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Quantum Aspects of the Equivalence Principle

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Two thought experiments are discussed which suggest, first, a geometric interpretation of the concept of a (say, vector) potential (i.e., as a kinematic quantity associated with a transformation between moving frames of reference suitably related to the problem) and, second, that, in a quantum treatment one should extend the notion of the equivalence principle to include not only the equivalence of inertial forces with suitable "real" forces, but also the equivalence of potentials of such inertial forces and the potentials of suitable real forces. The two types of cancellation are physically independent of each other, because of the Aharonov–Bohm effect. Finally, we show that the latter effect itself can be understood "geometrically" as a kinematic effect arising upon the transformation between the two reference frames.

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1. INTRODUCTION

It was previously predicted[1] and subsequently verified by experiment[2] that the electromagnetic potentials $A_\mu(\mathbf{x}, t)$ possess, within the framework of quantum mechanics, a well-defined (measurable) physical meaning of their own, even in regions where nonzero A_μ give rise to vanishing **B** and **E**. At the time[1] it was hinted that one may consider this physical effect as a *non local* action of **B** and **E** if one prefers to remain within a language using **B** and **E** only (e.g., one may say that the nonzero flux of **B** through a confined region affects the interference pattern of an electron beam moving strictly outside that region, i.e., in a domain where **B** and **E** are zero).

The purpose of this note is twofold. First, to provide a *geometric* understanding of this quantum effect of the potentials, i.e., as a straightforward result of the transformation formula for velocities from one moving reference frame to another. The second purpose is to point out some further conceptual conclusions from the above effect. These, we feel, should be of intrinsic interest, whether or not they will suggest an additional practical experiment. In short, these conclusions point to a quantum extension of the principle of equivalence: Not only can "inertial forces" arising upon transition to a noninertial system of reference be cancelled by the introduction of suitable "real forces" (and vice versa), but the "inertial potentials" belonging to such forces (even in "force-free" regions) can similarly be cancelled. Both facts together afford a "geometric interpretation" (i.e., as inertial effects produced by a change of reference frames) to *both* fields of forces *and* their potentials.

The two purposes stated are, as we shall see, connected in a natural way. Thus, if a frame somewhere in the cosmos would tentatively be considered as "inertial" by a classical physicist because, locally, no forces on particles are detectable, the *quantum* physicist could go one step further to determine whether this is really so. He would set up an experiment detecting, say, the presence of (force-free) potentials $A\mu$ in that region. This experiment cannot decide whether it occurs in an *inertial* system pervaded by "physical" nonzero A_μ, or in a noninertial system. A decision between these two versions could of course be attempted by checking the relative motion of the frame which has previously been decided (by convention) as being inertial (such a decision can be backed by measurements as far as forces go, but remains a convention as far as the potentials are concerned). In the next paper of this series[3] we show, however, that even when a frame B moves *uniformly* with respect to an *inertial* system A it need not be inertial itself. This is when the two systems are "quantum-related," e.g., if the relative velocity of B to A has a quantum spread which cannot be neglected. It will turn out that this relationship between the two frames can be expressed via a forceless vector potential **A** arising upon transformation from frame A to frame B, where **A**, too, has a quantum spread and therefore cannot be removed by a c-number transformation.

2. A THOUGHT EXPERIMENT

Consider a "laboratory" confined to an arbitrarily narrow ring of radii r_1 and r_2 (Fig. 1) centered at O. If the laboratory is rotated with angular velocity $\dot{\theta}_0$ around O, and the physicist fixed in that lab (call him the "rotonaut") observes the motion of particles which he releases (with initial r_0 and v_0 relative to his lab), he will conclude that there are two types of nonzero forces acting in his lab (explained as "centrifugal" and "Coriolis" by the stationary observer).

In a canonical formalism the velocity-dependent Coriolis force can be described via the curl of a suitable vector potential (while the centrifugal force is describable through the gradient of a "scalar" potential), rendering the situation completely isomorphic to an electromagnetic field. As a result, $p_\theta = m\ddot{\theta}r^2 - mr^2A_\theta = nh$, i.e., the angular velocity of a particle in the ring will be quantized in the form $\dot{\theta} = [(n + \alpha)/mr^2]\, h$ where $\alpha = A_\theta/h$ is in general a noninteger arising from the vector potential.

This can be understood in the following simple geometric way: The stationary observer, with respect to whom the ring rotates with angular velocity $\dot{\theta}_0 = \omega$, would find the discrete values $p_\theta^{st} = nh$ and $\dot{\theta}^{st} = nh/mr^2$ (where st indicates stationary) and he would thus predict that the rotonaut would find $\dot{\theta} = \dot{\theta}^{st} - \dot{\theta}_0$, i.e., he would explain A_θ as $A_\theta = \dot{\theta}_0$.

