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2010 J. Phys. A: Math. Theor. 43 354012

(http://iopscience.iop.org/1751-8121/43/35/354012)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **43** (2010) 354012 (11pp) doi:10.1088/1751-8113/43/35/354012

Nonlocal quantum dynamics of the Aharonov–Bohm effect

T Kaufherr[1] **and Y Aharonov**[1,2]

[1] Tel Aviv University, School of Physics and Astronomy, Tel Aviv 69978, Israel
[2] Chapman University, Schmid College of Sciences, Orange, CA 92866, USA

E-mail: trka@post.tau.ac.il

Received 28 January 2010, in final form 2 June 2010
Published 12 August 2010
Online at stacks.iop.org/JPhysA/43/354012

Abstract
The answer to the question 'when does the AB effect occur?' is elusive, for in every gauge the relative phase between the two wave packets evolves differently. Considering gauge-invariant modulo momentum, i.e., the displacement operator $e^{\frac{i}{\hbar}(\vec{p}-\frac{e}{c}\vec{A})\cdot\vec{L}}$ or its Hermitian counterpart $\cos\frac{1}{\hbar}\left(\vec{p}-\frac{e}{c}\vec{A}\right)\cdot\vec{L}$, it is found that when the external particle's two wave packets become co-linear with the solenoid, an abrupt nonlocal exchange of the conserved quantity occurs. Using the Heisenberg picture, we show that this exchange is responsible for the shift of the interference pattern of the AB effect. We also describe a gedanken experiment that shows that our prediction can, in principle, be tested experimentally. Finally, this exchange gives new insight into the famous two-slit quantum interference experiment.

PACS number: 03.65.−w

1. Introduction

Below we describe a nonlocal dynamical exchange of a gauge-invariant quantity that is responsible for the shift of the interference pattern of the AB effect.

The AB set-up [1] consists of a very long solenoid enclosing a flux Φ, surrounded by an impenetrable barrier, and a charged external particle confined to the field-free region beyond the barrier. Classically as well as quantum mechanically, no local measurement performed on the external particle will disclose the presence of the solenoid, since the fields are the only gauge-invariant locally observable quantities. But with a quantum particle, prepared in a superposition of two wave packets moving on the two sides of the solenoid, a shift of the interference pattern which is proportional to the flux is observed, thus revealing the presence of the solenoid. We regard this shift of the interference pattern as a nonlocal phenomenon, since the external particle is confined to the non-simply connected region which does not contain the solenoid. Classically, such an experiment is of course impossible, since a classical particle cannot move on both sides of the solenoid at once.

There is no way to describe quantum interference in the Heisenberg picture, the reason being that in this picture the state of the system is fixed to be the initial state. Thus there is no way to account for changes of the relative phase of a superposition. This gap is filled by modulo momentum [2, 3], represented here by the displacement operator $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\vec{p}\cdot\vec{L}}$ or its Hermitian counterpart $\cos\frac{\vec{p}\cdot\vec{L}}{\hbar}$. Modulo momentum is the dynamical variable which describes quantum interference in the Heisenberg picture. As such, it is a key to the understanding of the quantum two-slit interference experiment, as well as to the solution of the problem of which-way measurements. It is a conserved quantity and changes under a nonlocal equation of motion.

The AB effect is a semi-classical electromagnetic phenomenon (since only the external particle, but not the field, is quantized) as well as an inherent feature of all gauge theories. Therefore we focus on the gauge-invariant modulo momentum $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\vec{v}\cdot\vec{L}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}(\vec{p}-\frac{e}{c}\vec{A})\cdot\vec{L}}$. We find that an abrupt, nonlocal change of the gauge-invariant modulo momentum of the external particle occurs when the line connecting the two wave packets crosses the solenoid [3]. Equivalently, one can say that the velocity distribution of the external particle changes abruptly as it passes by the solenoid. Moreover, we find that this change of the velocity distribution is the only gauge-invariant change occurring throughout the entire evolution of the external particle. Thus it is the only candidate to explain the shift of the interference pattern.

Using the Heisenberg picture we show that the change of the velocity distribution manifests itself later on as the shift of the interference pattern of the external particle. In other words, the shift of the interference pattern in the AB effect can always be traced back to a change of the velocity distribution occurring when the charged particle passes by the solenoid or to a nonlocal exchange of the gauge-invariant modulo momentum.

