Consistency of the Aharonov-Bohm Effect with Quantum Theory.

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Summary. — A recent claim the inconsistency of the Aharonov-Bohm effect with the principles of quantum theory is shown to contain incorrect arguments.

Recent years have witnessed a resurgence in the number of papers which, in one form or another, challenge the conceptual validity of the Aharonov-Bohm (AB) effect. Among these one occasionally finds a paper containing arguments which are correct as far as they go, but do not go quite far enough to uncover the subtle means by which they actually support that which they purport to question (5). Other papers, however, present arguments which, to the extent that they are unambiguous, can be shown to be incorrect. Ordinarily one would ignore these papers, depending upon the authors themselves to reexamine their work. However, private communications have convinced us that a recent paper of this type (4) has actually resurged in some confusion. It is the purpose of this note to question the arguments put forth therein.

Boccardi and Lanfredi claim that the AB effect is conceptually inconsistent with, the fundamental principles of quantum theory. They support this claim with examples using the AB electric effect and the AB magnetic interference effect. We analyze their arguments in the context of the electric effect. The same analysis applies mutatis mutandis to the magnetic case and is left as an exercise for the reader.

We commence by paraphrasing the excellent description of the AB electric effect given by Boccardi and Lanfredi: An electron wave packet impinges on two slits $T_1$ and $T_2$; immediately behind $T_1$ and $T_2$ there are two cylindrical Faraday cages $C_1$ and $C_2$. These are long enough to contain for a given time interval $t$, to a very good approximation, the two coherent wave packets 1 and 2, emerging from $T_1$ and $T_2$.

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An electric potential difference \( U(t) \) is applied between \( C_1 \) and \( C_2 \) for a time interval \((t', t')\) contained in \( T \). Assume, for example, that \( C_1 \) is at zero potential and \( C_2 \) at potential \( U(t) \). If \( \psi_1(r, t) \) and \( \psi_2(r, t) \) are the wave functions of the two packets going, respectively,through \( C_1 \) and \( C_2 \) in the absence of any potential difference, the changes induced by \( U(t) \) result in the new wave packets being represented, for \( t > t' \), by (according to AB)

\[
\psi_1(r, t) = \psi_1^{(t)}(r, t),
\]

\[
\psi_2(r, t) = \exp\{-i\phi\} \psi_2^{(t)}(r, t),
\]

where \( \phi \) involves the time integral of \( U(t) \) over the interval \((t', t)\).

Now, using the complete wave function \( \psi(r, t) = \psi_1(r, t) + \psi_2(r, t) \), the expectation value at time \( t \) of any observable \( \mathcal{A} \) is

\[
\langle \mathcal{A} \rangle = \langle \psi_1^{(t)}, \mathcal{A} \psi_2^{(t)} \rangle + \langle \psi_2^{(t)}, \mathcal{A} \psi_1^{(t)} \rangle + 2 \cos \phi \text{Re} \langle \psi_1^{(t)}, \mathcal{A} \psi_2^{(t)} \rangle + 2 \sin \phi \text{Im} \langle \psi_1^{(t)}, \mathcal{A} \psi_2^{(t)} \rangle.
\]

It is the overlap term, \( \langle \psi_1^{(t)}, \mathcal{A} \psi_2^{(t)} \rangle \), which enters with \( \phi \)-dependent coefficients, which seems to cause concern to Bocchiere and Longen, and to lead to unwarranted conclusions. For example they state:

This implies that the position (if \( \mathcal{A} \) is the position observable) of the 'burying centres' of the system of the Young interference fringes depends on \( \phi \); manifestly, this is an astonishing result, even for the supporters of the Averesevic-Bohm point of view; we have a shift of the above 'burying centres' with respect to the situation for which there is no potential difference between \( C_1 \) and \( C_2 \), notwithstanding the absence of any force acting on the electron. An analogous conclusion holds true also for other physical observables \( F(x, \vec{p}) \) of the particle.

