Locality or Non-Locality in Quantum Mechanics: Hidden Variables without "Spooky Action-at-a-Distance"

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The folklore notion of the "Non-Locality of Quantum Mechanics" is examined from the point of view of hidden-variables theories according to Belinfante's classification in his *Survey of Hidden Variables Theories*. It is here shown that in the case of EPR, there exist hidden variables theories that successfully reproduce quantum-mechanical predictions, but which are explicitly local. Since such theories do not fall into Belinfante's classification, we propose an expanded classification which includes similar theories, which we term as theories of the "third" kind. Causal implications of such theories are explored. – Pacs: 03.65.Bz

Key words: Non-Locality; Hidden Variables; EPR.

I. Introduction

Quantum mechanics predicts correlations between causally disconnected events, as in EPR-type experiments, which can not be explained by a strictly local hidden-variable theory [1, 2]. Thus follows the standard folklore that "quantum theory is non-local".

Obviously, such statements must be taken with a few grains of salt. To wit, it is possible to construct hidden-variables theories yielding predictions in agreement with quantum theory, Bohm's for instance [3], but which explicitly violate the locality principle in the sense of Einstein [4]. In other words, quantum mechanics could be "explained", if we chose to, by postulating mechanisms providing superluminal information transfer or, in Einstein's words, "spooky action-at-a-distance" [5]. And so one may be tempted to take this "spookiness" to be the actual connotation of the term "non-local" in reference to quantum mechanical entanglement.

But non-locality when viewed in this sense is in opposition to our relativistic intuition, not only by challenging the very geometric interpretation of space-time, but also by countering every indication coming from quantum physics itself as to its "peaceful coexistence" [6] with relativistic causality. Indeed, nowhere in the mathematical structure of quantum dynamics, i. e., in the Heisenberg picture, is there any indication of non-covariant physical processes. Such questions enter in the context of the measurement process, and even then no explicit violation of relativistic causality (e. g., superluminal communication) is to be found. Such evidence is compelling enough to pursue other ways of comprehending the meaning of "non-locality" as it bears on hidden variables theories that purport to explain quantum entanglement.

Our contention is that in fact there is still enough latitude for such an alternative interpretation, a point that we wish to illustrate by deriving the EPR-correlation function in terms of a hidden variables theory that does not invoke superluminal communication mechanisms. As expected, any alternative hidden-variable approach must still entail some sort of bold departure from our cherished notions of causality. The theory presented here certainly does so, by calling into question the microscopic arrow of time. We emphasize, however, that our intention is not to argue in favor of the hidden variables approach per se (or much less of any particular hidden variables theory). Instead, our intention is to suggest that to the same extent that one may find hidden variables theories with trade-offs that may be argued to be more (or less) palatable than those entailed by explicitly non-local theories such as Bohm's, there are clearly other viable alternatives to the folklore of "non-locality".

II. Belinfante's HV_2 Hidden-variables Theories

Our interest in this section is on Hidden Variables theories of the second kind, according to Belinfante's classification in his *Survey of Hidden Variables Theories* [7], and particularly as they bear on the characteristic correlations of the two-spin singlet state. Consider the standard EPR setting with two Stern Gerlach Apparatus (SGA). For simplicity, we restrict the orientations of the SGA to the xz plane, assuming that the two particles fly off from the common source in the direction y. Denote by θ and ϕ the two angles giving the orientation of the SGAs relative to the z-axis (θ and ϕ will also be used as labels for the two devices). Finally, let us think of each SGA measurement as a "pass"/"no-pass" test, where "pass" for a given orientation θ will be denoted by θ_+ and will stand for a deflection consistent with the "up" direction relative to the orientation θ of the SGA.

When the particles are prepared in an initial singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\uparrow_1\downarrow_2\rangle - \frac{1}{\sqrt{2}}|\downarrow_1\uparrow_2\rangle, \tag{2.1}$$

quantum mechanics predicts an observed joint passage with probability

$$\mathcal{P}_{\text{QM}}\left(\theta_+\,\phi_+\,|\,\Psi\right) = \langle\Psi|\hat{\Pi}(\theta_+)\otimes\hat{\Pi}(\phi_+)|\Psi\rangle \tag{2.2}$$

$$= \frac{1}{4}\left[1-\cos(\theta-\phi)\right],$$

where $\hat{\Pi}(\theta_+) = |\theta_+\rangle\langle\theta_+|$. In a hidden variables treatment of EPR (e.g. Bell's [8]), one would like to reproduce the quantum statistical predictions by postulating additional microscopic variables λ (of unspecified dimensionality), which are made available to both particles at their common emission event, and which together with other reasonable assumptions as to their statistical properties allow for the joint distribution (2.2) to be separated in the form

$$\mathcal{P}_{\text{QM}}\left(\theta_+\,\phi_+\,|\,\Psi\right) = \int \mathrm{d}\,\lambda\;\rho(\lambda)\,\pi(\theta_+|\lambda)\,\pi(\phi_+|\lambda). \tag{2.3}$$

In this general form, $\rho(\lambda)$ is a hidden variables distribution, replacing the role of the density matrix $\hat{\rho} = |\Psi\rangle\langle\Psi|$, and $\pi(\theta_+|\lambda)$, henceforth denoted as the *microscopic passage probability*, is the probability for particle "1"to pass the θ-test, *given* the hidden variable λ, etc.

In his survey of hidden variables theories, Belinfante makes a distinction between two types of theories conforming to the above form, (2.3). These he calls theories of type I and type II (henceforth HV_1 and HV_2):

1.) HV_1 theories are those that give the same statistical predictions as quantum mechanics insofar as the hidden-variables are assumed to be in the equilibrium state associated with the vector $|\Psi\rangle$. Deviations from quantum mechanical predictions are possible under non-equilibrium situations in which the state of the system cannot be attached to any particular $|\Psi\rangle$. Type *I*-theories are, however, explicitly non-local, in the sense that the microscopic passage probability $\pi(\theta_+|\lambda)$ for particle "1" must depend not only on θ, but also on the *settings* of what is measured on particle "2", i.e., the angle ϕ (and vice-versa). In other words, $\pi(\theta_+|\lambda)$ is a conditional probability of the form

$$\pi(\theta_+|\lambda) = \mathcal{P}\left(\theta_+\;|\;\lambda\theta\phi\right) \neq \mathcal{P}\left(\theta_+\;|\;\lambda\theta\right). \tag{2.4}$$

2.) By contrast, in HV_2 theories it is assumed that $\pi(\theta_+|\lambda)$ is a conditional probability where the dependence on ϕ is irrelevant

$$\pi(\theta_+|\lambda) = \mathcal{P}\left(\theta_+\;|\;\lambda\theta\right) = \mathcal{P}\left(\theta_+\;|\;\lambda\theta\phi\right), \tag{2.5}$$

and is therefore explicitly local, i. e., in the sense of [4]. Type-II theories may agree with quantum mechanical prediction for single-particle statistics, but ultimately deviate from standard quantum predictions for EPR.