Finally, we describe a gedanken experiment that shows that our prediction can, in principle, be tested experimentally.

2. Modulo momentum

Let us consider a particle in a superposition of two non-overlapping wave packets of width Δx each, with a relative phase α and separated by a distance L:

$$\Psi_\alpha = \psi_1 + \mathrm{e}^{\mathrm{i}\alpha}\psi_2 = \psi(x) + \mathrm{e}^{\mathrm{i}\alpha}\psi(x-L). \tag{1}$$

In search of a dynamical variable describing quantum interference, we first note that the average position and momentum $\langle x\rangle$, $\langle p\rangle$ do not depend on the relative phase α. Thus also the averages of all polynomials in x and in p do not depend on α.[3] But the average of the displacement operator does depend on the relative phase[4]:

$$\langle\Psi_\alpha|\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}pL}|\Psi_\alpha\rangle = \frac{\mathrm{e}^{\mathrm{i}\alpha}}{2}, \tag{2}$$

[3] Evaluating $\langle\Psi_\alpha|ax^n + bx^n|\Psi_\alpha\rangle$ note that only the cross terms of the form $\langle\psi_1|ax^n + bx^n|\,\mathrm{e}^{\mathrm{i}\alpha}\psi_2\rangle$ depend on α. Since ψ_1 and ψ_2 by hypothesis do not overlap in space, and the action of any finite polynomial in x and in p on them does not change that fact, these terms will vanish.

[4] Note also that Ψ_α is a non-analytic function of x, and therefore

$$\left\langle \mathrm{e}^{\frac{\mathrm{i}}{\hbar}pL}\right\rangle = \left\langle \sum_n \frac{\left(\frac{\mathrm{i}pL}{\hbar}\right)^n}{n!}\right\rangle \neq \sum_n \frac{\left\langle\left(\frac{\mathrm{i}pL}{\hbar}\right)^n\right\rangle}{n!}.$$

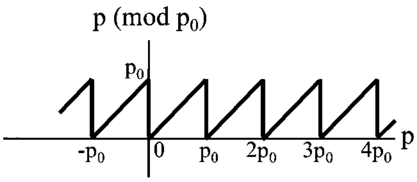

Figure 1. Modulo momentum.

since $e^{\frac{i}{\hbar}pL}\psi(x) = \psi(x+L)$ or $e^{\frac{i}{\hbar}pL}\psi_2 = \psi_1$. Similarly for the Hermitian counterpart of the modulo momentum $\langle\Psi_\alpha|\cos\frac{pL}{\hbar}|\Psi_\alpha\rangle = \frac{\cos\alpha}{2}$. Thus this is the dynamical variable that describes quantum interference. Note that for $L' - L \geqslant \Delta x$, $e^{\frac{i}{\hbar}pL'} = e^{\frac{i}{\hbar}p(\mathrm{mod}\ \frac{h}{L'})L'} \equiv e^{\frac{i}{\hbar}p(\mathrm{mod}\ p_0')L'}$ is completely uncertain. By definition, $p(\mathrm{mod}\ p_0')$ is completely uncertain if the probability $P_r[p(\mathrm{mod}\ p_0')] =$ constant for all $0 \leqslant p(\mathrm{mod}\ p_0') \leqslant p_0'$. $p(\mathrm{mod}\ p_0')$ is similar to an angle $\bar\theta \overset{\mathrm{def}}{=} \theta(\mathrm{mod}\ 2\pi)$. Complete uncertainty of $\bar\theta$ then means that all directions are equally probable. A necessary and sufficient condition for a constant probability $P_r[p(\mathrm{mod}\ p_0')]$ is that $\langle e^{\frac{i}{\hbar}npL'}\rangle = 0$ for $n \neq 0$.[5] Note that the eigenstates of the displacement operator $e^{\frac{i}{\hbar}pL}$ are also the eigenstates of the modulo momentum p mod p_0 (see figure 1) defined by

