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Further Discussion of Possible Experimental Tests for the Paradox of Einstein, Podolsky and Rosen.

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Summary. — In a previous article we have suggested the experiment of Wu-Shaknov on the annihilation radiation of positronium as a test for the paradox of Einstein, Podolsky and Rosen. In this article, we answer certain criticisms of our conclusions, raised by Peres and Singer. These criticisms are shown to be erroneous, being based on an incorrect interpretation of the polarization of electromagnetic radiation in the quantum domain.

1. – Introduction.

In a previous paper (1), we have discussed the paradox of Einstein, Podolsky and Rosen (2,3), and we have shown that the Wu-Shaknov experiment (4) on the polarization of the annihilation radiation of positronium provides an experimental confirmation of the features of the quantum mechanisms which are at the basis of the above paradox. In a recent article, Peres and Singer (5) criticize our conclusions and state that this experiment does not actually provide such a confirmation. We shall show with the aid of a more detailed analysis of the meaning of polarization of photons in quantum mechan-

(1) D. Bohm and Y. Aharonov: *Phys. Rev.*, **108**, 1070 (1957).
(2) A. Einstein, B. Podolski and N. Rosen: *Phys. Rev.*, **47**, 777 (1935).
(3) W. H. Furry: *Phys. Rev.*, **79**, 393, 476 (1936).
(4) C. S. Wu: *Phys. Rev.*, **77**, 136 (1950).
(5) A. Peres and P. Singer: *Nuovo Cimento*, **15**, 902 (1960).

ics that their conclusions are erroneous, being based on an unpermissible use of classical conceptions concerning the electromagnetic field in the quantum domain.

2. – Summary of essential features of paradox of Einstein, Podolsky and Rosen.

PERES and SINGER accept, as a legitimate example of the paradox of E.P.R., the case of the disintegration of a molecule consisting of two atoms having opposite spin, by some method which does not alter the total spin of the system [6]. We shall start therefore by summarizing briefly the main features of this example of the paradox, and we shall show later that there is no essential difference between it and the Wu-Shaknov experiment.

In such a system, the spin state is given, even after the particles are separated, by

$$\psi = 2^{-\frac{1}{2}}[\psi_+(A)\,\psi_-(B) - \psi_-(A)\,\psi_+(B)]\,, \tag{1}$$

where $\psi_+(A)\psi_-(B)$ refers to the state in which the particle A has spin $+\hbar/2$ and B has spin $-\hbar/2$, etc. This means that the spins of the two particles are correlated in a manner peculiar to the quantum theory. If the component of the spin of particle A is measured in *any* direction (for example z) then the same component of the spin of particle B is known to be opposite; and since the two particles are far from each other and do not interact, this information has been obtained without in any way disturbing particle B. In classical theory, such a correlation would be easy to understand, because all three components of the spins of each particle are defined simultaneously, and remain opposite to each other, so that the measurements of the spin of A simply gives information about a property already existent and well defined in B. In the quantum theory, however, only one component of the spin of each particles can be defined at a time, and the other two must be ambiguous (subject to uncontrollable « quantum fluctuations »). Therefore, before the spin of particle A in some direction is measured, we cannot suppose that all components of the spin of B are already well defined. On the other hand, it is possible, as we have seen, to choose to measure an arbitrary component of the spin of particle A, and in this way to determine the same component of the spin of B, without any interaction between B and A or between B and the measuring apparatus (after which the other two components of the spin of B will, of course, like those of A, be completely ambiguous). Such a result evidently

(6) See D. BOHM: *Quantum Theory* (New York. 1951), Chap. XXII.

contradicts the notion, commonly accepted before the paradox of E.P.R. was proposed, that the uncertainties of quantum mechanics represent only the effects of disturbances due to the measuring apparatus.

From a discussion of the properties described above, EINSTEIN, PODOLSKY and ROSEN [2] came to the conclusion that the quantum mechanics must be an incomplete theory. In doing this, they proposed the following criterion for an element of reality: «If, without in any way disturbing the system, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity». In the example of the spins, this criterion implies that in a complete theory, there would have to be a set of «elements of reality» corresponding to the simultaneous definition of all three components (and indeed of any component) of the spin of particle B. These elements of reality cannot be described in the quantum theory, because the different components of the spin do not commute. Therefore, some new theory is needed, which would give a more nearly complete description, in the sense that it contained these additional elements of reality.