This potential can be "isolated" in the ring as a forceless potential on which the quantum experiment described in Refs. 1 and 2 can be performed: It is possible in principle to introduce gravitational fields $g_{\mu\nu}$ which will cancel centrifugal and Coriolis forces just inside the ring. However, to simplify the following discussion, let us assume that all particles are charged with the same ratio e/m. The above forces could then be cancelled by radial electric (cancelling the centrifugal) and uniform magnetic forces, the latter perpendicular to the plane (cancelling the Coriolis force). The

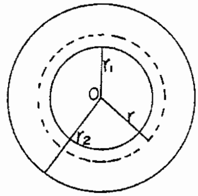

Fig. 1

rotonaut would then tend to conclude that his is an inertial system. Since we assume that the **B** fields are confined to the ring only, the inertial fields in the interior of the circle r_1 remain uncompensated and provide the "flux" which produces (forceless, in the ring) potentials A_μ outside the circle r_1. Thus the rotonaut would observe a nonvanishing $\oint \mathbf{A} \cdot d\mathbf{l} = \oint \mathbf{C} \cdot d\mathbf{S}$, where the path of integration is a circle r with $r_1 < r < r_2$. Here **C** is the Coriolis force inside the circle r_1 and **A** is its vector potential ($\mathbf{C} = \nabla \times \mathbf{A}$), and we assume $r \approx r_1$, so that the part of $\oint \mathbf{C} \cdot d\mathbf{S}$ lying inside the ring is arbitrarily small. Thus $\oint \mathbf{A} \cdot d\mathbf{l}$ is nonzero and could be detected by a shift in the interference pattern, as in Ref. 1. This could indeed be done by the rotonaut, to double-check whether his lab is indeed inertial (and he would find that it is not).

An alternative way to check this question is, of course, to measure $\oint \mathbf{v} \cdot d\mathbf{l}$ and see whether it is nh or $(n + \alpha)\, h$ (α is generally noninteger).

3. GEOMETRIC INTERPRETATION OF THE QUANTUM EFFECT OF POTENTIALS

Beyond the geometrical interpretation of the A_μ, one may attempt to interpret their quantum interference effect[1,2] geometrically. This can be done as follows.

Consider first an interference experiment of an electron in the ring when no cancelling **E** and **B** fields have yet been added. The stationary observer would describe the situation by saying that there exist no fields whatsoever, but that the rotonaut has sent two wave packets with (relative to the rotonaut) equal but opposite velocities to go around the ring and to meet (and interfere) after having traveled (according to the rotonaut) equal distances, i.e., at the point diametrically opposite to the starting point. Since the stationary observer does not consider the velocities of both packets as equal (but, say, $|\dot{\theta}_l| < |\dot{\theta}_r|$ if the rotonaut rotates counterclockwise), he will predict (and find) them meeting to the left of the point diametrically opposite the starting point.

From the point of view of the rotonaut (who would have expected a shift away from the diametrically opposite point in the interference pattern if his region had forceless potentials) this result corresponds to the fact shown by Werner and Brill[4] that, if a magnetic field is constant and homogeneous everywhere, there will be no shift of the interference pattern: The shifts produced by the A_μ and by the *local* **B** will cancel each other.

On the other hand, when the interference effect is carried out after the **E** and **B** fields have been added in the region of the ring (only), in order to cancel the effects of the inertial forces there, the stationary observe will say that he sees a **B** field in the ring and hence a shift in the interference pattern (equal but opposite in direction to what the **A** of such **B** within r_1 would have produced) toward what he considers as the diametrically opposite point to the starting point of the two packets.

The rotonaut, on the other hand, would claim that his is a stationary, nonsingly connected region with forceless potentials (produced by the "flux" of the Coriolis force through the disk r_1) and hence there is a (rightward) shift in the interference pattern. The fact that $A_\theta = \dot{\theta}_0$ kinematically accounts for the fact that the shift is just such as if to give the two packets equal angular velocities with respect to the *stationary* observer.

4. CONCLUSION

The thought experiments discussed show the following.

1. In a simple case the vector potential **A** can be interpreted geometrically as the relative angular velocity of one frame rotating with respect to another (with obvious generalizability to more complex geometric transformations.

2. One should extend the notion of the equivalence principle to include not only the equivalence of inertial forces with suitable "real" forces, but also the equivalence of potentials of such inertial forces (in regions where the forces themselves have been cancelled by suitable real forces) and the potentials of suitable "real" forces. The two types of cancellations are independent of each other because in a restricted, nonsimply connected laboratory the inertial forces may be cancelled locally, but not uitside, and hence at the same time the potentials may not be cancelled inside the lab. This is equivalent to a nonlocal effect in the lab of the uncancelled forces outside.

3. The quantum effect of potentials[1,2] can be interpreted geometrically and, together with the geometric interpretation of A_μ ($A_\theta = \dot{\theta}_0$ in the case considered here) implies as a necessary corollary the result of Brill and Werner: The shifts in the interference pattern caused by **A** and by the local **B** cancel each other in homogeneous **B** fields.

The considerations of this paper suggest a close connection between potentials and changes of frames of reference, which will be further analyzed in a subsequent paper.

REFERENCES

1. Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
2. R. G. Chambers, *Phys. Rev. Lett.* **5**, 3 (1960); H. A. Fowler, L. Marton, Y. A. Simpson,

and J. A. Suddeth, *J. Appl. Phys.* **32**, 1153 (1961); H. Boersch, H. Hamisch, D. Grohmann, D. Wohlleben, *Z. Phys.* **159**, 397 (1960); **164**, 55 (1961); **165**, 79 (1961); **167**, 72 (1962); **169**, 263 (1962).

3. Y. Aharonov and G. Carmi, Quantum related reference frames and the local physical significance of potentials, to appear in *Found. Phys.*
4. F. G. Werner and D. R. Brill, *Phys. Rev. Lett.* **4**, 349 (1960).