$$p(\mathrm{mod}\ p_0) = p - Np_0, \tag{3}$$

and so that its eigenvalues satisfy

$$0 \leqslant p(\mathrm{mod}\ p_0) \leqslant p_0. \tag{4}$$

N is an operator having integer eigenvalues and

$$p_0 = \frac{h}{L}. \tag{5}$$

Modulo momentum has two important attributes. First, it has a nonlocal equation of motion. With the Hamiltonian given by $H = \frac{p^2}{2m} + V(x)$ we have,

$$\frac{\mathrm{d}}{\mathrm{d}t}e^{ipL} = i[V(x), e^{ipL}] = i[V(x) - V(x+L)]\,e^{ipL} \tag{6}$$

using $e^{ipL}V(x)\,e^{-ipL} = V(x+L)$. The change of modulo momentum is proportional to a nonlocal potential difference. A periodic step potential, with step width of L i.e. $V(x+L) = V(x) + V_0$, (see figure 2), induces a change of the momentum modulo $\frac{h}{L}$ or the relative phase,

$$e^{ipL}(t) = e^{i(pL - V_0 t)}. \tag{7}$$

Modulo momentum is also a conserved quantity. Consider a collision between two particles. π_1 and π_2 are their respective modulo momenta:

$$\begin{aligned}\pi_1 &= \cos p_1 L\\ \pi_2 &= \cos p_2 L.\end{aligned} \tag{8}$$

[5] Switching to θ we have $\langle e^{in\theta}\rangle = \int_0^{2\pi} P_r(\theta)\,e^{in\theta}\,\mathrm{d}\theta$, where $P_r(\theta)$ is periodic in $-\infty \leqslant \theta \leqslant \infty$. $P_r(\theta)$ can therefore be expanded in a Fourier series $P_r(\theta) = \sum_{n=-\infty}^{\infty} a_n\,e^{in\theta}$. The above condition then means that for a constant $P_r(\theta)$, all the Fourier coefficients $a_n = \frac{1}{2\pi}\langle e^{-in\theta}\rangle$ except a_0 vanish, with $P_r(\theta) = a_0 = \frac{1}{2\pi}$.

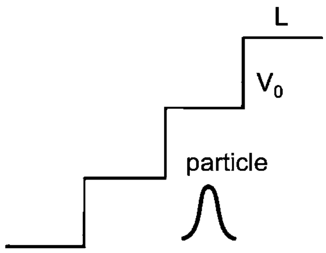

Figure 2. The periodic step potential with step width L induces a change of the momentum modulo $\frac{h}{L}$ of the particle.

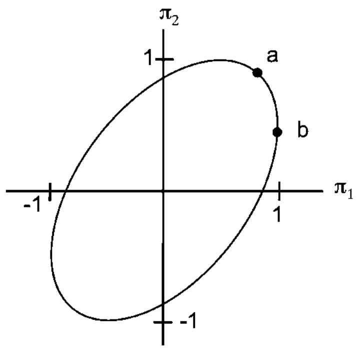

Figure 3. Conservation law for modulo momentum.

In the collision, modulo momentum is exchanged on an ellipse, and the system shifts from point a to point b[6] (see figure 3):

$$\pi_1^2 + \pi_2^2 - 2C\pi_1\pi_2 = 1 - C^2, \tag{9}$$

where

$$C = \cos(p_1 + p_2)L. \tag{10}$$

Finally, note that modulo momentum is the only way to describe interference in the Heisenberg picture. The reason for this is that in this picture the state of the system is the initial state. So there is no way to account for changes of the relative phase, unless one uses modulo momentum.

3. The AB effect

We now apply modulo momentum to the AB effect. In this context we have to consider the gauge-invariant modulo momentum $\mathrm{e}^{\mathrm{i}(\vec{p}-\frac{e}{c}\vec{A})\cdot\vec{L}}$. Below we show that an abrupt change occurs in the gauge-invariant modulo momentum of the external particle when the line connecting the two wave packets crosses the solenoid. See figure 4. We then proceed to show that this change of the gauge-invariant modulo momentum is responsible for the shift of the interference pattern of the AB effect.

[6] Momentum conservation, here in the x-direction, means $p_1 + p_2 = p_1' + p_2'$, from which $\cos[(p_1 + p_2)L] = \cos[(p_1' + p_2')L]$ follows, and from which (11) follows.