For instance, Bohm mechanics[3] is a type-I theory in which the role of λ is played by the (initial values of) coordinates and angular variables of the two particles, plus the wave function in configuration space (now interpreted as a real field). This wave function determines a so-called "quantum potential", mediating the interaction of the two particles even after they have been separated. Thus, the dynamics of particle "1"'s passage through its SGA become explicitly dependent on the choice of orientation ϕ for particle "2"'s SGA.

Here, we are interested on an example given by Belinfante of an HV_2 in his survey [9]. As a first building step, Belinfante considers a prototype single-particle theory based on the following situation: suppose a single particle successfully passes a first SGA filter, tilted from the z-axis by an angle α, and is subsequently passed through a second SGA filter tilted by an angle θ. Given the first passing, quantum mechanics predicts that the second passing will occur with a probability

$$\mathcal{P}_{\mathrm{QM}}(\theta_+ \mid \alpha_+) = \langle \theta_+ | \hat{\Pi}(\alpha_+) | \theta_+ \rangle = \frac{1}{2}[\,1 + \cos(\theta - \alpha)\,]\,. \tag{2.6}$$

At this stage, it should be clear that α is an external macroscopic parameter. However, one could choose to think of it as the prototype of some internal polarization variable of the particle on which the passage probability depends. The single particle theory therefore consists of (a) assuming that there exists a hidden internal polarization angle α, and (b) that the microscopic passage probability is

$$\pi(\theta_+|\alpha) = \frac{1}{2}[\,1 + \cos(\theta - \alpha)\,]\,. \tag{2.7}$$

The next step is to construct a local realistic theory for EPR-type correlations based on the single-particle hidden variables. Here, we assume that the generic hidden variable λ consists of two internal polarization angles α and β, for particles "1" and "2", respectively, and such that the microscopic passage probability $\pi(\theta_+|\phi)$ for particle "1" depends only on α according to (2.7), and that the microscopic passage probability for particle "2", $\pi(\phi_+|\beta)$, depends only on β in a similar manner. To account for the correlations, it is assumed that the hidden angles α and β are determined at the source of emission according to some joint distribution $\rho(\alpha, \beta)$, which within HV_2 plays the role of the singlet state $|\Psi\rangle$. Thus, in this example of HV_2, the canonical form (2.3) translates to

$$\mathcal{P}_{\mathrm{HV}_2}(\theta_+ \phi_+ \mid \Psi) = \int \mathrm{d}\alpha\, \mathrm{d}\beta \rho(\alpha\beta)\pi(\theta_+|\alpha)\pi(\phi_+|\beta). \tag{2.8}$$

Now, recalling from Eq. (2.2) that the correlation is maximum when the angles θ and ϕ lie π degrees from each other, a reasonable choice for $\rho(\alpha\beta)$ is then

$$\rho(\alpha\beta) = \frac{1}{2\pi}\,\delta(\alpha - \beta - \pi). \tag{2.9}$$

With this choice, the predicted joint passage probability from HV_2 is

$$\begin{aligned}\mathcal{P}_{\mathrm{HV}_2}(\theta_+ \phi_+ \mid \Psi) &= \int \frac{\mathrm{d}\alpha\,\mathrm{d}\beta}{2\pi}\,\delta(\alpha - \beta - \pi) \\ &\quad \times \frac{1}{2}[1 + \cos(\theta - \alpha)]\,\frac{1}{2}[1 + \cos(\phi - \beta)] \\ &= \frac{1}{4}\int \frac{\mathrm{d}\alpha}{2\pi}[1 + \cos(\theta - \alpha)]\,[1 - \cos(\phi - \alpha)] \\ &= \frac{1}{4}\left[1 - \frac{1}{2}\cos(\theta - \phi)\right].\end{aligned} \tag{2.10}$$

Clearly, the choice (2.9) cannot yield the correct quantum mechanical joint passage probability, i. e. (2.2).

In his book, Belinfante argues that the failure of this simple HV_2 example need not be too surprising, as in actuality there is no physical basis for the hidden polarization variables: "The polarization [spin] hidden-variable here introduced, is, of course, a quantity which does not exist in quantum theory. ...in quantum theory, no such thing as α even exists." [7]. Still, it is quite interesting to note that assumption (2.9) does in fact lead to a correlation curve that is qualitatively not too different in shape from the correct quantum mechanical result. We are thus prompted to investigate whether it may be possible to reproduce the observed quantum joint passage probability from some other form similar to that of (2.8), that is, by preserving the assumptions of (a) hidden polarization variables and (b) local microscopic passage distributions.

III. Explicit Locality via Wigner Distributions

Considerable insight towards this end is gained from a treatment in terms of spin quasi-distribution functions analogous to Wigner functions in phase space [10 - 12]. This approach allows us to automatically cast the quantum mechanical joint passage probability (2.2) in the form suggested by the hidden-variables approach, (2.3). Furthermore, we shall see that the quasi-distribution approach naturally defines a hidden angular variable α, similar to that of Belinfante's HV_2 example.

