Near-resonance absorption processes: A model study

C. K. Au

Department of Physics, University of South Carolina, Columbia, South Carolina 29208 and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 and Department of Physics, Hong Kong University, Hong Kong

Y. Aharonov

Department of Physics, University of South Carolina, Columbia, South Carolina 29208 and Department of Physics, University of Tel Aviv, Israel

(Received 29 June 1981; accepted for publication 31 March 1982)

We construct an exactly soluble model in which an otherwise bound system can make transitions to another channel with a continuous energy spectrum to simulate the decay of an excited atom by the emission of photons. We show that the resonance absorption line shape occurs naturally in such a model.

I. INTRODUCTION

It is well known that a physical atom in an excited state decays because of its coupling to the electromagnetic field. To account for such decays, the energy eigenvalues for the excited states are often phenomenologically replaced by a complex number whose real part represents the shifted energy level due to virtual interactions with the electromagnetic field and whose imaginary part is the half-width of the excited state in question.[1] Such replacements are justified from the scattering point of view.[2,3] It also explains the Lorentzian line shape of near-resonance photoabsorption by atoms and the decay of excited states.[4] While much attention is paid to the energy shift and the width, very often, the corrections to the wave functions of the excited atomic states do not receive the deserved attention. The excited state of the atom in the presence of coupling to the electromagnetic field is made up of a superposition of photon states and atomic bare states. The projection of the physical state upon each bare state depends mainly on the energy difference of the physical state and the bare state in question.

In the case of atoms, this fact is hard to demonstrate even for the simplest case, that of the nonrelativistic hydrogen atom coupled to the electromagnetic radiation field. This is due to the fact that the Hamiltonian is not exactly diagonalizable and that perturbation calculation involves infinite sums over intermediate states in an infinite dimensional Hilbert space. All this seems to leave a gap in our understanding of near-resonance absorption processes, at least from the pedagogical point of view. For this very reason, we have decided to investigate simple nonrelativistic models, in which an otherwise bound system is coupled to another particle with infinite and continuous eigenenergy modes. Such models may lack physical significance. However, their simplicity and the fact that they are exactly soluble should lead to a better understanding, at least pedagogically, of near-resonance absorption processes.

In Sec. II, we describe such a model and give its exact solution. In Sec. III, we show the resonance characteristics. In Sec. IV, we give the conditions for the existence of truly bound states. In Sec. V, we show how this model leads to resonance line shapes. In Sec. VI, we draw some concluding remarks and give an alternate model, but without the detailed solutions, that would lead to similar behavior.

II. SOLUBLE MODEL OF RESONANCE IN ONE DIMENSION

We consider a one-dimensional unit mass nonrelativistic particle with two internal degrees of freedom (channels) represented by the components of an isospinor. In the physical state, these two channels are connected via a weak coupling strength λ,[5] so that the total Hamiltonian can be written as ($\hbar = c = 1$):

$$H = H_0 + \lambda\sigma_1, \tag{2.1}$$

where H_0 is the Hamiltonian for the bare states, i.e., in the absence of the coupling λ:

$$H_0 = (p^2/2)1 + [(1+\sigma_3)/2]V(x) + U(x)1, \tag{2.2}$$

where $\sigma_1, \sigma_2, \sigma_3$ are the usual Pauli isospin matrices and 1 is the 2×2 unit matrix,

$$V(x) = -2V_0 < 0 \quad 0 < x < 1, \tag{2.3a}$$

$$= \infty \quad 1 \leqslant x; \tag{2.3b}$$

and

$$U(x) = 0 \quad 0 < x, \tag{2.4a}$$

$$= \infty \quad x \leqslant 0. \tag{2.4b}$$

The eigenstates of H_0 are

$$\psi_{\text{up}} = \theta(x)\theta(1-x)\sin k_n x \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{2.5}$$

where

$$k_n = \pi n \quad n = 1,2,3,..., \tag{2.6}$$

corresponding to discrete energy levels

$$E_n = (\pi^2 n^2/2) - 2V_0, \tag{2.7}$$

for the isospin-up state, and

$$\psi_{\text{down}} = \theta(x)\sin k' x \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.8}$$

where k' is any positive number corresponding to a continuous spectrum energy

$$E = k'^2/2, \tag{2.9}$$

for the isospin-down state. $\theta(x)$ is the unit step function which equals unity for positive arguments and zero otherwise.