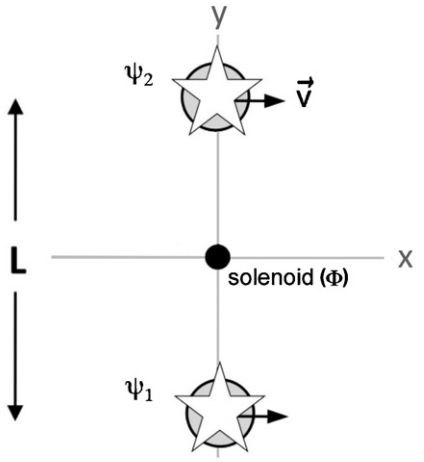

Figure 4. An abrupt change of the gauge-invariant modulo momentum or the velocity distribution of the external particle occurs when the wave packets pass by the solenoid.

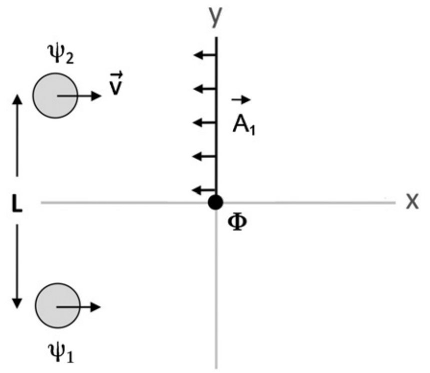

Figure 5. By crossing the potential line, the external particle acquires a relative phase $\mathrm{e}^{-\mathrm{i}\frac{e\Phi}{c}}$. Here $\vec{v}$ is the average velocity of the wave packet.

The abrupt change of the modulo momentum of the external particle follows immediately in the singular Shelenkov gauge. See figure 5. Here, the vector potential is non-vanishing only on the positive *y*-axis and is proportional to the flux:

$$\begin{aligned} A_{1x} &= \Phi\theta(y)\delta(x) \\ A_{1y}s &\equiv 0, \end{aligned} \tag{11}$$

where $\theta(y)$ is the step function.

If the external particle is initially prepared in a superposition of the two wave packets ψ_1 and ψ_2 (see figure 5), i.e. its state before crossing the potential line is

$$\Psi = \psi_1 + \psi_2, \tag{12}$$

then by crossing the potential line it acquires a relative phase that equals the full AB phase, and its state after the crossing is given by

$$\Psi' = \psi_1 + \mathrm{e}^{-\mathrm{i}\alpha}\psi_2, \tag{13}$$

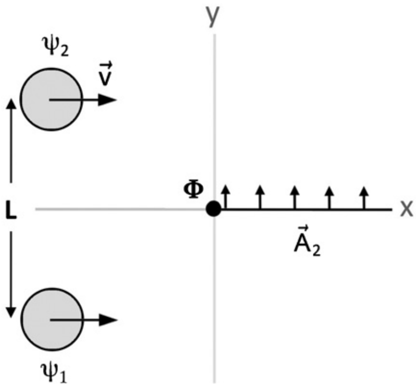

Figure 6. In this gauge, the relative phase does not change when the wave packets pass by the solenoid. And yet, the gauge-invariant modulo momentum changes just there. Here $\vec{v}$ is the average velocity of the wave packet.

$\alpha = \frac{e\Phi}{\hbar c} \equiv \frac{e\Phi}{c}$. Note also that in this gauge, both before and after crossing the potential line, the gauge-invariant modulo momentum is equal to the canonical momentum i.e. $m\vec{v} = \vec{p}$. It follows that to calculate the change in the gauge-invariant modulo momentum, one need only to consider the usual modulo momentum. Thus, when the external particle passes by the solenoid, its modulo momentum changes by

$$\delta\langle \mathrm{e}^{\mathrm{i}mv_y L}\rangle = \delta\langle \mathrm{e}^{\mathrm{i}p_y L}\rangle = \tfrac{1}{2}(\mathrm{e}^{-\mathrm{i}\alpha} - 1). \tag{14}$$

Note also that the velocity distribution of the external particle changes when it passes by the solenoid. This is so since the average modulo momentum is the Fourier transform of the velocity distribution:

$$\langle \mathrm{e}^{\mathrm{i}mv_y L}\rangle = \int P_r(v_y)\,\mathrm{e}^{\mathrm{i}mv_y L}\,\mathrm{d}mv_y. \tag{15}$$