We start with the single particle theory, by considering the distribution of outcomes from an SGA measurement of the spin along a certain direction, say z. Denoting the single particle initial state as $|\Psi\rangle$, this distribution may be written in the form

$$\tilde{\rho}(m_z) = \delta(m_z - 1)\,\langle\Psi|\hat{\Pi}(z_+)|\Psi\rangle + \delta(m_z + 1)\langle\Psi|\hat{\Pi}(z_-)|\Psi\rangle = \langle\Psi|\delta(m_z - \hat{\sigma}_z)|\Psi\rangle. \tag{3.1}$$

Next, in order define an angular variable in the xz plane, suppose that there exists a joint distribution $\tilde{\rho}(m_z m_x)$ such that $\mathcal{P}_{QM}(\,m_z \mid \Psi\,)$ is its marginal distribution, i. e.

$$\tilde{\rho}(m_z) = \int \mathrm{d}\, m_x \tilde{\rho}(m_z m_x). \tag{3.2}$$

It is true that such a joint distribution cannot be interpreted literally as the outcome distribution for joint measurements of $\hat{\sigma}_x$ and $\hat{\sigma}_z$, as the two operators correspond to incompatible measurements; however, the concept could still be meaningful outside the context of measurement.

A suggestive choice that will suffice for our purposes is

$$\tilde{\rho}(m_z m_x) = \langle\Psi|\delta(m_x - \hat{\sigma}_x)\delta(m_z - \hat{\sigma}_z)|\Psi\rangle, \tag{3.3}$$

although it is important to emphasize that other choices are possible, depending on operator ordering. What we wish to do now is consider a state $|\Psi\rangle = |\chi_+\rangle$ describing a polarization along some direction in the xz plane and tilted from z by an angle χ. In the case of spin "up", i. e. $\chi = 0$, the joint distribution is easily computed:

$$\tilde{\rho}(m_x m_z) = \langle z_+|\delta(m_x - \hat{\sigma}_x)\delta(m_z - \hat{\sigma}_z)|z_-\rangle = \frac{1}{2}\left[\delta(m_x - 1) + \delta(m_x + 1)\right]\delta(m_z - 1), \;\; \chi = 0. \tag{3.4}$$

Transforming to polar coordinates, $m_z = m\cos\alpha$, $m_x = m\sin\alpha$, we have

$$\tilde{\rho}(\alpha, m) = \left|\frac{(m_x, m_z)}{(\alpha, m)}\right| \tilde{\rho}(m_z(\alpha, m), m_x(\alpha, m)) = \frac{1}{2}\left[\delta(\alpha - \pi/4) + \delta(\alpha + \pi/4)\right]\delta(m - \sqrt{2}). \tag{3.5}$$

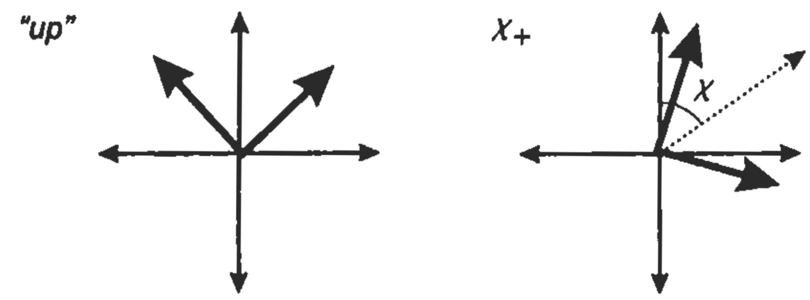

Fig. 1. Interpretation of the spin "up" and χ_+ states in terms of the hidden polarization angle α. The thick arrows represent equally probable directions.

Finally, marginalizing with respect to m, we find

$$\tilde{\rho}(\alpha) = \int \mathrm{d}\, m\, \tilde{\rho}(\alpha, m) = \frac{1}{2}\left[\delta(\alpha - \pi/4) + \delta(\alpha + \pi/4)\right], \;\; \chi = 0. \tag{3.6}$$

The generalization for arbitrary χ follows from noting that an active rotation of the state, i. e. $|\chi_+\rangle$ is equivalent to shifting the origin of the angle α by $-\chi$, i. e.,

$$\tilde{\rho}(\alpha) = \frac{1}{2}\left[\delta(\alpha - \chi - \pi/4) + \delta(\alpha - \chi + \pi/4)\right]. \tag{3.7}$$

Equation (3.7), illustrated schematically in Fig. 1, serves as an effective (single-particle) quantum mechanical distribution for the quasi-distribution parameter α.

The final step in the single-particle theory is to determine the spin-distribution representation of a given projection operator $\hat{\Pi}(\theta_+)$. For this, we expand the observed single-particle passage probability as

$$\begin{aligned}
\mathcal{P}_{QM}(\,\theta_+ \mid \Psi\,) &= \langle\Psi|\hat{\Pi}(\theta_+)|\Psi\rangle \\
&= \frac{1}{2}\langle\Psi|\left[1 + \hat{\sigma}_z\cos\theta + \hat{\sigma}_x\sin\theta\right]|\Psi\rangle \\
&= \int \mathrm{d}^2 m\, \langle\Psi|\delta(m_z - \sigma_z)\left[1 + \sigma_z\cos\theta + \sigma_x\sin\theta\right] \\
&\quad \times\; \delta(m_z - \sigma_z)|\Psi\rangle \\
&= \int \mathrm{d}^2 m\, \tilde{\rho}(m_x m_z)\left[1 + m_z\cos\theta + m_x\sin\theta\right] \\
&= \int \mathrm{d}\,\alpha\, \tilde{\rho}(\alpha)\,\frac{1}{2}\left[1 + \sqrt{2}\cos(\theta - \alpha)\right].
\end{aligned} \tag{3.8}$$

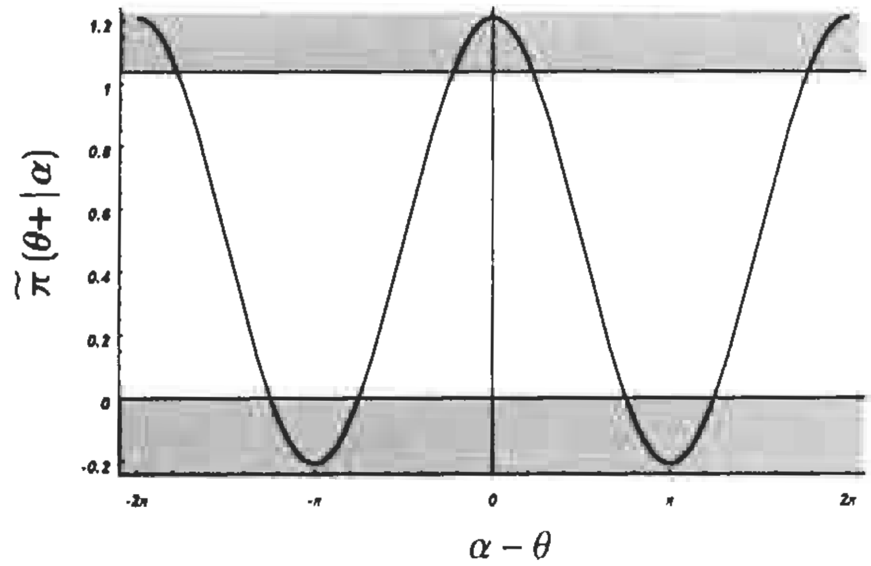

Fig. 2. Angular dependence of $\tilde{\pi}(\theta_+|\alpha)$, the quasi-distribution representation of the projector $\hat{\Pi}(\theta_+)$, and regions where it fails to yield acceptable probabilities (shaded).

