Spontaneously Broken SU(3) Symmetry in a Laser

Y. Aharonov
Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208, and Physics Department, Tel-Aviv University, Ramat-Aviv, Israel 41611

and

J. M. Knight
Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208
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Constants defining a quantum state can serve as macroscopic parameters specifying the spontaneous breaking of a symmetry. An example is given in which a laser possesses a SU(3) symmetry which is spontaneously broken above threshold. The parameters can be readily observed experimentally.

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A single spin in quantum mechanics cannot have a definite direction. For example, if the state with z component of spin $m_s = \frac{1}{2}$ is rotated to a new direction θ, φ, the resulting state

$$|\chi\rangle = |\tfrac{1}{2}\rangle \cos(\tfrac{1}{2}\theta) + |-\tfrac{1}{2}\rangle e^{i\varphi} \sin(\tfrac{1}{2}\theta) \quad (1)$$

has an overlap $\cos\theta$ with the unrotated state, and consequently is not independent of that state. It is thus impossible to speak in the classical sense of a definite spin direction, since there is no spin state which is strictly localized in direction. On the other hand, in a system containing a large number n of spins, the overlap $\cos^n(\frac{1}{2}\theta)$ between states with all spins aligned approaches zero as n becomes infinite. States differing in direction become independent in this limit, so that the defining parameters represent quantities with well-defined classical analogs. A frequently cited example of such a quantity is the magnetization vector of a simple ferromagnet, which has a nonzero value as a result of spontaneous breaking of the underlying rotational symmetry when dynamical conditions favor alignment of the spins.

The phenomenon described here is not peculiar to the spin angular momentum and the associated rotation group. Any parameters which serve to define a quantum state can generate macroscopic observables if the "alignment" of many microscopic systems in the same state is brought about by the dynamics of the system. In the examples known from thermal physics, the macroscopic variables have a simple transformation character, usually that of a vector quantity, as in a ferromagnet, or of a phase, as in a superconductor.[1] The significance of the example presented in this Letter lies in the fact that the group G involved is larger than the rotation group, and, in fact, contains the rotation group as a subgroup. The set of macroscopic variables generated is therefore more general in character than a rotational tensor, but nevertheless contains the purely rotational parameters as a subset. Thus, when we make a symmetry transformation from G, we intermix parameters defining the direction of the system in space with other parameters which do not have a direct spatial interpretation. In our example, the additional parameters are readily identified, as we show below, and are closely correlated with familiar properties of the system. It is tempting to speculate, however, that other examples might exist in which the mechanism described here gives rise to macroscopic variables of an unfamiliar and unsuspected nature.

The example that we propose is drawn from the field of laser physics and involves a symmetry which is spontaneously broken when the laser undergoes its threshold transition. When this occurs in a single-mode laser, the parameter which assumes a macroscopic significance is the phase[2-5] of the laser radiation. This phase is correlated with the relative phase of the upper and lower levels in the superposition which describes the emitting atoms. The single-mode laser thus provides another illustration of the mechanism which we are discussing. The case which we now proceed to consider, being based on a much larger symmetry group, offers correspondingly richer possibilities and promises to be more directly observable in the laboratory.

Consider a laser in which the active atoms make a $p \to s$ transition in a spherical cavity. When the pump intensity is not zero, individual atoms in the cavity will be found in states which are superpositions of the s state and a p state of the general form

$$|\psi\rangle = |1\rangle\cos(\omega/2) + |0\rangle e^{i\xi}\sin(\omega/2)\cos(\chi/2) + |-1\rangle e^{i\eta}\sin(\omega/2)\sin(\chi/2), \tag{2}$$

where the basis states are labeled by the eigenvalues m_z. This state has four "directional" parameters ω, χ, ξ, and η, analogous to the parameters θ and φ of state (1). If we rotate the single state $m=1$ to an arbitrary direction θ, φ in space as we did to obtain the state (1), we find instead

$$|\psi\rangle = |1\rangle\cos^2(\tfrac{1}{2}\theta) + |0\rangle e^{i\varphi}\sin\theta/\sqrt{2} + |-1\rangle e^{2i\varphi}\sin^2(\tfrac{1}{2}\theta), \tag{3}$$

which has the same form as (2), but which has only two parameters instead of four. Clearly, the states (2) are transformed among themselves by a group which is larger than the rotation group, but which contains the rotation group as a subgroup. The group is SU(3). It can be realized as the set of complex special unitary transformations of the basis vectors $|1\rangle$, $|0\rangle$, and $|-1\rangle$.

As in the spin example, the parameters ω, χ, ξ, and η become strictly local variables only when a large number of atoms become "aligned" in the state (2) with common values of these parameters. It is evident that ω, χ, ξ, and η specify a direction in the complex space in which the SU(3) transformations operate in the same way that θ and φ specify a direction in ordinary space. Although two independent combinations of the four parameters may be used to define the direction of the total dipole moment of the atomic system, there remain two combinations which have no direct interpretation in configuration space. Their significance will become evident below.