To solve the eigenvalue equation for the total Hamiltonian (2.1), it is more convenient to split the Hamiltonian into two domains: $x<1$ and $x>1$. The solution is then obtained by matching the solutions in the two domains at the boundary $x=1$. We rewrite

$$H=H_{<}\,\theta(x)\theta(1-x)+H_{>}\,\theta(x-1), \tag{2.10}$$

where

$$H_{<}=\left(\frac{p^2}{2}-V_0\right)\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}-V_0&\lambda\\\lambda&V_0\end{pmatrix} \tag{2.11}$$

and

$$H_{>}=\frac{p^2}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}\infty&\lambda\\\lambda&0\end{pmatrix}. \tag{2.12}$$

The eigenvalue problems for $H_{<}$ can be solved by exact diagonalization of the matrix. $H_{<}$ admits any positive energy eigenvalue and thus has a continuous spectrum for whatever the depth of the well $2V_0$. $H_{<}$ has a spectrum of bound states if the well is sufficiently deep, when $V_0>\pi^2/4$, a case which we will consider later. For a sufficiently shallow well, $H_{<}$ only has a positive energy spectrum with a continuous eigenvalue E and the eigenfunctions

$$\theta(x)\theta(1-x)\sin k_{\pm}x\begin{pmatrix}\alpha_{\pm}\\\beta_{\pm}\end{pmatrix}, \tag{2.13}$$

where

$$k_{\pm}=\{2[E+V_0\pm(\lambda^2+V_0)^{1/2}]\}^{1/2}, \tag{2.14}$$

such that

$$\alpha_{-}=\beta_{+}=\lambda/[2(\lambda^2+V_0^2)+2V_0(\lambda^2+V_0^2)^{1/2}]^{1/2} \tag{2.15}$$

and

$$\alpha_{+}=-\beta_{-}=\lambda/[2(\lambda^2+V_0^2)-2V_0(\lambda^2+V_0^2)^{1/2}]^{1/2}. \tag{2.16}$$

If we assume that the coupling strength λ is small compared to the well depth V_0, then the above can be expanded in powers of λ and we obtain

$$\alpha_{-}=\beta_{+}=\lambda/2V_0+O(\lambda^3) \tag{2.15'}$$

and

$$\alpha_{+}=-\beta_{-}=-1+\tfrac{1}{8}\lambda^2/V_0^2+O(\lambda^4). \tag{2.16'}$$

From (2.15) and (2.16), we have

$$\alpha_{-}\beta_{+}-\beta_{-}\alpha_{+}=1, \tag{2.17}$$

without any approximation. In the event that the well depth V_0 becomes zero, we have

$$\alpha_{-}=\beta_{-}=1/\sqrt{2}=\beta_{+}=-\alpha_{+}, \tag{2.18}$$

irrespective of the value of λ. Thus the depth of the well is a measure of the inertia against the isospin rotation due to the coupling λ. As for $H_{>}$, it only admits isospin down states with a continuous spectrum of positive eigenvalues. The general solution for the total Hamiltonian can now be found by matching the boundary conditions. The unnormalized eigenstate corresponding to a positive energy eigenvalue E is

$$\psi=\frac{2\theta(x)e^{-ik}}{1-iZ}\left\{\theta(1-x)\left[\frac{\alpha_{-}\sin k_{+}x}{\sin k_{+}}\begin{pmatrix}\alpha_{+}\\\beta_{+}\end{pmatrix}-\frac{\alpha_{+}\sin k_{-}x}{\sin k_{-}}\begin{pmatrix}\alpha_{-}\\\beta_{-}\end{pmatrix}\right]+\begin{pmatrix}0\\1\end{pmatrix}\theta(x-1)[\cos k(x-1)-Z\sin k(x-1)]\right\}, \tag{2.19}$$

where $k_{\pm}$, $\alpha_{\pm}$, and $\beta_{\pm}$ are as defined by (2.14), (2.15), and (2.16),

$$k=\sqrt{2E}, \tag{2.20}$$

and

$$Z\equiv(\alpha_{+}\beta_{-}k_{-}\cot k_{-}-\alpha_{-}\beta_{+}k_{+}\cot k_{+})/k. \tag{2.21}$$