Our result appears perhaps even more surprising in the gauge where the potential line lies along the positive x-axis (see figure 6). In this gauge, the relative phase does not even change when the wave packets pass by the solenoid. And yet the velocity distribution changes just there. In this case we have to calculate the change of $\langle \mathrm{e}^{\mathrm{i}mv_y L}\rangle$ using

$$\mathrm{e}^{\mathrm{i}mv_y L} = \mathrm{e}^{-\mathrm{i}\frac{e}{c}\int_y^{y+L} A_y \mathrm{d}y}\,\mathrm{e}^{\mathrm{i}p_y L}. \tag{16}$$

With $\vec{A}_2$, the line integral to the right of the flux line yields Φ, while the line integral to its left is equal to zero. Thus we obtain again the same result as before i.e. (15) above.

The most general expression for the change of modulo momentum or the velocity distribution is given by

$$\delta\langle \mathrm{e}^{\mathrm{i}m\vec{v}\cdot\vec{L}}\rangle = \tfrac{1}{2}\left(\mathrm{e}^{-\mathrm{i}\frac{e}{c}\oint \vec{A}\cdot\mathrm{d}\vec{l}} - 1\right) = \tfrac{1}{2}\left(\mathrm{e}^{-\mathrm{i}\frac{e\Phi}{c}} - 1\right), \tag{17}$$

following from

$$\mathrm{e}^{\mathrm{i}(\vec{p}-\frac{e}{c}\vec{A})\cdot\vec{L}} = \mathrm{e}^{-\mathrm{i}\frac{e}{c}\int_{\vec{r}}^{\vec{r}+\vec{L}} \vec{A}\cdot\mathrm{d}\vec{l}}\,\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{L}} \tag{18}$$

and Stokes's theorem. $\mathrm{e}^{\mathrm{i}(\vec{p}-\frac{e}{c}\vec{A})\cdot\vec{L}}$ is a gauge-invariant translation by $\vec{L}$ operator. $\vec{r}$ is the particle's position operator. The path of the integral on the rhs of (17) is a straight line joining the positions $\vec{r}$ and $\vec{r}+\vec{L}$. Above, we have shown that the velocity distribution changes near $x = 0$, where the particle passes by the flux line. From (18) it follows immediately that the

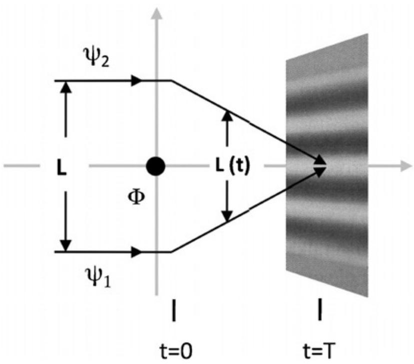

Figure 7. After the external particle has passed by the solenoid, the two wave packets are brought together to interfere.

velocity distribution changes nowhere else. This is so since at all other parts of the particle's trajectory the closed line integral of the vector potential encloses no flux. Thus, the change of the velocity distribution in the vicinity of the flux line is the only candidate to explain the shift of the interference pattern.

Note also that the abrupt change of the velocity distribution occurs via a nonlocal interaction with the flux line from which the external particle is excluded. In the previous section, a scalar potential analog of such an interaction has been shown, namely (8), (9) above.

In the above, we have assumed a very thin solenoid, its diameter tending to zero, as well as very small wave packets. For this case we obtained the abrupt change of the velocity distribution. When the solenoid has a finite diameter or the wave packets finite width, the change occurs continuously since the flux and the probability are additive in the direction of motion. To see how this works in detail consider a modification from one thin solenoid to a distribution of thin solenoids, and take the special case of two thin solenoids. Apply our above considerations to one solenoid after the other. Similarly we deal with a finite wave packet as a sum of many very small wave packets. Thus, when the solenoid and wave packets have finite width, the exchange takes a time $\Delta t = \frac{W}{v}$, where W is the width of the wave packets and solenoid combined. This must be compared to the time of excursion of the particle throughout the experiment $\sim T_1$. We have here two time scales with $\Delta t \ll T_1$.