With suitable choice of parameters and conditions, the dynamical equations for the laser are indeed invariant with respect to the group of SU(3) transformations described above. In a spherically symmetric cavity, each characteristic electric dipole frequency will be represented by three modes, labeled by an integer $m=1, 0, -1$ proportional to the z component of the field angular momentum. If one characteristic frequency is centrally tuned to the atomic transition frequency $(E/\hbar)$, and the atoms are placed at the center of the cavity in a region with dimensions which are small compared to the wavelength, then all other modes of the cavity will remain inactive.

The symmetry of the system can then be exhibited at the most fundamental level by restricting the Hamiltonian to the subspace spanned by the four atomic states and the three active field modes. Introducing the atomic operators $b_m^\dagger$ defined by $b_m^\dagger|s\rangle = |m\rangle$ and $b_m^\dagger|m'\rangle = 0$, and the creation operators for the field modes, we find

$$H = \epsilon\sum_m (b_m^\dagger b_m + a_m^\dagger a_m) + g\sum_m (\langle m|Y_1^m|s\rangle a_m b_m^\dagger + \text{H.c.}), \tag{4}$$

where the matrix element of Y_1^m between the atomic states arises from the dipole approximation to the $\vec{A}\cdot\vec{p}$ term in the full Hamiltonian. This matrix element is independent of m. In writing (4) we have made the rotating-wave approximation by omitting terms containing $a_m b_m$ and $a_m^\dagger b_m^\dagger$. The SU(3) group which transforms the states (2) among themselves acts directly on the triplet (b_1, b_0, b_{-1}) and simultaneously transforms the triplet (a_1, a_0, a_{-1}) of field operators, leaving the Hamiltonian (4) invariant.

The particular symmetry considered here operates in a small subspace of the states of the system, and will therefore not be evident upon inspection of the full atom-field Hamiltonian. It will hold in any atomic subspace spanned by states connected by a specific dipole transition, provided that the dipole approximation is valid, and assumes a simple form only when the atomic transition involves an s and a p state.

When certain conditions are met, the full laser equations exhibit the same symmetry possessed by the Hamiltonian (4). The derivation of these equations, to be presented elsewhere, proceeds in a straightforward manner from standard semiclassical laser theory.[6] The complex mode amplitudes E_1, E_0, and E_{-1} are found to satisfy the equations

$$\dot{E}_m = aE_m - bE_m\sum_{m'}|E_{m'}|^2, \tag{5}$$

where a and b are parameters depending on the pump intensity or injection rate, on the level widths, and on the matrix element for the atomic dipole transition. We have made a number of assumptions in arriving at (5). In addition to the rotating-wave approximation, we have supposed (i) that the field-damping mechanism is spherically symmetric and thus affects all three modes equally, (ii) that the atoms are pumped or injected into the cavity in a symmetrical manner with initial density matrix proportional to $\delta_{mm'}$, and (iii) that there is no significant dephasing among the three m states. The last assumption will be valid if the lifetime of the upper level is much less than the collision time.

A special unitary transformation of the complex field vector (E_1, E_0, E_{-1}) leaves Eqs. (5) invariant, and reflects the invariance of the Hamiltonian (4). Equation (5) implies that when the pump intensity exceeds its threshold value, making the parameter a positive, the square of the magnitude of the complex field vector, $\sum|E_m|^2$, reaches a stable equilibrium value a/b. The direction of this vector in the complex space, however, is not determined, and the equilibrium is neutral with respect to the parameters defining this direction. The situation can be described by saying that above threshold there is a spontaneous breaking of the underlying SU(3) symmetry. Under the completely symmetrical conditions contemplated here, the direction in an actual experiment would be determined by a random fluctuation which would be amplified above threshold. Alternatively, a small symmetry-breaking interaction or a small asymmetry in the pump, injection, or damping parameters could define the direction.

The significance of the symmetry transformation and the corresponding macroscopic parameters becomes evident when we examine the field at the center of the cavity:

$$\vec{E}(0)=E_1(\hat{x}+i\hat{y})/\sqrt{2}+E_0\hat{z}+E_{-1}(\hat{x}-i\hat{y})/\sqrt{2}. \quad (6)$$

In addition to the purely rotational transformations represented by Eq. (3), there are transformations which modify the polarization states of the field from one type of elliptical polarization to another. For example, a real 45° rotation in the E_1-E_{-1} plane changes a state of linear polarization into a state of circular polarization and vice versa. This set of arbitrary polarization transformations is isomorphic to the rotation group, but forms a subgroup of G distinct from the rotation subgroup. The general transformation of G is a combination of the two. We note that all transformations of G induce changes in the distribution of intensity and polarization of the emitted radiation, and that these changes can be monitored experimentally by allowing a fraction of the radiation to escape from the cavity.