III. RESONANCE BEHAVIOR TO ORDER λ^2/V_0

If not for the coupling λ, the spin-up states are all bound states corresponding to the energy eigenvalues

$$E_{n,\mathrm{res}}=(\pi^2n^2/2)-2V_0. \tag{3.1}$$

We therefore expect the system to exhibit resonance behavior as the energy E of the system approaches $E_{n,\mathrm{res}}$. For a sufficiently shallow well, when $V_0<(\pi/2)^2$, there will be no true bound states, only resonance states (as well as antiresonance states). For a sufficiently deep well, when $V_0>(\pi/2)^2$, bound states are possible. However, to simplify the discussion, we shall exclude the situation where $V_0=(\pi n/2)^2$ for any integer n. This corresponds to the situation where a resonance energy is identical to the onset of the continuum, where we may expect to encounter a pole embedded in a cut, and coincident with the branch point in the complex E plane. With this in mind, we let

$$E=(\pi^2n^2/2)-2V_0+\Delta, \tag{3.2}$$

where Δ is the amount of detuning from resonance. Then to order λ^2/V_0, and Δ, we have

$$k_{+,n}=\pi n+(2\Delta+\lambda^2/V_0)/2\pi n, \tag{3.3a}$$

$$\cos k_{+,n}=(-1)^n, \tag{3.3b}$$

$$\sin k_{+,n}=\tfrac{1}{2}[(2\Delta+\lambda^2/V_0)/\pi n](-1)^n, \tag{3.3c}$$

$$\cot k_{+,n}=2\pi n/(2\Delta+\lambda^2/V_0), \tag{3.3d}$$

$$k_{-,n}=(\pi^2n^2-4V_0)+\frac{1}{2}\frac{(2\Delta-\lambda^2/V_0)}{(\pi^2n^2-4V_0)^{1/2}}, \tag{3.4a}$$

$$\cos k_{-,n}\cong\cos(\pi^2n^2-4V_0)^{1/2}, \tag{3.4b}$$

$$\sin k_{-,n}\cong\sin(\pi^2n^2-4V_0)^{1/2}\equiv S_n, \tag{3.4c}$$

which is nonzero, and

$$\cot k_{-,n}\cong\cot(\pi^2n^2-4V_0)\equiv T_n, \tag{3.4d}$$

which is nonsingular. Also,

$$k_n=(\pi^2n^2-4V_0)^{1/2}+\Delta/(\pi^2n^2-4V_0)^{1/2}. \tag{3.5}$$

Thus

$$k_n\cong k_{-,n}\cong(\pi^2n^2-4V_0)^{1/2}. \tag{3.6}$$

In the above, we have made the reasonable assumption that the detuning Δ is comparable to the width of the state λ^2/V_0. We have also attached a subscript n to the related wavenumbers to associate them to the nth resonant state. The corresponding quantity Z_n as defined by (2.21) is

$$Z_n = -\left(T_n + \frac{(\pi n)^2}{(\pi^2 n^2 - 4V_0)^{1/2}} \frac{\lambda^2}{(4V_0^2\Delta + 2\lambda^2 V_0)}\right), \tag{3.7}$$

which is a finite quantity. The coefficient in front of the predominantly spin-up isospinor $\begin{pmatrix}\alpha_+\\ \beta_+\end{pmatrix}$,

$$\alpha_-/\sin k_{+,n} \cong (-1)^n \lambda \pi n/2V_0/\Delta + \lambda^2/2V_0 \tag{3.8}$$

exhibits a resonance structure, whereas the coefficient in front of the predominantly spin-down isospinor $\begin{pmatrix}\alpha_-\\ \beta_-\end{pmatrix}$,

$$\alpha_+/\sin k_{-,n} \cong -1/S_n, \tag{3.9}$$

is finite. Thus only the spin-up component shows a resonance structure as the energy E approaches the bare state eigenenergy of the spin-up states, $\pi^2 n^2/2 - 2V_0$.