The above argument was answered by BOHR [7], who showed that quantum mechanics implies an inseparability of observing apparatus and observed object which contradicts the criterion of E.P.R. for elements of reality.

3. – A more detailed analysis of the experiment of Wu-Shaknov.

In order to demonstrate that the experiment of Wu-Shaknov is also a valid example of the paradox of E.P.R., we shall first briefly review the discussion of this experiment given in our previous article, and then we shall add a more detailed mathematical analysis, which will help to clarify our answers to the criticisms of PERES and SINGER.

This experiment tests for the correlation of polarization of pairs of photons emitted in the annihilation of positronium. In order to write the wave function for this problem, we first consider a single photon moving in the $+z$ direction. Let ψ_k^x represent the wave function of such a photon, with wave vector k, and linearly polarized in the x direction, ψ_k^y the same for the y direction. Then the wave function of a photon polarized in an arbitrary direction is

$$\psi_k^\alpha = a_1 \psi_k^x + a_2 \psi_k^y , \tag{2}$$

where $a_1 = \cos\alpha$ and $a_2 = \sin\alpha$. There is a formal analogy here to the spin problem, an analogy that we shall develop in detail presently. Thus, the two

(7) N. BOHR: *Phys. Rev.*, 98, 696 (1935).

states ψ_k^x and ψ_k^y correspond to the two opposite spin states φ_+ and φ_- in some direction, say z. The wave function for a spin defined at an arbitrary angle relative to z (in the $x-z$ plane) is

$$\Phi^\beta = \cos\frac{\beta}{2}\,\Phi_+ + \sin\frac{\beta}{2}\,\Phi_- \tag{2'}$$

so that Φ^β corresponds to ψ_k^α.

It is clear that a rotation, $\beta = 180°$, of the spin vector corresponds to a rotation, $\alpha = 90°$, of the polarization vector. Therefore, what corresponds to the two possible spin states of an atom are two perpendicular possible directions of polarization of the photons; and just as the component of the spin in the direction β does not, in general, commute with that in the z direction, so the component of the polarization in the direction α does not, in general, commute with that in the x or y directions.

As stated in our article, the wave function of the pair of photons from the annihilation of positronium takes the form

$$\Phi = 2^{-\frac{1}{2}}(\psi_k^x\psi_{k'}^y - \psi_k^y\psi_{k'}^x)\,, \tag{3}$$

where k represents the wave vector of photon A and k' that of photon B (in a direction opposite to that of A).

From the similarity of (1) and (3), one can conclude that there will be a type of correlation in the polarization directions of the two photons, which is analogous to that of the spin directions. We can measure the polarization of photon A in a pair of directions, say x and y. If there is a single photon, then it will be found to be polarized either in the direction x or y. Whatever the direction is, we can deduce that photon B will be polarized in the other direction. This is analogous to measuring one component of the spin of particle A and deducing that the same component of B is opposite. But we can, instead, rotate the pair of axes (x, y) through some angle α and once again we will obtain the same kind of correlation between photons A and B for polarization observables that do not commute with the original set. This is analogous to measuring the spin of particle A in a direction at angle $\beta = 2\alpha$. and obtaining the corresponding correlation to the rotated spin operators for particle B.

At first sight, there may seem to be a difficulty in the above formal analogy between polarization and spin. If one applies the classical idea of a well defined polarization vector too literally, one comes to the conclusion that when the linear polarization in the x and y directions is defined, then the state of polarization cannot be ambiguous. As a result it might seem inconsistent to state that when we rotate the apparatus, new polarization operators not commuting with original ones will arise, which represent uncertainties in the polarization vector. It seems likely, indeed, that this problem is at the root of the main criticism of PERES and SINGER (as we shall explain in the next section). In order to clarify this point, we shall therefore develop here a further analysis of how the polarization vector must be described in quantum mechanics.