Next, we show that the change of the velocity distribution is responsible for the shift of the interference pattern of the AB effect. We now relate to that part of the evolution when the wave packets are brought together to interfere. The two wave packets are accelerated in the y direction, acquiring velocities $\pm\frac{\hbar k_0}{m}$ respectively. See figure 7. In the gauge $\vec{A}_1$ that we are considering, we now have a free Hamiltonian, $H = \frac{p^2}{2m}$. Note that the time $t = 0$ is defined as the time when the two wave packets start to move toward each other. This occurs after the particle has passed by the solenoid. This means that at $t = 0$, the modulo momentum has already changed. T is the time when the wave packets meet. $L(t)$ is the instantaneous separation between the packets.

We now transfer to the Heisenberg picture, and, in particular, we shall consider the motion in the y direction only. The Heisenberg state is the initial state $\Psi(0)$ which, in this gauge, has already acquired the AB relative phase:

$$\Psi(0) = \psi_1 + \mathrm{e}^{-\mathrm{i}\alpha}\psi_2 \tag{19}$$

$$= \psi\left(y + \frac{L}{2}\right)\mathrm{e}^{\mathrm{i}k_0 y} + \mathrm{e}^{-\mathrm{i}\alpha}\psi\left(y - \frac{L}{2}\right)\mathrm{e}^{-\mathrm{i}k_0 y}. \tag{20}$$

$\Psi(0)$ is an eigenstate of the function of modulo momentum given by

$$A(0) = \cos[p_y L + 2k_0 y(0)]. \tag{21}$$

The evolution of the position operator in the Heisenberg picture is given by $y(t) = y(0) + \frac{p_y}{m}t$. Substituting $y(0) = y(t) - \frac{p_y}{m}t$ in (22) and defining the instantaneous separation $L(t) = L - \frac{p_y}{m}t$, we obtain

$$A(0) = \cos[p_y L(t) + 2k_0 y(t)] = A(t). \tag{22}$$

A is a constant of the motion since both the momentum p_y and $y(0)$ are constants. The time evolution of A is given by $A(t)$. Note that it is proportional to the instantaneous modulo momentum $\cos[p_y L(t)]$ since $L(t)$ is the instantaneous separation. By the time $T = \frac{mL}{2k_0}$, A has transformed into the local function $A(T) = \cos 2k_0 y(T)$ describing interference in the Heisenberg picture. We have

$$\langle A(0)\rangle = \frac{\cos\alpha}{2}, \tag{23}$$

where A is a constant of the motion. This means that $A(T) = A(0)$ or that $\cos 2k_0 y(T) = A(0)$. It follows that

$$\langle \cos 2k_0 y(T)\rangle = \frac{\cos\alpha}{2} \tag{24}$$

which means a shift of the interference pattern. We have thus seen that the change of modulo momentum, which occurred when the external particle passed by the solenoid and the two wave packets were still apart, manifests itself later on as a shift of the interference pattern of the AB effect. And conversely, the shift of the interference pattern of the AB effect can always be traced back to a change of the velocity distribution that occurred earlier on, when the wave packets passed by the solenoid.

We now relate briefly to modulo angular momentum. See figure 8. For a full discussion of this subject, the reader should refer to [3]. Note that in the figure, the motion is described in the rest frame of the external particle. The gauge-invariant modulo angular momentum about the z-axis changes abruptly when the solenoid enters the circle, and again as it leaves it:

$$\langle \mathrm{e}^{\mathrm{i}I v_\theta \pi}\rangle = 1 \rightarrow \frac{1 + \mathrm{e}^{-\mathrm{i}\alpha}}{2} \rightarrow \cos\alpha. \tag{25}$$

Here I is the moment of inertia and v_θ is the angular velocity.