Thus we see that in the single-particle case, the quantum mechanical passage probability may be written as

$$\mathcal{P}_{QM}(\theta_+\,|\,\Psi) = \int \mathrm{d}\alpha\,\tilde{p}(\alpha)\,\tilde{\pi}(\theta_+|\alpha), \tag{3.9}$$

where

$$\tilde{\pi}(\theta_+|\alpha) = \frac{1}{2}\left[1+\sqrt{2}\cos(\theta-\alpha)\right]. \tag{3.10}$$

Note that although $\tilde{\pi}(\theta_+|\alpha)$ is similar to a microscopic passage probability in hidden variables theory, strictly speaking it is a spin quasi-distribution representation of the projector $\hat{\Pi}(\theta_+)$. A simple look at this $\tilde{\pi}(\theta_+|\alpha)$ (Fig. 2) shows that this function cannot be interpreted in terms of standard probabilities, as it exceeds the allowed range of probability $[0,1]$ in two regions, specifically

$$\begin{aligned}&\text{(a) } |\alpha-\theta| > 3\pi/4, \quad \text{where } \tilde{\pi}(\theta_+|\alpha) < 0,\\ &\text{(b) } |\alpha-\theta| < \pi/4, \quad \text{where } \tilde{\pi}(\theta_+|\alpha) > 1.\end{aligned} \tag{3.11}$$

We avoid here the delicate subject of interpreting these results in terms of extended probabilities [13, 14].

Let us then turn to the construction of the spin quasi-distribution EPR theory. For this, we wish to express the joint passage probability in local form as

$$\mathcal{P}_{QM}(\theta_+\,\phi_+\,|\,\Psi) = \int \mathrm{d}\alpha\,\mathrm{d}\beta\,\tilde{p}(\alpha\beta)\,\tilde{\pi}(\theta_+|\alpha)\,\tilde{\pi}(\phi_+|\beta), \tag{3.12}$$

where $\tilde{\pi}(\theta_+|\alpha)$ and $\tilde{\pi}(\phi_+|\beta)$ are given by expressions of the form (3.10), and where $\tilde{p}(\alpha\beta)$ is a joint hidden

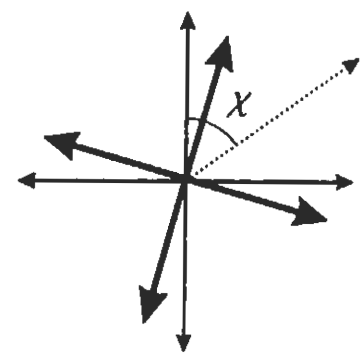

Fig. 3. Interpretation of the singlet distribution for α when the singlet is parameterized in terms of the angle χ.

quasi-distribution appropriate to the singlet state. This distribution is constructed in a manner similar to that of the single particle case, namely, by looking at the joint distribution

$$\begin{aligned}\tilde{p}(\vec{m}_1\vec{m}_2) = \langle\Psi|\delta(m_x^{(1)}-\sigma_x^{(1)})\,\delta(m_z^{(1)}-\sigma_z^{(1)})\\ \times\,\delta(m_x^{(2)}-\sigma_x^{(2)})\,\delta(m_z^{(2)}-\sigma_z^{(2)})|\Psi\rangle\end{aligned} \tag{3.13}$$

and then marginalizing with respect to $|\vec{m}_1|$, $|\vec{m}_2|$. A key point to note is that in the case of a singlet state, we are entirely free to parameterize as

$$|\Psi(\chi)\rangle = \frac{1}{\sqrt{2}}|\chi_+\chi_-\rangle - \frac{1}{\sqrt{2}}|\chi_-\chi_+\rangle, \tag{3.14}$$

where χ stands for some arbitrary angle in the xz plane. It is then a matter of computation to show that

$$\begin{aligned}\tilde{p}_\chi(\alpha\beta) = \frac{1}{4}\sum_{n=0}^{3}\delta\Big[\alpha-\chi-\Big(\frac{\pi}{4}+n\frac{\pi}{2}\Big)\Big]\\ \times\,\delta\Big[\beta-\chi-\Big(\frac{\pi}{4}-n\frac{\pi}{2}\Big)\Big],\end{aligned} \tag{3.15}$$

which, incidentally, can also be written in the form

$$\tilde{p}_\chi(\alpha\beta) = \delta(\beta-\alpha-\pi)\times\frac{1}{4}\sum_{n=0}^{3}\delta\Big[\alpha-\chi-\Big(\frac{\pi}{4}+n\frac{\pi}{2}\Big)\Big], \tag{3.16}$$

showing that, while it shares with Belinfante's $\tilde{p}(\alpha\beta)$ the spin anti-correlation factor $\delta(\beta-\alpha-\pi)$, it also includes an explicit distribution of directions determined by the arbitrary angle χ (illustrated in Figure 3). Note therefore that, while χ is irrelevant as a parameter of the singlet state, i.e. $|\Psi(\chi)\rangle = |\Psi(\chi')\rangle$, the mapping from the singlet state to the corresponding spin distribution is not unique: different choices of χ map to different spin-distributions.