To show exactly what operators of the electromagnetic fields are measured when a polarization experiment is done, we first write the well known expansion for the vector potential operator, from which the fields can be derived.

$$A(x, t) = \sqrt{2\pi c}\sum_{k,i}\left(C_{k,i}\exp\left[i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)\right]+C^{*}_{k,i}\exp\left[-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)\right]\right)\frac{\varepsilon_{k,i}}{k^{\frac{1}{2}}}, \tag{4}$$

where $\varepsilon_{k,i}$ is a unit vector normal to $\boldsymbol{k}$ and where i has two values, corresponding to two possible directions of polarization of the k-th wave. C_{ki} and $C^{*}_{k,i}$ satisfy the commutation relation

$$[C^{*}_{k,i}, C_{k',j}] = \delta_{ij}\,\delta(\boldsymbol{k}-\boldsymbol{k}')\hbar\,. \tag{5}$$

If one introduces the Hermitean operators

$$q_{k,i} = \frac{C_{k,i}+C^{*}_{k,i}}{\sqrt{2}}, \qquad p_{k,i} = \frac{C_{k,i}-C_{k,i}}{\sqrt{2}}, \tag{6}$$

which satisfy the commutation relations

$$[p_{k,i}, q_{k,i}] = -i\hbar\,. \tag{7}$$

We obtain

$$A(x, t) = 2\sqrt{\pi c}\sum_{k}\left[q_{k,i}\cos(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)+p_{k,i}\sin(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)\right]\frac{\varepsilon_{k\,i}}{k^{\frac{1}{2}}}\,. \tag{8}$$

A linearly polarized photon is represented in the above equation as a superposition over a small range of $\boldsymbol{k}$ with i given a certain value. This leads to a wave packet describing the localization of the electromagnetic energy associated with the photon. If in the Wu-Shaknov experiment, the two photons are far enough apart, we can use wave packets with a broad range of z and therefore with a very small range of k. We can then simplify the problem by approximating the packet by a plane wave having a definite k (as is in fact done in all calculations concerning this experiment). The only part of the vector potential that is relevant for our problem is then a sum of two operators, one for k, the other for k' (which is opposite to k). Let us consider one of these operators,

$$A_k(x, t) = \frac{2\sqrt{\pi c}}{k^{\frac{1}{2}}}\left[(q_{k_1}\cos(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)+p_{k_1}\sin(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t))\varepsilon_{k_1}+ +(q_{k_2}\cos(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)+p_{k_2}\sin(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t))\varepsilon_{k_2}\right], \tag{9}$$

which represents a photon of direction k polarized in an arbitrary direction.

In classical theory q_{ki} and p_{ki} can be simultaneously well defined, so that the polarization vector for a given k can be specified unambiguously by specifying the four numbers, q_{k_1}, p_{k_1}, q_{k_2}, and p_{k_2}. Note that q_{ki} and p_{ki} determine both the intensity of the wave $I_{k.i}=(q^2_{ki}+p^2_{ki})/2$ and its phase $\varphi_{ki} = = \text{tg}^{-1}\, q_{ki}/p_{ki}$. With general values of q_k, p_k, one obtains elliptical polarization, which reduces to linear, if only one of the I_k is not zero, and to circular, if $I_{k_1}=I_{k_2}$ and $\varphi_{k_1}-\varphi_{k_2}=\pm\pi/2$.

In the quantum theory, q_{ki} and p_{ki} do not commute so that they cannot be defined simultaneously. Thus, as in the case of the spin, the direction of the polarization vector is, in general, ambiguous. In addition, with polarization, we must take into account the phase of the wave, which is also, in general, ambiguous.