4. The gedanken experiment

Finally, we describe a gedanken experiment that shows that our prediction can, in principle, be tested experimentally. Consider the set-up described in figure 9. Initially, the charged particle, the 'ball', is in the wave packet $\psi(y_1)$, at rest, and the uncharged external particle is in a superposition of the two incoming wave packets $\phi_1(y_2)$ and $\phi_2(y_2)$:

$$\Psi_{\text{initial}} = (\phi_1 + \phi_2)\psi(y_1). \tag{26}$$

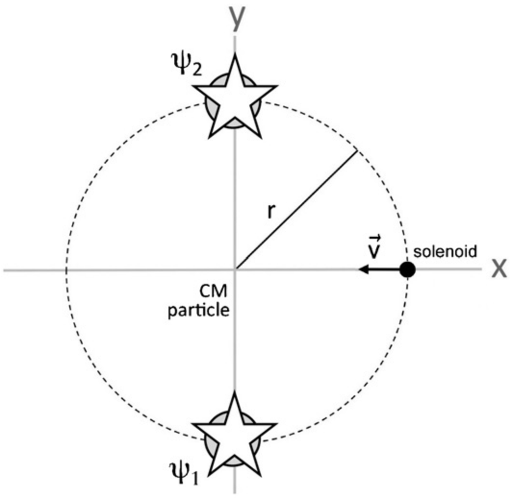

Figure 8. View from the rest frame of the external particle. The gauge-invariant modular angular momentum about the z-axis of the external particle changes abruptly when the solenoid enters the circle, and again as it leaves it.

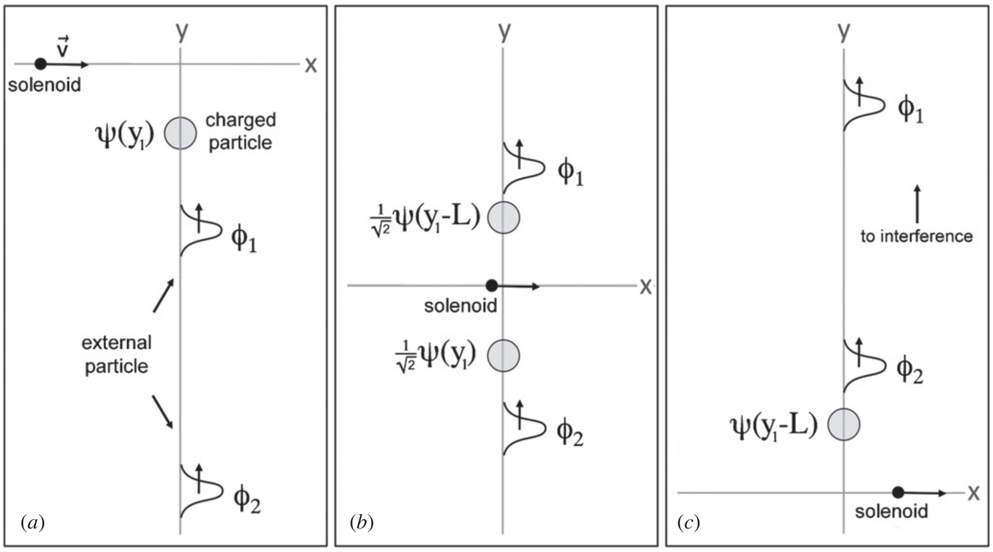

Figure 9. The gedanken experiment. The collision between the external, uncharged particle (in ϕ_1 and ϕ_2) and the charged particle, the 'ball' (in ψ), occurs at an uncertain time. (a) Initial state. (b) Set-up at an intermediate time. After ϕ_1 has hit the ball and before ϕ_2 has hit it, the solenoid passes between the two wave packets of the charged particle, $\psi(y_1)$ and $\psi(y_1 - L)$. (c) Final state.

See figure 9(a). When the external particle hits the ball, the latter moves for a distance L and then stops, ending up in the wave packet $\psi(y_1 - L)$.[7] Because the external particle

[7] The external particle exerts a force on the ball twice. First the ball is accelerated, shortly afterwards it is decelerated. This can be done with the help of a suitable square well potential describing the interaction between the particles. $V(|y_2 - y_1|) = -V_0$ for $|y_2 - y_1| \leqslant L'$, and $= 0$ otherwise.

