Consistency of the Aharonov-Bohm Effect with Quantum Theory.

Y. AHARONOV (*), C. K. AU, E. C. LERNER and J. Q. LIANG (**)

Department of Physics and Astronomy, University of South Carolina Columbia. SC 29208, USA

(ricevuto il 29 Novembre 1983)

PACS. 03.65. – Quantum theory; quantum mechanics.

Summary. – A recent claiming the inconsistency of the Aharonov-Bohm effect with the principles of quantum theory is shown to contain incorrect arguments.

Recent years have witnessed a resurgence in the number of papers which, in one form or another, challenge the conceptual validity of the Aharonov-Bohm (AB) effect. Among these one occasionally finds a papers containing arguments which are correct as far as they go, but do not go quite far enough to uncover the subtle means by which they actually support that which they purport to question (¹). Other papers, however, present arguments which, to the extent that they are unambiguous, can be shown to be incorrect. Ordinarily one would ignore these papers, depending upon the authors themselves to reassess their work. However, private communications have convinced us that a recent paper of this type (²) has actually resulted in some confusion. It is the purpose of this note to question the arguments put forth therein.

BOCCHIERI and LOINGER claim that the AB effect is conceptually inconsistent with the fundamental principles of quantum theory. They support this claim with examples using the AB electric effect and the AB « magnetic interference » effect. We analyse their arguments in the context of the electric effect. The same analysis applies « mutatis mutandis » to the magnetic case and is left as an exercise for the reader.

We commence by paraphrasing the excellent description of the AB electric effect given by BOCCHIERI and LOINGER: An electron wave packet impinges on two slits Y_1 and Y_2; immediately behind Y_1 and Y_2 there are two cylindrical Faraday cages C_1 and C_2. These are long enough to contain for a given time interval T, to a very good approximation, the two coherent wave packets 1 and 2, emerging from Y_1 and Y_2.

(*) Joint appointment with Tel Aviv University, Ramat Aviv, Israel.

(**) Permanent address: Physics Department, Shanxi University, People's Republic of China.

(¹) A paper of this type is G. CASATI and I. GUARNERI: *Phys. Rev. Lett.*, **42**, 1579 (1979), which considers the « hydrodynamical » formulation of wave mechanics. We hope to publish a discussion of this problem shortly. It brings out some extremely interesting aspects of nonlinear equations.

(²) P. BOCCHIERI and A. LOINGER: *Lett. Nuovo Cimento*, **35**, 469 (1982).

An electric-potential difference $U(t)$ is applied between C_1 and C_2 for a time interval (t', t'') contained in T. Assume, for example, that C_1 is at zero potential and C_2 at potential $U(t)$. If $\psi_1^{(0)}(\boldsymbol{r}, t)$ and $\psi_2^{(0)}(\boldsymbol{r}, t)$ are the wave functions of the two packets going, respectively, through C_1 and C_2 in the absence of any potential difference, the changes induced by $U(t)$ result in the new wave packets being represented, for $t > t''$, by (according to AB)

$$\psi_1(\boldsymbol{r}, t) = \psi_1^{(0)}(\boldsymbol{r}, t)\,, \tag{1}$$

$$\psi_2(\boldsymbol{r}, t) = \exp[-i\theta]\,\psi_2^{(0)}(\boldsymbol{r}, t\,, \tag{2}$$

where θ involves the time integral of $U(t)$ over the interval (t', t'').

Now, using the complete wave function $\psi(\boldsymbol{r}, t) = \psi_1(\boldsymbol{r}, t) + \psi_2(\boldsymbol{r}, t)$, the expectation value at time t of any observable A is

$$\langle A\rangle_t = \langle\psi_1^{(0)}, A\psi_1^{(0)}\rangle + \langle\psi_2^{(0)}, A\psi_2^{(0)}\rangle + \\ + 2\cos\theta\,\mathrm{Re}(\langle\psi_1^{(0)}, \psi A_2^{(0)}\rangle) + 2\sin\theta\,\mathrm{Im}(\langle\psi_1^{(0)}, A\psi_2^{(0)}\rangle)\,. \tag{3}$$

It is the « overlap » term, $\langle\psi_1^{(0)}, A\psi_2^{(0)}\rangle$, which enters with θ-dependent coefficients, which seems to cause concern to BOCCHIERI and LOINGER, and to lead to unwarranted conclusions. For example they state:

> This implies that the position (if A is the position observable) of the « barycentre » of the system of the Young intereference fringes depends on θ: manifestly, this is an astonishing result, even for the supporters of the Aharonov-Bohm point of view: we have a shift of the above « barycentre » with respect to the situation for which there is no potential difference between C_1 and C_2, notwithstanding the absence of any force acting on the electron. An analogous conclusion holds true also for other physical observables $F(x, \dot{x})$ of the particle.
>
> Further, when $\langle\psi_1^{(0)}\,\nabla^2\,\psi_2^{(0)}\rangle$ is different from zero, even the particle energy is not conserved, although the external source of the electric-potential difference does not perform any work.