Although it would, in principle, be possible to measure q_{ki}, or p_{ki} or some linear function of them, by measuring the Fourier components of the electromagnetic field, this would require that the number of photons became indeterminate. Such an observation is not what is actually done, when polarization is measured optically, nor is it what is done in the Wu-Shaknov experiment. Rather, what is done is to make measurements under conditions in which the number of photons is well defined, so that q_{ki} and p_{ki} must be to some extent ambiguous. What is usually called the « direction of polarization » then correspond only to a kind of average orientation of the field vector itself. In fact, what is measured is I_{ki} which is proportional to the energy of the part of the wave associated with the direction i. Thus, we measure the commuting pair of operators,

$$\begin{cases} I_{k_1} = \dfrac{p^2_{k_1}+q^2_{k_1}}{2} \\ I_{k_2} = \dfrac{p^2_{k_2}+q^2_{k_2}}{2} \end{cases}. \tag{10}$$

Classically, I_{k_1} and I_{k_2} are always well defined numbers with a continuous range of possible values (from 0 to ∞). Quantum-mechanically they need not, in general, be well defined (*i.e.* unambiguous because they are operators), but if the system is an eigenstate of these operators, their values are discrete and restricted to $(n+\frac{1}{2})h$.

In the experiment that we are discussing, we need consider only two possible values of I_{k1} and of I_{k2}, *viz.*, $h/2$ and $3h/2$. The value $h/2$ corresponds to the ground state (*i.e.*, the vacuum), while $3h/2$ corresponds to the first excited state (*i.e.*, one photon is present). It must be emphasized, however, that the I_{ki} are not components of vectors, rather they are components of tensor of the second rank. Their relation to the field vector is rather indirect. Indeed, when the I_{ki} are defined, the field vector still has a considerable ambiguity, not only in direction, but also in phase, so that it may be thought of as having an indeterminate degree of elliptical polarization. This follows basically from the fact that even the components of $\boldsymbol{A}_k$ with no photon in it still has a « zero point » energy of $h/2$. As a result, the component of A_k is still not zero, but may be said to « fluctuate » in the quantum mechanical sense about an average value of zero. Although this fluctuation is associated with the vacuum properties of the field, it is, nevertheless, of experimental significance, since it would be detected, if a precise measurement of the Fourier component of the field were actually made.

It is clear then that as we have already stated, I_{ki} reflects only some average property of the polarization. In the classical limit, when the number of photons can be very large, the zero point fields can be neglected and the polarization vector approaches a well defined direction. This justifies the usual procedure in the classical domain of identifying the direction of polarization with the orientation of the apparatus by which I_{ki} is measured. In the quantum domain, however, we must be careful not to become confused by the loose application of classical language, which, for the sake of brevity, has customarily been carried over into the description of quantum mechanical experiments.

The fact that after I_{k_1} and I_{k_2} are measured the polarization vector $\boldsymbol{A}_k$ is still ambiguous, is reflected in the experiment in which one measures the operators I'_{k_1} and I'_{k_2} which represent the average polarization along a pair of axes, rotated at an angle α relative to the original set. If the first measurement of I_{k_1} and I_{k_2} were able to determine the polarization vector, $\boldsymbol{A}_k$, unambiguously (as is suggested by the uncritical application of the classical language), then there could be no uncertainty in this vector, so that the result of the measurement in the second set of axes would have been determined without any ambiguities. Actually, however, as we shall now show, the operators, I'_{k_1} and I'_{k_2} do not commute with I_{k_1} and I_{k_2}. They are therefore not determined by I_{k_1} and I_{k_2} without uncertainty. To show this, we recall that $\boldsymbol{A}_k$ transforms as a vector under rotation, so that (q_{k_1}, q_{k_2}) and (p_{k_1}, p_{k_2}) each separately behave as a vector. We therefore have

$$(10)\qquad \begin{cases} q_{k_1} = q'_{k_1}\cos\alpha + q'_{k_2}\sin\alpha \\ q_{k_2} = q'_{k_2}\cos\alpha - q'_{k_1}\sin\alpha \\ p_{k_1} = p'_{k_1}\cos\alpha + p'_{k_2}\sin\alpha \\ p_{k_2} = p'_{k_1}\cos\alpha - p'_{k_1}\sin\alpha \end{cases}$$

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and from this, we obtain

$$(11)\quad \begin{cases} I'_{k_1} = \dfrac{(p'^2_{k_1} + q'^2_{k_1})}{2} = I_{k_1}\cos^2\alpha + I_{k_2}\sin^2\alpha + \sin 2\alpha(p_{k_1}q_{k_1} + p_{k_2}q_{k_2})\,, \\ I'_{k_2} = \dfrac{p'^2_{k_2} + q'^2_{k_2}}{2} = I_{k_1}\sin^2\alpha + I_{k_2}\cos^2\alpha - \sin 2\alpha(p_{k_1}q_{k_1} + p_{k_2}q_{k_2})\,. \end{cases}$$

(Note that the I'_{k2} do not transform as a vector.)