It is interesting to note that, in this model, antiresonance occurs as the energy approaches $\pi^2 n^2/2$, which would have been the bound state eigenenergies of the bare spin-up state had the depth of the well been zero. To show this, we let

$$E = \pi^2 n^2/2 + \Delta', \tag{3.10}$$

where now Δ' represents the amount of detuning from the antiresonance energy. Then to order λ^2/V_0 and Δ', we have

$$k'_{-,n} = \pi n + (\Delta' - \lambda^2/2V_0)/\pi n, \tag{3.11a}$$

$$\cos k'_{-,n} \cong (-1)^n, \tag{3.11b}$$

$$\sin k'_{-,n} \cong (-1)^n[(\Delta' - \lambda^2/2V_0)/\pi n], \tag{3.11c}$$

$$\cot k'_{-,n} \cong \pi n/(\Delta' - \lambda^2/2V_0), \tag{3.11d}$$

$$k'_{+,n} \cong (\pi^2 n^2 + 4V_0)^{1/2} + \frac{\Delta' + \lambda^2/2V_0}{(\pi^2 n^2 + 4V_0)^{1/2}}, \tag{3.12a}$$

$$\cos k'_{+,n} \cong \cos(\pi^2 n^2 + 4V_0)^{1/2}, \tag{3.12b}$$

$$\sin k'_{+,n} \cong \sin(\pi^2 n^2 + 4V_0)^{1/2} \equiv S'_n, \tag{3.12c}$$

which is nonzero, and

$$\cot k'_{+,n} \cong \cot(\pi^2 n^2 + 4V_0)^{1/2} \equiv T'_n. \tag{3.12d}$$

which is nonsingular. Also

$$k'_n \cong k'_{-,n} \cong \pi n. \tag{3.13}$$

The quantity Z'_n corresponding to (2.21) is

$$Z'_n = \pi n/(\lambda^2/2V_0 - \Delta'). \tag{3.14}$$

Thus the coefficient in front of the predominantly spin-up isospinor $\begin{pmatrix}\alpha_+\\ \beta_+\end{pmatrix}$,

$$\frac{\alpha_-}{(1 - iZ'_n)\sin k'_{+,n}} \cong \frac{\lambda/2V_0}{S'_n[1 + i\pi n/(\Delta' - \lambda^2/2V_0)]}, \tag{3.15}$$

goes to zero, whereas the coefficient in front of the predominantly spin down isospinor within the region of the well, $\begin{pmatrix}\alpha_-\\ \beta_-\end{pmatrix}$,

$$\alpha_+/(1 - iZ'_n)\sin k'_{-,n} \cong i \tag{3.16}$$

and the coefficient of the strictly spin-down spinor in the region outside the well is just $-i$. In fact, as E approaches $\pi^2 n^2/2$, the wave function ψ approaches

$$\psi_{n,\text{anti}} = \theta(x)e^{-ik}\left[\theta(1-x)i(-1)^{n+1}\sin k'_{-,n}x\begin{pmatrix}\lambda/2V_0\\ 1\end{pmatrix} - \theta(x-1)i\sin k'_n(x-1)\begin{pmatrix}0\\ 1\end{pmatrix}\right] \tag{3.17}$$

and is obviously predominantly spin-down.

IV. EXISTENCE OF A TRULY BOUND STATE

In Sec. III, we saw that for a shallow well, $V_0 < (\pi^2/2)^2$, only resonance states appear. We now show that for a sufficiently deep well, $V_0 > (\pi/2)^2$, truly bound states can appear. As explained earlier, we avoid considering the situation $V_0 = (\pi n/2)^2$ since this corresponds to the case where a resonant energy coincides with the decay threshold and we have to face a situation where a pole is embedded in a cut and coincident with the branch point. To be specific, we assume $(\pi/2)^2 < V_0 < \pi^2$, and we consider the situation where E is negative and approaches $\pi^2/2 - 2V_0$. In this case, the unnormalized eigenstate wave function has the form, to order λ^2/V_0,

