Aharonov-Bohm effect without closing a loop

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We discuss the consequences of the Aharonov-Bohm (AB) effect in setups involving several charged particles, wherein none of the charged particles encloses a closed loop around the magnetic flux. We show that in such setups, the AB phase is encoded either in the *relative* phase of a bipartite or multipartite entangled photons states, or alternatively, gives rise to an overall AB phase that can be measured relative to another reference system. These setups involve processes of annihilation or creation of electron-hole pairs. We discuss the relevance of such effects in "vacuum birefringence" in QED, and comment on their connection to other known effects.

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I. INTRODUCTION

In the usual setup of the Aharonov-Bohm effect (AB) [1], a charged particle encircles a flux tube of total magnetic flux Φ, and collects the phase

$$\phi_{AB} = \frac{e}{\hbar c}\oint_C \vec{A}\cdot dl = \frac{e\Phi}{\hbar c}. \tag{1}$$

The AB phase, ϕ_{AB}, has two important features. It depends only on the topology of the trajectory via the winding number n. Additionally the effect is "nonlocal"; at any intermediate point along the trajectory, the magnetic fields vanish, and hence the presence of the flux is locally undetectable. This is consistent with the fact that the line integral of the vector potential is gauge invariant only along closed trajectories.

We shall discuss some interesting consequences of the AB effect in setups involving several charged particles, none of which encloses a complete loop around the flux. Under such circumstances, the AB effect has different manifestations: the AB phase is encoded in the *relative* phase of a bipartite and multipartite entangled state. The AB topological nonlocality gets transformed here into the nonlocal property of the resulting entangled state.

Alternatively, the AB effect can give rise to an *overall* phase of the system, which can be measured with respect to another reference system. An electron-hole–positron pair is formed at one location, and recombines at another location after encircling a flux. The resulting photon then carries an overall AB phase.

In a related idea [2], the AB phase has been recently manifested in current-current correlations of electrons in a Hanbury-Brown-Twiss interferometer. In this proposal, however, the effect is based on the indistinguishability of the interfering electrons.

II. TRANSLATING THE AB PHASE TO ENTANGLED STATES

Consider an electron and a hole that approach the fluxon from opposite upwards and downwards directions and pass through beam splitters, as depicted in Fig. 1. The beam splitter transforms the electron and the hole to a superposition of left and right movers. The electron and hole can recombine into a photon either on the left side or the right side. Adding up the phases collected in each of the four parts of the circle in Fig. 1, we find that the two parts of the photon wave function have a relative phase equal to the full AB phase. The postselected state with no electron or hole, namely when a photon was created, is then

$$|1_L 0_R\rangle + e^{i\phi_{AB}}|0_L 1_R\rangle, \tag{2}$$

where $|n_L n_R\rangle$ is the state with n_L photons on the left and n_R photons on the right parts. Thus the flux becomes encoded in the relative phase of a maximally entangled state.

It is instructive to compare between the usual measurement of the AB phase in the standard setup and the present case. In our case, the final photon state can be converted to a bipartite entangled state of a pair of two atoms. $(1/\sqrt{2})(|e,g\rangle + e^{i\phi_{AB}}|g,e\rangle$. The flux can then be used to control nonlocally the relative AB phase. This phase cannot be observed by performing measurements on only one atom. It

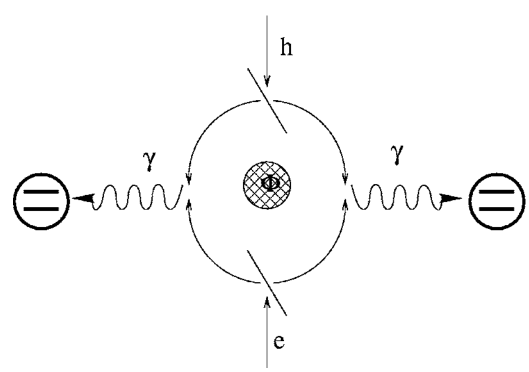

FIG. 1. The electron and the hole each completes half a loop and annihilate to a photon. The photon can be absorbed by either the left or the right atom. The bipartite entangled atom's final state depends on the AB phase.