It is clear that I'_{k_1} and I'_{k_2}, which both contain the operators $(p_{k_1}q_{k_1}+p_{k_2}q_{k_2})$, do not commute with I_{k_1} and I_{k_2}. This demonstrates that the measurement of what is usually called « polarization » in a pair of orthogonal directions does not completely determine the result of a similar measurement in another pair of orthogonal directions.

In our original article, we have indicated this property of the polarization (*) but without going into as much detail as we did here. As we pointed out there one could already infer this property from equation (2) of the present article which states that the wave function (and not the I_{ki} operator) transforms as a vector. Whenever any measurement whatsoever yields two possible results, corresponding to orthogonal wave functions, then a linear combination of these wave functions represents an eigenstate of another operator, (or commuting set of operators), which fails to commute with the first operator, (or commuting set of operators). Since the polarization measurement in a rotated frame leads to just such a linear combination of wave functions, we concluded that the measurement of what is usually called linear polarization in the rotated frame corresponds to a set of operators that do not commute with the original set. A more familiar case of this behaviour is found with spin, where as equation (1) shows, the measurement of a given component of spin in a rotated frame leads to a similar linear combination of eigenfunctions corresponding to measurements in the original frame.

On the basis of the above discussion, we see that the experiment of Wu-Shaknov is an example of the paradox of E.P.R. As in the case of spins, we can here measure what is usually called the « linear polarization » of one of the photons, say A, and from this, we conclude that B has an orthogonal « linear polarization ». This can be done with the orthogonal pair of directions (x, y), equivalent to finding that the spin is $\pm\hbar/2$ in the z-direction; or else is can be done in any rotated pair of orthogonal directions (equivalent to measuring the component of the spin along a rotated axis). Since the operators corresponding to these sets of measurements do not commute, we have again

(*) See reference [1], p. 1073, first and second paragraph after equation (4).

the same kind of quantum mechanical correlation which was described in Section **2** in connection with the spin, and which is at the basis of the paradox of E.P.R.

4. – Discussion of criticism of Perez and Singer.

As we stated in the introduction to this article, the criticisms of the conclusions of our previous paper by PERES and SINGER are based on an erroneous application of classical conceptions of polarization of electromagnetic waves in the quantum domain. This is brought out most clearly in their main criticism (*), which occurs near the beginning of Section 3 of their article ([5]). They state that « linear polarization is useless (for the purpose of representing the paradox E.P.R. in terms of photons) because the two *directions* of linear polarization are similar to the two *values* of the spin in some given direction. Thus, obviously there can exist no uncertainty relations for linear polarization alone. »

The above quotation clearly implies that PERES and SINGER regard the usual linear polarization experiments as capable of determining the direction of linear polarization unambiguously.

As we have shown in Section **3**, however, such an implication is erroneous, because these experiments do not measure the field vector, but only certain functions I_{k_1} and I_{k_2}, which still leave a great deal of ambiguity in the phase, magnitude, and direction of this vector, as well as in the degree of ellipticity of the polarization. This ambiguity is reflected, as we have shown, in the fact that measurements of « linear polarization » in some other pair of directions does not commute with that in the original pair.

Because of the above misconception concerning the nature of polarization in the quantum domain, PERES and SINGER went on to suggest an alternative formulation of the paradox E.P.R. for photons, which they (also erroneously) claimed to be inadmissible. To do this, they introduce what are called « Stokes operators » which satisfy commutation rules (**) similar to that of spin. One of these operators corresponds to the circular polarization of a photon, and

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(*) PERES and SINGER begin this section with a statement that we « have overlooked the important fact that the polarization of photons is physically different from the spin of fermions, as photons have zero at mass, their spin... is always oriented in the direction of propagation. » Since we have never attempted to formulate the paradox of E.P.R. by considering directly the spin operator of photons, the above statement has no connection with our article, and is indeed quite irrevelant to the point at issue.