In spite of this dependence, one can nevertheless see upon substituting (3.16) into (3.12), that the correct quantum mechanical prediction for the joint passage probability is obtained,

$$\mathcal{P}_{\mathrm{QM}}\big(\,\theta_+\,\phi_+\,|\,\Psi(\chi)\big) = \int \mathrm{d}\,\alpha\,\mathrm{d}\,\beta\,\frac{1}{4}\sum_{n=0}^{3}\delta(\alpha-\chi-(\pi/4+n\pi/2)\,\delta(\beta-\chi-(\pi/4-n\pi/2)) \times \frac{1}{2}\left[1+\sqrt{2}\cos(\theta-\alpha)\right]\frac{1}{2}\left[1+\sqrt{2}\cos(\phi-\beta)\right] = \frac{1}{4}\left[1-\cos(\theta-\phi)\right]. \tag{3.17}$$

After all, we are still doing quantum mechanics, but in a different representation. Note therefore that what essentially guarantees the independence of the final result on the singlet parameterization angle χ is the cross-shape (i.e. Z_4) symmetry of $\tilde{\rho}(\alpha\beta)$.

The above symmetry in fact provides an interesting connection with Belinfante's HV_2 theory, as one could suggest that χ plays the role of an additional hidden angle the initial distribution of which is beyond our control. Consequently, as there is no reason to prefer one value of χ over another, it becomes natural to define an "unbiased" joint hidden variable probability $\mathcal{P}_{\mathrm{QM}}(\,\alpha\beta\,|\,\Psi\,)$, obtained after averaging over a uniform *a-priori* distribution for χ:

$$\langle\rho(\alpha\beta)\rangle_\chi = \int\frac{\mathrm{d}\,\chi}{2\pi}\,\rho_\chi(\alpha\beta) = \frac{1}{2\pi}\delta(\alpha-\beta-\pi). \tag{3.18}$$

As we can see, the "unbiased" quasi-distribution for the singlet state proves to be nothing more than Belinfante's joint hidden-variable distribution.

In conclusion, the quasi-distribution approach allows us to draw a number of interesting observations: first, it becomes clear that in fact that both the hidden parameter angle α as well as an object reminiscent of a *local* passage probability dependent on α quite naturally emerge from quantum mechanics itself, from the viewpoint of spin quasi-distribution functions; secondly, in the case of EPR, the quasi-distribution approach suggests a "free" hidden parameter, the singlet parameterization angle χ; finally, in terms of this free angle, it is possible to interpret Belinfante's hidden variable HV_2 distribution as an "unbiased" distribution based on equal a-priori probabilities for χ.

IV. HV3: Another Type of Hidden Variables Theory

Thus, we have seen that Belinfantes's HV_2 example theory is more similar in form to the quantum distribution approach than may have heretofore been suspected. This similarity leads us to explore another type of theory that, while closely related to both of these treatments, nevertheless succeeds in reproducing the correct quantum mechanical results with local microscopic passage probabilities (albeit at the expense of other causal elements related to non-locality). As we saw in the quasi-distribution approach, parameterization of the singlet state in terms of hidden variables involves an arbitrary angle χ. Is it then possible to exploit this freedom so that for some values of χ we can find appropriate joint hidden-variables distributions and positive-definite local passage probabilities yielding the correct quantum mechanical joint passage probabilities?

We first take our lead from the single-particle quasi-distribution treatment, where, as we have seen, the polarization α naturally arises. Recall that in this treatment, an initial state with orientation angle χ corresponds to a quasi-distribution function with equal weights on two possible orientations for α, namely $\alpha = \chi \pm \pi/4$. We can therefore see that χ plays the role of the distribution's "mean orientation" $\overline{\alpha}$. But one also notes that if this mean angle is replaced into the HV_2 microscopic passage probability, $\pi(\theta_+|\alpha) = \frac{1}{2}\,[1+\cos(\theta-\alpha)]$, we obtain the correct observed single-passage probability for a particle prepared in the initial state $\Psi(\chi)$, i.e.

$$\pi(\theta_+|\alpha)\big|_{\alpha=\overline{\alpha}=\chi} = \frac{1}{2}\left[\,1+\cos(\theta-\chi)\,\right] = \mathcal{P}_{\mathrm{QM}}(\theta_+|\,\theta\chi_+)\,. \tag{4.1}$$

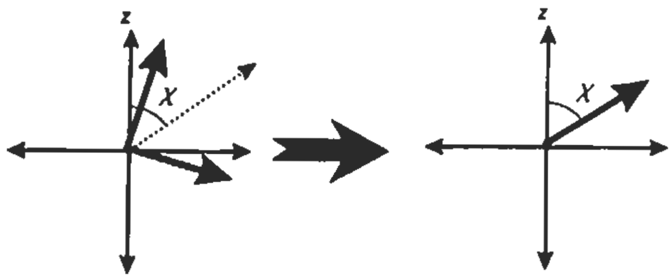

Fig. 4. Conversion of a single-particle quantum distribution function into an HV_3 distribution.

This suggests another hidden variables theory, call it HV_3, in which we keep from HV_2 the microscopic passage probability, $\pi(\theta_+|\alpha = \overline{\alpha}) = \frac{1}{2}[1 + \cos(\theta - \alpha)]$, but replace the single-particle hidden variable distribution $\rho(\alpha)$ by a delta function on χ, the "quasi-distribution mean angle":

$$\rho'(\alpha) = \delta(\alpha - \chi), \tag{4.2}$$

as schematically depicted in Figure 4.

Construction of the joint hidden-variable distribution for the EPR singlet now follows in a similar manner. For this, we go back to the schematic idea of the joint quasi-distribution depicted in Fig. 3 and its Z_4 symmetry. The four directions are now be broken up into two sets of directions, each corresponding to "up" or "down" (i. e., $\chi\pm$). Each of these pairs is then "collapsed" into its respective mean angle, as shown in Figure 5. The resultant joint hidden-variable distribution therefore becomes the Belinfante anticorrelation, but now multiplied by a pair of deltas representing "up" and "down" according to our HV_3 interpretation, i. e.

$$\rho'_\chi(\alpha\beta) = \delta(\alpha - \beta - \pi) \times \frac{1}{2}[\delta(\alpha - \chi) + \delta(\alpha - \chi - \pi)]. \tag{4.3}$$

The net effect of this conversion is then to break the Z_4 symmetry of the original quasi-distribution, in which case we should expect an explicit dependence on χ in our final results.