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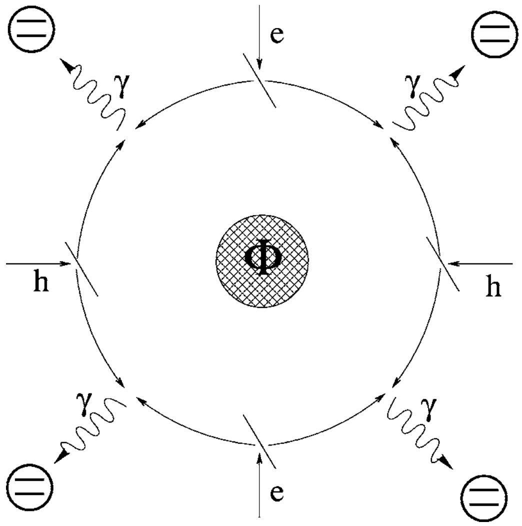

FIG. 2. The two electrons and the two holes each complete quarter of a loop and together collect the topological phase which is encoded in the phase of the emitted photons. The final state has either two photons in arms 1 and 3, or in arms 2 and 4.

is manifested, however, in the correlations between the results of the measurements performed locally on both atoms.

From the quantum information point of view, this setup provides an interesting method to encode a classical bit into an entangled state [3]. For example, the observer that controls the enclosed flux can encode "0" in $\psi^{+}=(1/\sqrt{2})(|e,g\rangle+|g,e\rangle$ and "1" in $\psi^{+}=(1/\sqrt{2})(|e,g\rangle-|g,e\rangle$ by changing the enclosed flux from $\Phi_0=0$ to $\Phi_1=hc/2e$.

The above scheme can be extended to n electrons and n holes. For example, in Fig. 2, two electrons and two holes approach the flux from four different directions. If a pair recombines then the two neighboring pairs cannot recombine, thus either two opposite photons are emitted or the other two opposite photons are emitted. The output state is then $|1010\rangle+e^{i\phi AB}|0101\rangle$, where 0 and 1 designate the Fock state of the four output channels of the photons.

In order to compute the resulting states in the above and similar setups, we make the following assumptions. The dynamical evolution of the system at the creation and annihilation vertexes can be obtained by applying creation-annihilation operators to the wave function. In particular, in the vertex where a photon creates an electron hole pair, or when an electron hole creates a photon, we have $a_{photon}a_e^{\dagger}a_h^{\dagger}$, and $a_e a_h a_{photon}^{\dagger}$, respectively. In these process, the net energy and momentum exchange of the charged particle with with the matter can be made small enough, so that the coherence of the process is maintained. In order to calculate the effect of the AB flux on the different charged particle trajectories, it is useful to use a particular gauge. In the singular gauge, the vector potential vanishes except along a singular line that emanates from the fluxon. In this gauge, only a charged particle that crosses the line accumulates a phase, for example, $a_e^{\dagger}\rightarrow a_e^{\dagger}e^{-i\phi}$, $a_h^{\dagger}\rightarrow a_h^{\dagger}e^{i\phi}$.

III. AB EFFECT WITH PHOTONS

In a different variant the AB phase is transferred to a photon as depicted in Fig. 3. The photon creates an electron and a hole. The latter can move on both sides of the flux and then annihilate back into a photon carrying the topological phase. The symmetry between electron-up, hole-down and electron-down, hole-up could be broken using an external electric field. To measure this phase we need the two beam splitters and mirror shown in Fig. 3 yielding two alternative paths for the photon, only one of which is affected by the AB phase. This yields a final output signal with small modulations periodic in the flux due to interference.

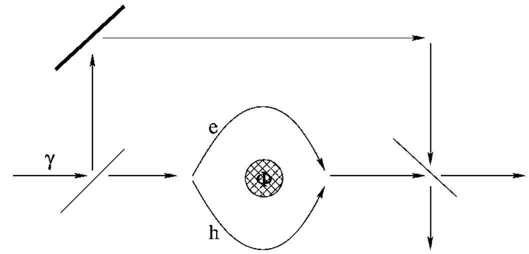

FIG. 3. The photon creates an electron and a hole and collects the topological phase on recombination. The process takes place in one of the arms of a Mach-Zender interferometer.

IV. VACUUM BIREFRINGENCE

The well-known "vacuum birefringence" in QED can be related to the above setups. Thus (PVLAS experiment [4]) envision laser light propagating in vacuum along the z direction, say in a transverse B field. The virtual electron box diagram generates for the low energy $(E_\gamma \ll m_e c^2)$ an effective Euler-Heisenberg Lagrangian [5–7] (in naturalized Gaussian units $\hbar=c=1$):