(**) See reference ([5]), equation (7).

another to its linear polarization. Because these operators do not commute, one can obtain the result already known by other methods that linear and circular polarization cannot be defined together. They then assert that this set of operators cannot be used to provide an example of the paradox of E.P.R. For precisely defined elements of reality would then have to exist in photon B, corresponding to the simultaneous definition of the states of linear and circular polarization. This, they say is « non-sensical because if the circular polarization of a photon is precisely defined, its linear polarization cannot be precisely defined, and vice-versa. » Thus, they conclude that the attempt to regard the Wu-Shaknov experiment as an example of the paradox of E.P.R. « does not lead to a paradox, but to an inconsistency ».

The above argument is likewise based on the unjustifiable use of the classical description of polarization in the quantum domain. Indeed, from their statement that there can exist no uncertainty relations for linear polarization alone, it would follow that once the wave is defined as « linearly polarized », its field vector would have a well defined direction, so that by definition it could not at the same time be circularly polarized. The error in this point of view is, as we have already pointed out, that the measurement of « linear polarization » does not define the field vector without uncertainties, but leaves a residual fluctuating part of undefined phase and amplitude. In other words, a better idea of this field vector is obtained by regarding it as elliptically polarized, with an indeterminate degree of ellipticity, and with an *average* polarization in the direction of the measurement. Clearly, such an elliptically polarized wave can be regarded as made up of a linearly polarized wave plus a circularly polarized wave. In the classical limit, both could be well defined and measured together, so that it would have meaning to specify the intensities and phases of both, without any self-contradiction. Quantum-mechanically, however, the two kinds of polarization cannot, as we shall see, be measured together, so that if one is well defined the other must be ambiguous.

We shall now discuss this problem more formally. In order to define the field vector completely, we need (as shown in Section 3) four operators, of which only two can be defined together. Instead of the original set (p_{k_i}, q_{k_i}), we can take the intensities, I_{k_1} and I_{k_2}, corresponding to « linear polarization » and another set, I_{k_+} and I_{k_-}, corresponding to « circular polarization ». These latter are defined by

$$(10)\qquad \begin{cases} I_{k_+} = \left[\dfrac{(q_{k_1} - p_{k_2})^2 + (q_{k_2} + p_{k_1})^2}{2}\right] = \dfrac{I_{k_1} + I_{k_2} + (p_{k_1} q_{k_2} - p_{k_2} q_{k_1})}{2}\,, \\[2ex] I_{k_-} = \left[\dfrac{(q_{k_1} + p_{k_2})^2 + (p_{k_1} - q_{k_2})^2}{2}\right] = \dfrac{I_{k_1} + I_{k_2} - (p_{k_1} q_{k_2} - p_{k_2} q_{k_1})}{2} \end{cases}$$

It is clear that I_{k_1} and I_{k_2} do not commute with I_{k_+} and I_{k_-}.

On photon A, we can then measure either the set I_{k_1} and I_{k_2} or the set I_{k_+} and I_{k_-} (which classically could, of course, be defined together). Thus, as in the case of spin, we have a set of non-commuting operators, subject to correlations at long distances, without any interactions. The analogy is that when « linear polarization » is measured, the « circular polarization » is indeterminate and vice-versa; while with spin, when one component is measured, the others are indeterminate and vice-versa. The fact that there are three components of the spin and only two for polarization is evidently not relevant here, nor is it relevant that polarization has phase as well as amplitude, while spin has no phase.