$$\psi_{\text{bound}} = \theta(x)\left\{\theta(1-x)\left[\sin k_+ x\begin{pmatrix}\alpha_+\\ \beta_+\end{pmatrix} + A\sinh k_- x\begin{pmatrix}\alpha_-\\ \beta_-\end{pmatrix}\right] + \theta(x-1)Be^{-kx}\begin{pmatrix}0\\ 1\end{pmatrix}\right\}, \tag{4.1}$$

where A and B are coefficients to be determined by matching boundary conditions, $\alpha_\pm$ and $\beta_\pm$ are as defined by (2.15) and (2.16), and

$$k_+ = [2(E + 2V_0 + \lambda^2/2V_0)]^{1/2}, \tag{4.2}$$

$$k_- = (2|E - \lambda^2/2V_0|)^{1/2}, \tag{4.3}$$

and

$$k = (2|E|)^{1/2}. \tag{4.4}$$

The coefficients A and B are found to be

$$A = -(\alpha_+ \sin k_+/\alpha_- \sinh k_-) \tag{4.5}$$

and

$$B = (\sin k_+/\alpha_-)e^k. \tag{4.6}$$

We arrive at the following transcendental equation:

$$\alpha_-\beta_+ k_+ \cot k_+ - \alpha_+\beta_- k_- \coth k_- = -k, \tag{4.7}$$

which shows that the equation admits a solution only for particular values of E, the eigenenergies of the truly bound states. In the particular case we are considering, $(\pi/2)^2 < V_0 < \pi^2$, there is only one bound state, and for a weak coupling constant λ, we know that the eigenenergy corresponding to this only bound state is very close to $\pi^2/2 - 2V_0$. Therefore we set

$$E_{\text{bound}} = \pi^2/2 - 2V_0 + \delta, \tag{4.8}$$

substitute this into (4.7) and solve δ to order λ^2/V_0. We then obtain, after some tedious algebra

$$\delta = \frac{\lambda^2}{2V_0(1-X)}\left(\frac{\pi^2}{2V_0(\pi^2 - 4V_0)^{1/2}} + X - 1\right), \tag{4.9}$$

where

$$X \equiv \coth(\pi^2 - 4V_0)^{1/2}. \tag{4.10}$$

Thus the eigenfunction and the eigenenergy are both determined to order λ^2/V_0.

V. TRANSITIONS AND LINE SHAPES

We are finally in a position to examine the transitions and line shapes in this simple model. Since our primary concern is to illustrate the resonance absorption process, we shall not make any attempt in adjusting the normalization constants nor in evaluating the transition oscillator strengths. To be specific, we consider the transition of this particle from the ground state to the nth state in the presence of an oscillating electric field, with an angular frequency ω, assuming that this particle is charged, so that the interaction Hamiltonian is

$$H_{\text{int}} = \eta E x e^{i\omega t}, \tag{5.1}$$

where η is the coupling constant. If the coupling between the two channels is switched off by letting λ go to zero, then a real transition to an excited state is possible only if the frequency of the oscillating electric field is equal to the on-shell energy between bare states

$$\omega_n \equiv \pi^2(n^2 - 1)/2, \tag{5.2}$$

as is demanded by the energy conserving delta function in Fermi's golden rule of time-dependent perturbation theory. However, when we consider the actual physical states where λ is not zero, a resonant transition is still possible when the frequency of the oscillating electric field is detuned from the resonant frequency ω_n since the coupling between the two channels forces the physical spectrum of the particle to be a continuous one for the positive energy states. Thus for an oscillating field frequency ω that appears off-resonant as far as the bare states are concerned, there is a physical state of the particle with precisely the energy that would make the transition an on-shell process, as is demanded by energy conservation in the Fermi's golden rule for every physical process.[6]