We investigate this further by calculating the predicted correlations for a given value of the singlet parameterization angle χ. The joint passage probability now becomes

$$\mathcal{P}_{HV_3}(\theta_+\phi_+|\Psi(\chi)) = \int \mathrm{d}\,\alpha\,\mathrm{d}\,\beta\,\rho'_\chi(\alpha\beta)\pi(\theta_+|\alpha)\,\pi(\phi_+|\beta)$$

$$= \frac{1}{2} \times \frac{1}{2}[1 + \cos(\theta - \chi)]\,\frac{1}{2}[1 + \cos(\phi - \chi + \pi)]$$

$$+ \frac{1}{2} \times \frac{1}{2}[1 + \cos(\theta - \chi + \pi)]\,\frac{1}{2}[1 + \cos(\phi - \chi)]$$

$$= \frac{1}{4}[1 - \cos(\theta - \chi)\cos(\phi - \chi)]. \tag{4.4}$$

So indeed we see that our result now depends explicitly on the parameterization angle χ and thus, for arbitrary values of χ, differs from the quantum mechanical prediction (2.2). The interesting point, however, is that there do exist two values of χ, namely

$$\chi = \theta \quad \text{and} \quad \chi = \phi, \tag{4.5}$$

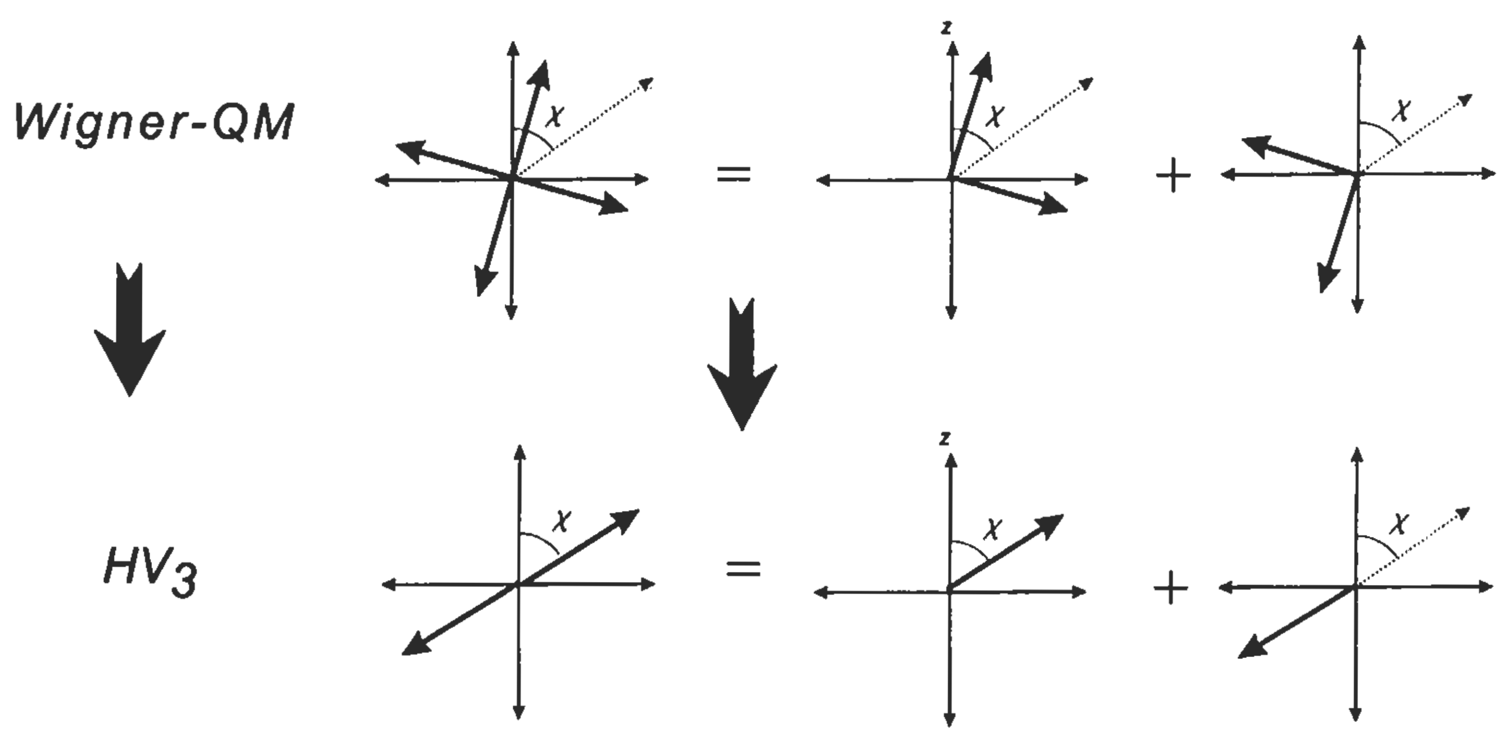

Fig. 5. Conversion of the singlet quantum distribution function into an HV_3 distribution.

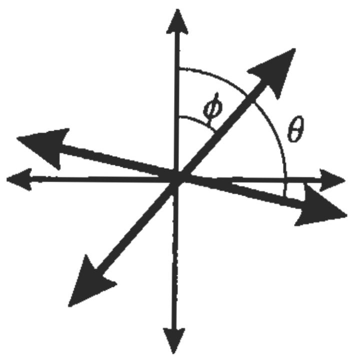

Fig. 6. "Unbiased" HV_3 representation of the hidden variables distribution for the singlet state

for which $\mathcal{P}_{HV_3}(\theta_+ \phi_+ \mid \Psi(\chi))$ indeed coincides with the quantum mechanical result $\mathcal{P}_{QM}(\theta_+ \phi_+ \mid \Psi)$.

Thus, we are in the position of completing our HV_3 theory by letting the "hidden" singlet parameterization angle χ, which is a free parameter in the quasi-distribution approach, to instead take either of two fixed values, θ or ϕ. Since neither of these two possibilities can be favored over the other *a priori*, we again follow the spirit of hidden variables by assuming that they are both equally likely. Thus, we obtain an "unbiased" hidden-variables distribution which is an equal-proportion admixture of the two angles, i. e.

$$\rho'(\alpha\beta) = \delta(\alpha-\beta-\pi)\frac{1}{4}\Big[\delta(\alpha-\theta)+\delta(\alpha-\theta-\pi) + \delta(\alpha-\phi)+\delta(\alpha-\phi-\pi)\Big], \quad (4.6)$$

as illustrated schematically in Figure 6.