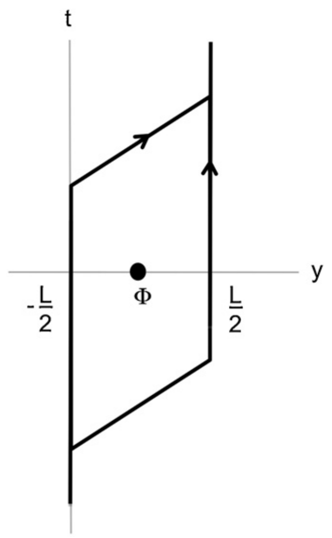

Figure 10. Time evolution of the charged particle.

hits the ball at an uncertain time, there is an intermediate time interval during which the charged particle is in the two wave packets $\psi(y_1)$ and $\psi(y_1 - L)$. The two particles are then entangled:

$$\Psi_{\text{intermediate}} = \frac{1}{\sqrt{2}}[\phi_1\,\mathrm{e}^{\mathrm{i}p_1L}\psi(y_1 - L) + \phi_2\psi(y_1)]. \tag{27}$$

See figure 9(*b*). p_1 is the momentum of the wave packet ψ. It is uncertain with $\Delta p_1 \sim \frac{\hbar}{\Delta y_1}$. This introduces an uncertainty $\Delta\varphi_{\text{rel}} = \Delta p_1 L \sim \hbar\frac{L}{\Delta y_1}$, into the relative phase of the intermediate state. At this stage, the solenoid, coming in from the left with velocity v in the x-direction, passes between the two wave packets $\psi(y_1)$ and $\psi(y_1 - L)$ of the charged particle, the latter thus acquiring an additional definite relative phase $\mathrm{e}^{-\mathrm{i}\alpha}$ due to the AB effect:

$$\Psi'_{\text{intermediate}} = \frac{1}{\sqrt{2}}[\phi_1\mathrm{e}^{-\mathrm{i}\alpha}\mathrm{e}^{\mathrm{i}p_1L}\psi(y_1 - L) + \phi_2\psi(y_1)]. \tag{28}$$

After the solenoid has passed between the two wave packets of the charged particle, the second wave packet of the external particle, ϕ_2, hits the ball. The ball ends up at $y_1 = L$. The particles are now disentangled and the external particle is back in a coherent superposition of the two wave packets ϕ_1 and ϕ_2:

$$\Psi_{\text{final}} = (\phi_1 + \mathrm{e}^{\mathrm{i}\alpha}\phi_2)\psi(y_1 - L). \tag{29}$$

See figure 9(*c*). Note that the AB phase $\mathrm{e}^{\mathrm{i}\alpha}$ has been transferred to the external particle, and can be measured later, in an interference experiment performed on it. Note also that while the relative phase was uncertain when the particles were entangled, in the final state the relative phase of the external particle is definite.

We now proceed to consider the time evolution of the charged particle. It is described in the y-t plane, specifically the plane $x \equiv 0$. See figure 10. Now the solenoid, in its motion along the x-axis, crosses this plane at a definite time, say $t = 0$. Because of its motion in the x direction, the solenoid encloses also an electric field in the y direction, $E_y = B_z v$. Thus, when the solenoid crosses the y-t plane, there appears at the origin an electric flux that is equal to Φ. Thus, the charged particle experiences an electric AB effect, and it acquires a relative phase that is equal to $\frac{e\Phi}{c}$. This occurs in the plane $x \equiv 0$. Thus its modulo momentum has changed abruptly. This change of the modulo momentum is observable as a shift of the interference pattern of the external particle. As we have seen, it cannot but have happened at a definite x, i.e. when the solenoid passed between the two wave packets of the charged particle.

Note that, owing to the finite width of the solenoid and wave packets, the above y-t plane is not a geometric plane but has a finite, but very small, thickness. This does not impair the validity of our argument. A full analysis of the gedanken experiment is beyond the scope of this paper and will be published separately.

5. Conclusions

Above, we have taken a dynamical approach to the AB effect. We have shown that the shift of the interference pattern is a manifestation of an abrupt, nonlocal exchange of gauge-invariant modulo momentum that occurred earlier on, when the external charged particle passed by the solenoid. We also described a gedanken experiment that shows that our prediction can, in principle, be tested experimentally. In the gedanken experiment, the exchange is viewed in the rest frame of the charged particle. Here, the exchange occurs at a definite time, when the solenoid passes between the two wave packets of the charged particle. The implication of this is that we have here a new kind of a 'quantum leap'. But, unlike the abrupt exchange of momentum and energy carried locally through spacetime by a photon, this exchange occurs nonlocally. The connection of modular momentum to the two-slit interference experiment has been discussed in [4].

Acknowledgment

This work has been supported in part by the Wolfson Family Charitable Trust.

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