After discussing the above described example, PERES and SINGER state further that « it seems that the impossibility of constructing an E.P.R. paradox for photons is connected with the impossibility of describing them without second quantization, while *this paradox can be raised only for that part of quantum mechanics having a classical counterpart.* »

With reference to the above statement, we would like to emphasize that the essential characteristic of the paradox of E.P.R. is that there must be two or more non-interacting dynamical systems separated in space so that it will be certain that if one of these variables is measured, the other is not disturbed in any way. Then, if the properties of the above systems are correlated in the way that has been described (such that the measurement of one of a set of non-commuting observables of one of them provides the value of the corresponding observable in the other), then we have an example of the paradox. In the example of the Wu-Shaknov experiment, we describe the systems in terms of a pair of wave packets of the vector potential operator, representing photons, which are clearly very distant from each other in space, and which are not connected with each other in any way, while the measurements under discussion are taking place.

Finally, it should be stated that the quantum theory of the electromagnetic field, (*i.e.*, the theory involving second quantization) does evidently have a classical counterpart, namely, the classical theory of this field. Indeed, in defining our polarization operators we were guided by the classical limit, replacing dynamical variables of the field vectors by operators, etc. The only difference between our example and that given in the original paper of E.P.R. is that we have used a field example, while there, a particle example was used. But as shown above, this difference is not relevant.

Elsewhere in their article (*), PERES and SINGER criticize our conclusions along a different line, claiming that our analysis of the Wu-Shaknov experiment would, in any case, not be of importance, because the conclusions that

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(*) See reference (5), section (2).

we drew could have been obtained immediately by considering the problem of parity conservation in electrodynamics.

Briefly, this point is concerned with the Furry hypothesis (*), which was aimed at avoiding the « paradoxical » features of the quantum mechanical treatment of this problem. This hypothesis involved the assumption that the many-body Schrödinger equation is correct for atomic orders of distance (where it has been tested quite well), but breaks down in a fundamental way at macroscopic distances (where the paradox of E.P.R. is relevant and where there has previously been no clear experimental test). Briefly, FURRY considers the possibility that in the latter case, the wave function for the system becomes a product of the wave functions of the parts, with no superposition, but with a certain probability that a particular product will appear. In our original article we calculated the results of the Wu-Shaknov experiment according to the Furry hypothesis with all possible assumptions concerning the polarization states with which the system separates, and we have shown that they are all inconsistent with the results of the experiment (which were already known to agree within experimental error with the predictions of the quantum theory).

PERES and SINGER remark that for the case of a pair of photons, the Furry hypothesis would lead to non-conservation of parity. They then assert that if parity were not conserved, this would have been noticed long ago, because quantum electrodynamics allows experiments of the highest accuracy.

It must be stated, however, that no test of conservation of parity in electrodynamics with regard to macroscopic orders of distances was possible, previous to the Wu-Shaknov experiment (and similar experiments). First of all, it is evident that no purely classical observations of the electromagnetic field can possibly test for the conservation of parity. For the parity is defined as a reflection property of the *wave function* of the whole system (in this case the electromagnetic field). Since the wave function does not appear in the classical limit, no classical property can depend on the parity of this function. The fact (to which PERES and SINGER have alluded) that parity is a discrete quantum number, for which there is no meaning to statistical conservation, is therefore not relevant in the classical limit.

Thus far, parity conservation has been tested most accurately in spectroscopy, but here the many-body systems (*e.g.* atoms) have not extended over very long distances, so that these experiments do not test for the possibility of a breakdown of conservation of parity in the case of the paradox of E.P.R. The only way to test for this breakdown is to consider an experiment carried out to a quantum-mechanical level of accuracy on a many-body problem, in

(*) See reference (3).

which there is a correlation of the properties of systems at a macroscopic order of distance; and as we have already stated, experiments of the general type that we have described here are the first case where such a test can be made.

RIASSUNTO (*)

In un lavoro precedente abbiamo suggerito come prova per il paradosso di Einstein, Podolsky e Rosen, l'esperimento di WU-SHAKNOV sulla radiazione di annichilazione del positronio. In questo articolo rispondiamo ad alcune critiche alle nostre conclusioni, avanzate da PERES e SINGER. Si mostra come queste critiche siano erronee, essendo basate su una scorretta interpretazione della polarizzazione della radiazione elettromagnetica nel dominio quantistico.

(*) *Traduzione a cura della Redazione.*