Note therefore that in contrast to HV_1 and HV_2 theories, in the HV_3 approach the joint hidden-variables distribution is in fact determined not only by the initial state $|\Psi\rangle$, but also by two macroscopic parameters, the orientations of the two SGA's. It is precisely by allowing this dependence of the joint hidden variables on the experimental parameters θ, ϕ that we can maintain locality and positivity in the passage probabilities and still reproduce the observed quantum mechanical correlation joint passage probabilities.

V. An Expanded Classification of Hidden Variables Theories

Before discussing the possible implications of an HV_3-type theory, it will be important to show how such theories do have a natural place in a more general formal classification of hidden-variables theories. The classification follows from a detailed derivation of the general probabilistic forms for the joint passage probability which can be derived using standard probability calculus and the hidden variables assumption. We can then show that there is in fact a richer taxonomy of hidden variables theories than pre-supposed in Belinfante's classification.

Now, in all cases considered so far, we were investigating the joint passage probability given three relevant microscopic conditions, namely the preparation of the initial state $|\Psi\rangle$, and the two orientations of the SGAs, θ, ϕ. Thus, we are investigating the probability $\mathcal{P}(\theta_+ \phi_+ \mid \theta\phi\Psi)$, where we now emphasize all relevant macroscopic conditions. Now recall the sum and product rules of probability:

$$\mathcal{P}(A \mid B) = \sum_i \mathcal{P}(X_i \mid B), \text{ sum rule}, \quad (5.1)$$

$$\mathcal{P}(AB \mid C) = \mathcal{P}(A \mid BC)\,\mathcal{P}(B \mid C), \text{ product rule},$$

where the propositions X_i are disjoint and exhaustive. Application of these two rules allows us to expand $\mathcal{P}(\theta_+ \phi_+ \mid \theta\phi\Psi)$ in terms of an exhaustive set of hidden variables $\{\lambda\}$ as

$$\mathcal{P}(\theta_+ \phi_+ \mid \theta\phi\Psi) = \int \mathrm{d}\lambda\, \mathcal{P}(\lambda \mid \theta\phi\Psi) \times \mathcal{P}(\theta_+ \phi_+ \mid \theta\phi\lambda\Psi)\,. \quad (5.2)$$

As before, we take the hidden variables assumption to be that there exists a set of variables λ, (again of unspecified dimensionality), and satisfying the following conditions (i. e., in the context of EPR):

1.) that they are made available to both particles at their common emission event (e. g. λ may stand for the initial values of some time dependent set of hidden variables),

2.) that they "complete the wave-function", in other words, that $|\Psi\rangle$ becomes irrelevant as a condition if it is specified in conjunction with λ,

3.) that they constitute the minimal *microscopic* conditions by which each particle's passage through one SGA apparatus becomes independent of the other particle's passage.

Given these assumptions on the hidden variables, the joint microscopic passage probability factors out as

$$\mathcal{P}(\theta_+ \phi_+ \mid \theta\phi\,\lambda\,\Psi) = \mathcal{P}(\theta_+ \mid \theta\phi\,\lambda)\ \mathcal{P}(\phi_+ \mid \theta\phi\,\lambda)\,. \quad (5.3)$$

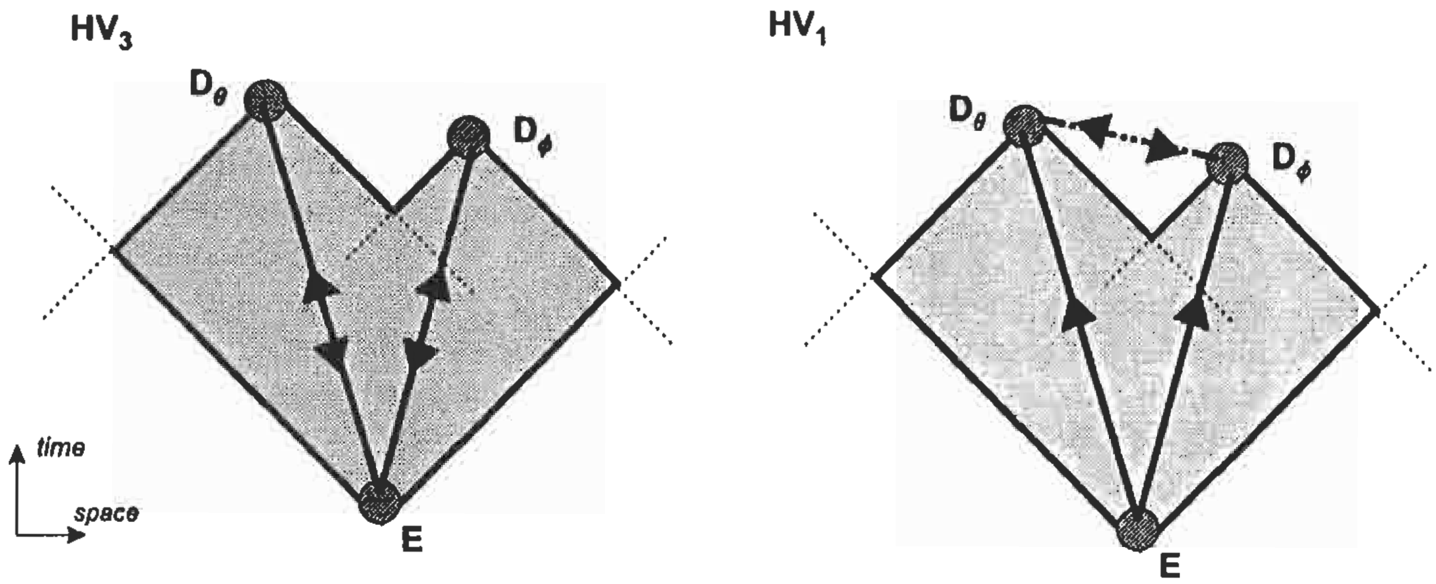

Fig. 7. Arrows of microscopic causation in HV_3 vs. HV_1 theories. The causal region (shaded) is defined by the forward light cone emanating from the emission event E, and the backward cones of the two detection events D_θ and D_ϕ. In HV_3 all causal mechanisms lie within this region along time-like curves. In contrast, the space-like segment $D_\theta D_\phi$, representing action-at-a-distance lies outside the causal region.