The growth of the field amplitudes to their above-threshold value is accompanied by an "alignment" of the atoms of the cavity. The macroscopic parameters arising from this alignment describe not only the direction of the collective atomic dipole moment in space but also the phase relations between the states in the superposition (2) which determine the polarization state of the laser radiation. This is the precise implication of the enlarged symmetry as far as the atoms are concerned. The semiclassical calculation leading to Eq. (6) allows identification of the atomic state involved in terms of the complex field amplitudes. The atomic density operator contains, in second order, a contribution $\rho_{mm'}=CE_mE_{m'}^*$, in addition to contributions from the pump and decay processes which are diagonal in the (m, m') subspace. This contribution may be interpreted as arising from a superposition of s and p states in which the p state has the form $|\psi\rangle \sim E_m|m\rangle$, establishing a correspondence of the complex field amplitudes with the parameters of Eq. (2) for the state of the active atoms in the cavity.

The complex field vector and the associated complex vector describing the atomic state undergo fluctuations due to spontaneous emission and thermal noise. Although the magnitude of this vector is stable with respect to such fluctuations, its direction in the complex space is subject to diffusion.[7] This is analogous to the diffusion of the phase in a single-mode laser, but here the domain in which it takes place is multidimensional. Furthermore, the diffusion is correlated with directly observable variations in the intensity and polarization of the laser radiation. It is reasonable to expect that the time scale on which it takes place, being inversely related to the laser linewidth, can be made long enough to be readily observable. Low frequency is indicated in order to reduce spontaneous emission noise, and to allow enough atoms within a wavelength of the cavity center to achieve high intensity. Thermal noise would impose a lower limit on the frequency.

The point of view advanced here and the particular configuration proposed immediately suggest further lines of investigation, both theoretical and experimental. We plan to present later a fuller discussion of the system including the formulation of conservation laws which follow from the symmetry and a treatment of fluctuations and the effects of symmetry breaking. On the experimental side, in addition to observing the spontaneous breaking of the higher symmetry SU(3), one could study the gradual diffusion in direction of the field vector, and the possible fixing of this direction by small asymmetries in the system. These inherent asymmetries could be either compensated or enhanced by introducing similar deviations from symmetry in pumping the atoms or in injecting them into the cavity. The neutrality of the system with respect to direction in SU(3) space would make it a sensitive indicator of small deviations from symmetry which might not easily be observable by other means.

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Acollinearity Distribution in Lepton-Hadron Scattering

J. Cleymans and M. Kuroda

Department of Theoretical Physics, University of Bielefeld, D-4800 Bielefeld 1, Germany

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The phenomenological consequences of soft-gluon radiation are considered for the acollinearity distribution in deep-inelastic lepton-hadron scattering. It is emphasized that energy-weighted acollinearity distributions provide a good way for studying the deflection of quarks by soft gluons.

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The effects of hard-gluon bremsstrahlung have been demonstrated very clearly by the observation of three-jet events in e^+e^- annihilation[1] and, more recently, in the observation of a three-jet structure in the final state of the scattering of neutrinos on nuclei.[2] The agreement between the experimental distributions and lowest-order calculations based on the radiation of one gluon from one of the quark lines is very satisfactory[3] and provides great confidence in the correctness of the underlying ideas.

The region where two of the jets overlap can, however, not be treated with such a simple procedure for the simple reason that the lowest-order calculation becomes very big and higher-order terms can no longer be neglected in the region where gluons become soft. It is therefore essential to sum over the emission of all soft gluons in this region. This was first done for the leading doubly logarithmic terms.[4] A thorough phenomenological analysis of the acollinearity distribution of hadrons in e^+e^- annihilation, as measured by Berger *et al.*,[5] in terms of the emission of soft gluons has recently been done by Baier and Fey,[6] who showed that once the data are fitted at a lower energy [9.5 GeV in the center-of-mass system (c.m.s.)] the effects of soft gluons could account for the evolution of the data at a higher energy (30 GeV in the c.m.s.). Similar analyses based on the Sudakov form factor[7,8] and on a Monte Carlo calculation[9] also gave good descriptions of the data. These provide support for the correctness of the treatment of soft-gluon emissions.

In this paper we want to extend the calculations of Baier and Fey to the cases of deep-inelastic lepton-hadron scattering where very interesting data are presently being analyzed which may provide further evidence for the effects of soft-gluon emission. We emphasize the importance of studying experimentally the energy-weighted acollinearity distribution in lepton-hadron scattering as a very good way to look for soft-gluon radiation by quarks.

The dynamics of the process is schematically