$$L_{eff}=\frac{2\alpha^2}{45(4\pi)^2m^4}[(E^2-B^2)^2+7(EB)^2] \tag{3}$$

with the four E,B factors representing external fields and or photons. L_{eff} generates in particular vacuum birefringing, namely a relative phase between the x and y polarizations of the photon. Can we heuristically understand this in a manner emphasizing the role of an AB-type phase? The photon can virtually convert into an electron-positron pair which after free propagation (in configuration space) recombine back into the original photon. If a B flux threads the path closed jointly by the electron and positron then the amplitude picks up an extra AB phase. For the case of a uniform B field $\Phi_{AB}=AB\sin\theta$ with θ the angle between the "plane" where e^+e^- move and B with A the "net" signed area enclosed. During their lifetime $\Delta t=h/2mc^2$, the electron and positron travel distances $l=h/mc$ and $A=l^2=(h/mc)^2$. The "dipole" interaction $E_{photon}(p^+-p^-)$ tends to create or annihilate the pair in the polarization plane of the photon. The amplitude of polarization perpendicular to the B field picks an AB phase relative to the other, orthogonal, polarization adding a small circular polarization to the primarily linearly polarized light or a small "ellipticity." The above argument fails, however. The explicit Euler-Heisenberg effective Lagrangian and more generally Furry's theorem [8,9] forbid trilinear photon coupling or polarization changes linear in B external. Indeed for every loop traversed by the e^+ and e^- in a given sense there is loop of equal amplitude with $e^+ \leftrightarrow e^-$ traversed and the opposite AB phase! A finite effect ensues in next order: The $U = \mu B$ interaction with the loops' magnetic moment: $\mu = eA\omega = eAmc^2/h$ enhances the probability amplitude of one orientation of the loop relative to the other by U/mc^2, i.e., by $BA \approx B(h/mc)^2$ avoiding the above cancellation of the AB phases and yielding a net effect of the correct form e^4B^2/m^4.

V. SOME COMMENTS ON THE AB ZEEMAN AND AB FARADAY EFFECT

In Secs. II and III above different variants of the AB effect where no single charge particle encircles the flux were utilized to transfer the AB phase to photons. Here we note that aloso the conventional AB effect can yield such a phase transfer via an "AB Faraday" effect. For extended B fields the "classical," nonanomalous Zeeman and Faraday effects are well understood: the $\vec{\mu}\cdot\vec{B} \propto g\vec{l}\cdot\vec{B}$ interaction splits the m sublevels. The resonant absorption frequency of left and right circularly polarized light separate by $2gB$ and the corresponding indices of refraction differ accordingly by $n_R - n_L \propto gB_\parallel/\omega$, with $B_\parallel$ the (say z) component of B parallel to the light propagation. Along a distance L this does in turn rotate the initial plane (of the linearly polarized light) by $(n_R - n_L)L/\lambda$ (λ is the wavelength divided by 2π).

In the AB effect the electron picks up the magnetic-field-induced phase despite being at all times in the $B=0$ region. The "AB Zeeman" effect is the energy shift of such a particle. Let a cylindrical shell of inner or outer radii r, R be threaded by a flux ϕ along its (z) axis. The states of noninteracting electrons in this shell where a cylindrically symmetric potential exists $\Psi_{k,m,n}$ have energies $E_{k,m,n}$ depending on the z components of the linear and angular momenta $k = p_z/h$, $m = l_z/h$ and a remaining "radial" quantum number n. The introduction of the flux shifts changes the angular momentum quantum number: $l_z' = l_z - \alpha$. It leaves the single valued wave functions changing the energies via the substitution $E_{k,n,m} \rightarrow E_{k,n,m+\alpha}$. The AB flux is a "modular" variable and the levels cross for $|\alpha| = 1/2$, hence the shift above is by the smaller of noninteger part of the flux or its complement to an integer.

The Zeeman shift is linear in B (for "small" B) and is not periodic. The AB Zeeman effect *is* periodic in the flux and, for fixed area of fluxons, in the field B. When the field is uniform and the sample continuous we find that the AB shift becomes the Zeeman effect. It is interesting to note that in a cylindrical sample with mobile electrons (and/or holes) with a common (small) radius a, the levels corresponding to paths enclosing the hole with radii peaked near $r=a$, and a periodic dependence on B, see Ref. [12].

This is illustrated in excitonic states at the rim of the holes bound by modified Coulomb $1/|\theta - \theta'|$ potential. This is equivalent to looking for bound states in the one-dimensional problem on an interval $[0, 2\pi a]$ where for say the even parity sector we demand that $\psi'(0) = \psi'(2\pi a) = 0$ which can be solved with and without the fluxon the introduction of which changes the $d/id\theta$ into $d/id\theta - \alpha/2\pi$ [10].