Thus, we arrive at a canonical form for the joint passage probability

$$\mathcal{P}(\theta_+ \phi_+ | \theta\phi\Psi) = \int \mathrm{d}\lambda\, \mathcal{P}(\lambda | \theta\phi\Psi) \times \mathcal{P}(\theta_+ | \theta\phi\lambda)\, \mathcal{P}(\phi_+ | \theta\phi\lambda), \tag{5.4}$$

which, of course, is the form (2.3) already presented in Section II.

Based on this form, it is now straightforward to classify hidden variables theories into at least three kinds, thus amplifying Belinfante' s classification. In the first two kinds, we assume that the hidden variables' distribution is exclusively determined by the wave function Ψ, and is therefore independent of the SGA settings, i. e.

$$\mathcal{P}(\lambda | \theta\phi\Psi) = \mathcal{P}(\lambda | \Psi). \tag{5.5}$$

The difference between HV_1 and HV_2 has to do instead with the dependencies of the microscopic passage probabilities: Whereas

$$\mathcal{P}_{HV_2}(\theta_+ | \theta\phi\lambda) = \mathcal{P}_{HV_2}(\theta_+ | \theta\lambda), \tag{5.6}$$

in HV_2, the orientation ϕ of the other SGA is not an irrelevant condition in $\mathcal{P}_{HV_1}(\theta_+ | \theta\phi\lambda)$, making it an "action-at-a-distance" theory. On the other hand, HV_3 departs form HV_1 and HV_2 in that the SGA setting is not irrelevant in the initial distribution of the hidden variables, whereas the microscopic passage probabilities are local, as in HV_2. In summary then, the classification is as

$$\mathcal{P}(\theta_+ \phi_+ | \theta\phi\Psi) = \tag{5.7}$$

$$\int \mathrm{d}\lambda \begin{cases} \mathcal{P}(\lambda | \Psi) \times \mathcal{P}(\theta_+ | \theta\phi\lambda)\, \mathcal{P}(\phi_+ | \theta\phi\lambda), & HV_1, \\ \mathcal{P}(\lambda | \Psi) \times \mathcal{P}(\theta_+ | \theta\lambda)\, \mathcal{P}(\phi_+ | \phi\lambda), & HV_2, \\ \mathcal{P}(\lambda | \theta\phi\Psi) \times \mathcal{P}(\theta_+ | \theta\lambda)\, \mathcal{P}(\phi_+ | \phi\lambda), & HV_3, \end{cases}$$

Obviously, the classification is not exhaustive; as additional theories combining elements from the three above may be envisaged. What is useful about these forms is that each one captures a distinct type of mechanism by which the correlations are presumably generated. Explicitly, HV_1 theories are "action-at-a-distance" involving superluminal transfer of microscopic information, HV_2 are local-realistic with no violation of our common definition of causality; finally, HV_3 theories may be regarded as local but "retro-causal", as we shall now discuss in the following section.

VI. Causal Implications of HV_3 Theories

In our HV_3 theory, the break with our ordinary preconceptions of causality comes from the fact that the initial hidden-variables distribution, (4.6), becomes explicitly dependent on the settings θ and ϕ of the two SGA measurements. What is important to note then is that while the outcome (i. e. θ_+ vs. θ_-) of an SGA measurement is determined microscopically, the actual SGA orientations, θ and ϕ, are *choices* that are entirely left open to the experimentalist. Now, as such choices can be made after the particles have left their common source of emission, it follows that the initial hidden variables distribution already carries the imprint of its macroscopic control parameters, *even before some of those parameters have actually been decided upon*. What is called into question within an HV_3 theory is therefore the microscopic time arrow of causation. In other words, in an HV_3 theory, macroscopic events (i. e., "choice of parameter") in the future may be causes of microscopic events (i.e. "establishment of an HV distribution") in the past. Thus, the term "quantum non-locality", when seen in the light of an HV_3 theory, does not convey the idea of "action-at-a-distance", but rather of "microscopic retro-causation".

Note, however, that retro-causation does not violate relativistic locality (in contrast to action-at-a-distance), as the hidden-variables distribution is established at an event that lies within the backward light-cones of both observation events. All information transmission happens along time-like, albeit sometimes past-oriented, curves, as opposed to space-like curves as in HV_1 (Figure 7). Thus, while the net effect of an HV_3 is that between the two final observation events an *indirect* casual connection is established (i. e., mediated by the prior emission event), this violation of the "spirit of relativity" nevertheless happens through microscopic mechanisms which are always time-like, in consistency with relativistic space-time geometry.

That quantum mechanics may have retro-causal implications at a more fundamental level is a questions that has already been explored, with particular emphasis on EPR, by Costa de Beauregard and others [15]. Also noted by one of us together with Bergmannn and Lebowitz [16], is the fact that the mathematical structure of quantum mechanics already contains the elements for its own interpretation in terms of a time-symmetric theory at the microscopic level. This idea has been further elaborated in recent years in a "Two-Vector" formulation of quantum mechanics [17] and a new type of observable physical property, the "weak" value of quantum observables [18]. It remains to be seen, however, as to whether one can encompass all quantum predictions from a consistent hidden-variables theory with a microscopic arrow of time running in both directions, similar perhaps to Wheeler and Feynmann absorber-theory of radiation [19].

VI. Conclusion

We have argued in this paper that the standard folklore that "quantum mechanics is non-local" deserves closer examination when the locality of quantum mechanics is assessed against the sort of hidden variables theories that can successfully account for EPR-type correlations. Indeed, we have shown in this paper that there exist hidden-variables theories, here termed HV_3 theories, which do not invoke superluminal action-at-a-distance as a basic mechanism, but rather retro-causation.

Now, whether backward causation is a preferable mechanism within the hidden-variables context to action-at-distance is a question that lies beyond the scope of this paper and is ultimately left open to the reader. Nevertheless, we believe the question is interesting enough to merit some consideration. A reassessment of "quantum non-locality" from the wider perspective suggested here may serve to stress the deeper significance of the role that time plays in quantum theory.

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