We now qualitatively study the behavior of the transition oscillator strength when the frequency of the oscillating field is given by

$$\omega = \omega_n + \Delta - \delta, \tag{5.3}$$

as a function of detuning $\omega - \omega_n = \Delta - \delta$, where δ is as given by (4.9). The final state has a corresponding energy of

$$E_{\text{final},\psi} = n^2\pi^2/2 - 2V_0 + \Delta, \tag{5.4}$$

so that the results derived in Secs. II and III are applicable. If we ignore terms of order λ/V_0 compared to 1, then from (2.15)–(3.9) and (4.1)–(4.7), the wave functions for the final and initial states are

$$\psi_{\text{final}} = \theta(x)\theta(1-x)\frac{\alpha_-}{\sin k_{+,n}}\frac{\sin k_{+n}x}{1 - iZ_n}\begin{pmatrix}1\\0\end{pmatrix} \tag{5.5}$$

and

$$\psi_{\text{initial}} = \theta(x)\theta(1-x)\sin k_+ x\begin{pmatrix}1\\0\end{pmatrix}, \tag{5.6}$$

where $\alpha_-, k_{+,n}$, and Z_n are given by (2.15), (3.3a), and (3.7) and k_+ is given by (4.2). Thus the transition oscillator strength for this absorption, which goes like the square of the interaction matrix element with the interaction given by (5.1), would behave like

$$f_{gn} \sim \left|\int \psi_{\text{final}} \times \psi_{\text{initial}}\right|^2 \equiv L_n M_n, \tag{5.7}$$

where

$$M_n = \left|\int_0^1 \sin k_{+n}x \sin k_+ x\right|^2. \tag{5.8}$$

The factor L_n comes from the square of the coefficient in front of $\sin k_{+n}x$ in (5.5):

$$L_n = \left|\frac{\alpha_-}{\sin k_{+n}}\frac{1}{1 - iZ_n}\right|^2. \tag{5.9}$$

and yields information about the variation of the transition oscillator strength with Δ, i.e., the line shape. On substituting (3.8) for $\alpha_-/\sin k_{+n}$ and (3.7) for Z_n, we get, after some tedious algebra,

$$L_n \cong \left|\frac{(\pi n\lambda/2V_0)}{(\Delta + \lambda^2/2V_0) + i[(\Delta + \lambda^2/2V_0)\cot Y_n + (1/Y_n)(\pi n\lambda/2V_0)^2]}\right|^2, \tag{5.10}$$

where

$$Y_n \equiv (\pi^2 n^2 - 4V_0)^{1/2}. \tag{5.11}$$

Now from (4.9), we see that

$$\delta \sim -\lambda^2/2V. \tag{5.12}$$

Thus

$$\Delta + \lambda^2/2V_0 \sim (\omega - \omega_n) + O(\Gamma), \tag{5.13}$$

where

$$\Gamma \sim \lambda^2/V_0 \tag{5.14}$$

is the width of the excited states. From (5.13) and (5.10), the line shape factor L_n is seen to exhibit Lorentzian characteristics in the detuning variable $(\omega - \omega_n)$ and goes inversely as the width of the excited state λ^2/V_0, as the detuning vanishes. For small n,

$$Y_n \sim \cot Y_n \sim \text{unity}. \tag{5.15}$$

For large n,

$$Y_n \sim \pi n \tag{5.16}$$

and

$$\cot Y_n \sim \pi n/V_0. \tag{5.17}$$

In either case, as the detuning goes to zero,

$$L_n \sim (V_0/\lambda)^2 \sim 1/\Gamma, \tag{5.18}$$

since V_0 is of order unity due to the restriction $(\pi/2)^2 < V_0 < \pi^2$.

VI. CONCLUDING REMARKS

We have introduced a simple model for a particle in two possible channels (the two isospin states) whereby in the absence of any coupling between the two channels, the energy spectrum for one is completely discrete with positive and negative values and that for the other one is completely continuous for all positive energy values. In the presence of a coupling between the two channels, the energy spectrum

of the physical particle becomes continuous for the positive energy states and remains discrete for negative energy values, and the physical state rotates from a pure bare state (either isospin-up or isospin-down) to a superposition of the bare states. This rotation can be in any amount, and is determined by the strength of the coupling and the energy detuning of the physical state from the bare state. This rotation also causes the physical state to have a nonzero projection on other bare states. Near a resonance, the projection on the corresponding bare state has a Lorentzian structure (3.8), giving rise to Lorentzian spectroscopic line shapes (Sec. V).