The second setup (Fig. 3) can be manifested in a photon exciton system [11] wherein angular momentum conservation simplifies the calculations. In semiconductors when a photon creates an exciton an R photon creates an X_+ exciton and an L photon creates an X_- exciton (where L, R are orthogonal circular polarizations and X_- and X_+ are orthogonal states of the exciton with different angular momentum). Each exciton collects the phase with a different sign since in the relative coordinates the charge rotate in a different direction. Thus for the proper choice of flux the relative phase between X_+ and X_- is 180^o. Since the angular momentum is in the direction of the propagation of the photon, in this setup the magnetic field should be parallel to the momentum of the photon. Hence a photon with polarization in the x direction would change polarization to the y direction. In this scheme the AB phase is manifested in the rotation of polarization.

The AB Faraday effect is the rotation of polarization plane for light propagating in the z direction i.e., along the fluxon and axis of the cylindrical sample. For the exciton the idea is the same as for the normal effect except that the energy shifts are of the exciton and not of the electron. The energy shift explanation is valid for weak magnetic fields, for stronger magnetic fields the reason for the rotation is analogous to the explanation of the AB rotation of polarization. To avoid a strong decline of the effect with the distance from the above axis or fluxon, the wavelength of the light can be of order of the radius of the cylinder.

In order to estimate the magnitude of the effect we calculate the regular AB Faraday effect for a charged particle condensate constrained to a narrow ring, with a flux passing through the ring axis. Let ϕ be the angle on the ring, $\psi(\phi,t)$ be the condensate wave function on the ring, and assume the form

$$\psi(\phi,t) = \sqrt{n}e^{(i/h)S(\phi,t)}, \tag{4}$$

where n is a constant particle density, integrating to a total number of particles N on the ring. We assume a flux $\Phi = \beta\Phi_o$ along the z axis, and an incident circularly polarized electromagnetic plane wave along the same direction,

$$\vec{A}_{inc} = A_\pm \hat{\varepsilon}_\pm e^{i(kz-\omega t)}, \tag{5}$$

where $\hat{\varepsilon}_\pm = (1/\sqrt{2})[\hat{x} \pm i\hat{y}]$. In a low-density approximation, the phase satisfies the Hamilton-Jacobi equation

$$-\dot{S} = \frac{1}{2m}\left(\frac{1}{R}\frac{\partial S}{\partial \phi} - \frac{e}{c}A_\phi\right)^2, \tag{6}$$

where A_ϕ is the ϕ component on the ring of the total vector potential $\vec{A}_{tot} = (1/2\pi R)\beta\Phi_o\hat{\phi} + \vec{A}_{inc}$. Note that $(e/2\pi c)\Phi_o = \beta\hbar$, so that, assuming linear response to A_{inc}, we get response components $S(\phi,t) = S_\pm e^{-i\omega t \pm i\phi t}$,

$$S_\pm(\omega) = \frac{1}{\sqrt{2}}\left(\frac{eR}{c}\right)\frac{\beta\omega_o}{\beta\omega_o \pm \omega}A_\pm, \tag{7}$$

with $\omega_o = \hbar/2mR^2$. The associated current densities on the ring are

$$\vec{J}_{\pm} = \pm i \frac{ne^2}{mc} \left(\frac{\omega}{\omega \mp \beta\omega_o} \right) \frac{A_{\pm}}{\sqrt{2}} \hat{\phi}. \tag{8}$$

The scattered fields preserve the incident polarization, resulting in forward scattering amplitudes $f_{\pm}(\Theta=0) = (N/2)(e^2/mc^2)[\omega/(\omega \mp \beta\omega_o)]$. Designating by $r_0 = e^2/mc^2$ the classical radius of the electron, using the dimensionless s matrix

$$S_{\pm} = 1 + if_{\pm}k = 1 + \frac{ir_0\omega}{\lambda(\omega \pm \beta\omega_0)} \tag{9}$$

and specializing to the limit case of $N=1$, i.e., a single electron in the ring, we finally find a rotation angle of order $\Delta\theta = \Delta S = (r_0\beta/R^2)(\hbar/mc) \approx (\beta 10^{-24}\ \mathrm{cm}^3)/R^2$; for $R=\lambda \approx 10^{-4}$ cm the angle is very tiny, $\Delta\theta \approx 10^{-15}$.

In conclusion, we have discussed some interesting features of the AB effect, and showed that the AB phase can manifest itself without any loops being closed by a single particle. We have discussed several variations of this idea, and showed that the nonlocal AB phase can be stored either in an entangled bipartite or multipartite state, or in the overall phase of photons or in the direction of polarization.

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[12] The above expression for the flux induced energy shift depends on the azimuthal (θ) symmetry when l_z is a good quantum number. A strong θ dependent potential breaking this symmetry can generate bound states localized in θ. Only the small overlap of the left and right tails of the wave function allows such states to enclose the fluxon and the energy shift is accordingly reduced.