the results stated in (18a) and (18b). The two-state vector (24) gives also the outcomes of possible intermediate *weak measurements* [39], which (in the limit of extreme weakness) do not disturb quantum systems.

For instance, at intermediate times after the pre-selection of $|\Psi_{\text{EPR}}\rangle$ and before the post-selection of $\langle\Psi_{\text{FIN}}|$ experimenter A could make a weak measurement of $\sigma_x^{(1)}$ and B of $\sigma_z^{(2)}$. They would both find -1, and thus $\sigma_x^{(1)}$ and $\sigma_z^{(2)}$ are *weak-measurement elements of reality* [43] for these pre- and post-selected pair of atoms.

This is in marked contrast to (18c), which amounts to claiming that the final bra vector (23) is equivalent to

$$\frac{1}{\sqrt{2}}\Big(\langle\uparrow| - \langle\downarrow|\Big) \otimes \langle\downarrow| = \frac{1}{\sqrt{2}}\Big(\langle\uparrow\downarrow| - \langle\downarrow\downarrow|\Big)\,. \tag{26}$$

Clearly, it is not, and this supports what is said in Sec. II B about (18c).

The two-state vector (24) is Lorentz covariant by construction. Only local measurements and interactions are under consideration here, so no contradiction with relativistic causality can happen. Moreover, the two-state formalism avoids the apparent paradoxes of "nonlocal collapse." Indeed, in the example considered, the measurement $\sigma_z^{(1)} = 1$ leads to $\sigma_z^{(2)} = -1$. This, in turn, in another Lorentz frame leads to $\sigma_z^{(1)} = 1$ even before the measurement performed on the first particle. This sounds paradoxical in the standard approach that regards state reduction as a real physical process, but it is a natural part of the time-symmetric approach in which the status of measurements at a later time is equivalent to the status of measurements at an earlier time.

The WA and QE sorting of the preceding section can also be discussed in the framework of the two-state formalism. We can consider the measurement by B as partial post-selection because in some Lorentz frame the measurement of σ_ϕ by A is performed before the final measurement of B. The modified ABL formula for the initial state $|\Psi_{\text{EPR}}\rangle$ and partial post-selection $B = b$ is [44]

$$\text{prob}\,(C = c_n) = \frac{\left\|\mathbf{P}_{B=b}\mathbf{P}_{C=c_n}|\Psi_{\text{EPR}}\rangle\right\|^2}{\sum_i \left\|\mathbf{P}_{B=b}\mathbf{P}_{C=c_i}|\Psi_{\text{EPR}}\rangle\right\|^2}\,. \tag{27}$$

Equations (21) and (22) are obtained by a straightforward application thereof.

SUMMARY

After recalling standard arguments supporting the view that EPR correlations are a manifestation of quantum nonlocality, we have given counter arguments that are at least as convincing in our judgment. It is true that there are correlations at-a-distance stronger than any classical correlation can be, but there are no effects at-a-distance.

We conclude that the EPR trio's "elements of reality" must be revised. Only when the outcome of a control measurement can be predicted with certainty — and the prediction verified — should the status of reality be granted. In the case of statements about imaginary results of measurements that have not been made and can no longer be made, extreme caution is advised.

The process of quantum erasure exploits EPR correlations but, as we have discussed, it does not rely on nonlocal actions. All procedures involved in this kind of data sorting are local. Further, the description of EPR correlations in the two-state formalism is Lorentz covariant and, therefore, it does not accommodate nonlocal elements.

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