Further, when \( \langle \psi_1^{(t)}, \mathcal{A} \psi_2^{(t)} \rangle \) is different from zero, even the particle energy is not conserved, although the external source of the electric-potential difference does not perform any work.

Let us first consider the question of the energy. Writing the Hamiltonian as \( H = H_0 + H(t) \), where \( H(t) = 0 \) when \( U(t) \) is turned off, we note that \( H_0 \) itself is quite complicated, involving as it does the various potential barriers representing the slit, Faraday cages, etc. However, one very simple fact remains: Once \( U(t) \) is turned off, \( H_t \) is thereafter a constant of the motion; furthermore, any matrix element of \( H_0 \), involving \( \langle \psi_1^{(t)}, H_0 \psi_2^{(t)} \rangle \), is constant. Therefore, if \( \langle \psi_1^{(t)}, H_0 \psi_2^{(t)} \rangle \) is ever zero for \( t > t' \), it remains zero. The statement that the external source of the electric-potential difference does not perform any work really implies (as does the whole wave packet argument) that \( \langle \psi_1^{(t)}, H_0 \psi_2^{(t)} \rangle \) vanishes at the instant that \( U(t) \) is switched off. Thus the subsequent overlap of the two wave packets in no way leads to violation of conservation of energy, either in the chronological evocation of the motion, or with respect to the situation for which there is no potential difference between \( C_1 \) and \( C_2 \). To the extent that the wave packets eventually get completely clear of the Faraday cages, i.e. when

\[
\langle \psi_1^{(t)}(r), H_0 \psi_2^{(t)}(r) \rangle = \langle \psi_1^{(t)}(r), \frac{\lambda}{2m} \psi_2^{(t)}(r) \rangle,
\]
we have $\langle \psi^a, \psi^a \rangle \psi^B = 0$. We note this last fact because Bocchiari and Longoni apparently simply assume that the matrix element of $\psi^A$ does not vanish when the wave packets overlap (\textsuperscript{1}).

The question of position expectation is somewhat more complicated, but does not lead to any contradiction with the laws of quantum theory. It should be stated first that $\langle \psi^A, \psi^A \rangle \psi^B$ obviously does depend on $\theta$ when $\psi^B$ and $\psi^A$ overlap; this is the essence of the AB effect for this case, and is not in conflict with any fundamental principles. It further follows from this that the system is subsequently subjected to a position-dependent force, the effect on the expectation value of the particle position could be different for the two cases, i.e. those where the telescope had or had not, been applied in the past. An example would be if the region of overlap of the two wave packets (after $U(\theta)$ is switched off) included the boundaries of the Faraday cages themselves.

On the other hand, if one envisions the situation where there is a time interval in which the wave packets have completely emerged from the Faraday cages, but still do not overlap, then $\langle \psi^A, \psi^A \rangle \psi^B$ will vanish for all subsequent times and the statement of Bocchiari and Longoni with respect to the $\psi^B$-curve quoted above is incorrect. One sees this by noting that once the Faraday cages are out of the picture we have a free-particle situation. Then if $t_1$ is the time before overlap (but after emergence from the cages), for any later time $t$ elementary quantum theory gives

$$\langle \psi^A, \psi^A \rangle \psi^B = \langle \psi^A, \psi^A \rangle \psi^B + \frac{\hbar}{2m} \langle \psi^A, \psi^B \rangle \psi^A \psi^B.$$

Both of the matrix elements on the right-hand side of the above equation vanish. None of the above discussion of position expectation, or energy, is at variance with the laws of quantum mechanics. As stated above, we leave the discussion of the magnetic case to the reader. The present authors do not accept the suggestion of Bocchiari and Longoni that the AB effect has to be abandoned as a means of explaining the growing experimental evidence which verifies it.

\textsuperscript{1} We must be careful for invoking the correct procedure of simple complex conservation of energy (as used in a direct consideration of the time-independent Hamiltonian, and then drawing conclusions from that). The converse procedure of assuming, incorrectly, the vanishing of a matrix element and then pointing out a vanishing contradiction is the one that is at least here.