There are quite a few other models one can construct similar to the one we have discussed here. For example, the Hamiltonian

$$H = p^2/2 + (1-\sigma_3)V_1 + (\sigma_1 - 1)V_2 + U, \tag{6.1}$$

where

$$\begin{aligned} V_1 &= \infty \quad x < 1 \\ &= 0 \quad \text{elsewhere,} \end{aligned} \tag{6.2}$$

$$\begin{aligned} V_2 &= \infty \quad x > 1 + \epsilon \\ &= 0 \quad \text{elsewhere,} \end{aligned} \tag{6.3}$$

and

$$\begin{aligned} U &= \infty \quad x < 0 \\ &= 0 \quad \text{elsewhere,} \end{aligned} \tag{6.4}$$

not only exhibits resonance structure, but also demonstrates mass renormalization. The parameter ϵ takes the place of the coupling constant. But this model does not demonstrate any more about line shapes than the one we have discussed. It is our hope that our model fills a pedagogical gap and demonstrates an alternative view to understand the Weisskopf–Wigner line shape.

ACKNOWLEDGMENTS

This work is supported by the National Science Foundation under Grant No. ISP-80-11451. The work of C. K. Au is also supported in part by the National Science Foundation under Grant No. PHY79-01053, and in part by a University of South Carolina Faculty Research and Productive Scholarship Grant, and in part by a University of South Carolina Summer Faculty Research Appointment. CKA thanks Walter Kohn for his hospitality at the Institute for Theoretical Physics at Santa Barbara where part of the writing of this paper was done. CKA also thanks the Physics Department of Hong Kong University for their hospitality.

[1] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1939); **65**, 18 (1930).

[2] F. Low, Phys. Rev. **88**, 53 (1952).

[3] M. Goldberger and K. Watson, *Collision Theory* (Wiley, New York, 1968).

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford, London, 1954).

[5] A somewhat similar "coupled-channels" model is discussed by S. D. Doyle, J. S. Eck, W. J. Thompson, and O. L. Weaver, Am. J. Phys. **43**, 677 (1975).

[6] See, for example, G. Baym, *Lectures on Quantum Mechanics* (Benjamin, New York, 1969).

A simple, accurate alternative to the minimum deviation method of determining the refractive index of liquids

B. P. Chandra and S. C. Bhaiya[a)]

Department of Physics, Government Science College, Raipur (M.P.) 492002, India

(Received 19 November 1981; accepted for publication 2 April 1982)

A fixed angle of incidence method, which is an alternative to the minimum deviation method of determining the refractive index of liquid, is used. This method is very simple as compared to others. The value of refractive index determined by this method is nearly the same as that determined by other methods.

I. INTRODUCTION

It is known that the refractive index plays a vital role in many branches of physics. Recently, Waldenstrøm and Naqvi[1] have suggested an alternative to the classical minimum deviation method for the determination of the refractive index of the prism. They have reported that the use of an equilateral hollow prism would allow the study of refractive index of most ordinary liquids ($1.3 < n < 1.6$) by the same method. In this connection we have checked the accuracy of the method and have found that the fixed angle of incidence method can also be suitable for determining the refractive index of the liquids.

II. PRINCIPLE OF THE METHOD

Let us discuss in short the principle of the fixed angle of incidence method of determining the refractive index. Figure 1 shows a prism *ABC*. Let the angle of prism *CAB* be equal to ϕ and the angle of incidence *DEM* be equal to α_1, which is indirectly equal to ϕ. It is seen from the figure that

$$\beta_1 + \beta_2 = \phi.$$

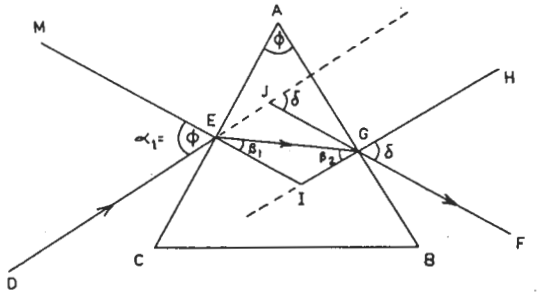

Fig. 1. Basic principle of the fixed angle of incidence method of determining the refractive index.