Let us first consider the question of the energy. Writing the Hamiltonian as $H = H_0 + H_1(t)$, where $H_1(t) = 0$ when $U(t)$ is turned off, we note that H_0 itself is quite complicated, involving as it does the various potential barriers representing the slits, Faraday cages, etc. However, one very simple fact remains: *Once $U(t)$ is turned off, H_0 is thereafter a constant of the motion*; *furthermore, any matrix element of H_0, including $\langle\psi_1^{(0)}, H_0\psi_2^{(0)}\rangle$, is constant.* Therefore, if $\langle\psi_1^{(0)}, H_0\psi_2^{(0)}\rangle$ is ever zero for $t \geqslant t''$, it remains zero. The statement that the external source of the electric-potential difference does not perform any work really implies (as does the whole wave packet argument) that $\langle\psi_1^{(0)}, H_0\psi_2^{(0)}\rangle$ vanishes at the instant that $U(t)$ is switched off. Thus the subsequent overlap of the two wave packets in no way leads to violation of conservation of energy, either in the chronological evolution of the motion, or with respect to the situation for which there is no potential difference between C_1 and C_2. To the extent that the wave packets eventually get completely clear of the Faraday cages, *i.e.* when

$$\langle\psi_1^{(0)}, H_0\psi_2^{(0)}\rangle = \left\langle\psi_1^{(0)}, -\frac{\hbar^2}{2m}\nabla^2\psi_2^{(0)}\right\rangle,$$

we have $\langle\psi_1^{(0)}, \nabla^2\psi_2^{(0)}\rangle = 0$. We note this last fact because BOCCHIERI and LOINGER apparently simply *assume* that the matrix element of ∇^2 does not vanish when the wave packets overlap (³).

The question of position expectation is somewhat more complicated, but also does not lead to any contradiction with the laws of quantum theory. It should be stated first that $|\psi(\boldsymbol{r}, t)|^2$ (as opposed to $\langle\psi, \psi\rangle$) obviously does depend on θ when $\psi_1^{(0)}(\boldsymbol{r}, t)$ and $\psi_2^{(0)}(\boldsymbol{r}, t)$ overlap; this is the essence of the AB effect for this case, and is not in conflict with any fundamental principle. It further follows from this that if the system is subsequently subjected to a position-dependent force, the effect on the expectation value of the particle position could be different for the two cases, *i.e.* those where the potential had, or had not, been applied in the past. An example would be if the region of overlap of the two wave packets (after $U(t)$ is switched off) included the boundaries of the Faraday cages themselves.

On the other hand, if one envisages the situation where there is a time interval in which the wave packets have completely emerged from the Faraday cages, but still do not overlap, then $\langle\psi_1^{(0)}, \boldsymbol{r}\psi_2^{(0)}\rangle$ will vanish for all subsequent times and the statement of Bocchieri and Loinger with respect to the « barycentre » quoted above is incorrect. One sees this by noting that once the Faraday cages are out of the picture we have a free-particle situation. Then if t_0 is the time before overlap (but after emergence from the cages), for any later time t elementary quantum theory gives

$$\langle\psi_1^{(0)}, \boldsymbol{r}\psi_2^{(0)}\rangle_t = \langle\psi_1^{(0)}, \boldsymbol{r}\psi_2^{(0)}\rangle_{t_0} + \frac{(t-t_0)}{m}\langle\psi_1^{(0)}, \boldsymbol{p}\psi_2^{(0)}\rangle_{t_0}. \tag{4}$$

Both of the matrix elements on the right-hand side of the above equation vanish.

None of the above discussion of position expectation, or energy, is at variance with the laws of quantum mechanics. As stated above, we leave the discussion of the magnetic case to the reader. The present authors do not accept the suggestion of Bocchieri and Loinger that the AB effect has to be abandoned as a means of explaining the growing experimental evidence which verifies it.

(³) We must be excused for invoking the correct procedure of accepting conservation of energy (in time) as a direct consequence of a time-independent Hamiltonian, and then drawing conclusions from that. The converse procedure of assuming, incorrectly, the nonvanishing of a matrix element and then pointing out a resulting « contradiction » is the one that is at fault here.