

# Take home examination in Family Economics

Receive Monday July 17 2005, 12.00

Return Monday July 24 2005, 12.00

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Sources:

1. Shapley, L. and M. Shubik (1972), "The Assignment Game I: The Core," *International Journal of Game Theory*, 1, 11-130.
2. Roth A. and M. Sotomayor (1990), *Two-Sided Matching*, Cambridge University Press, Chapter 8.
3. B. C.W. chapters 8 and 9.

Let  $m$  be a male attribute and  $f$  a female attribute. If a man with attribute  $m$  marries a woman with attribute  $f$  they can receive together

$$z = g + m + f - \frac{1}{2}(m - f)^2,$$

where  $g$  is a positive constant. The total marital output can be divided between the two partners and utility is linear in the shares (transferable utility). A single man with attribute  $m$  receives  $m$  and a single woman with attribute  $f$  receives  $f$ .

1. Prove that the household production function is super modular and single peaked in  $m$  and  $f$ .

2. Consider first a case with two men with attributes

men	$m$
1	2
2	3

and three women, with attributes

women	$f$
1	1
2	2
3	3

What is the stable assignment? Let  $v_i$  be the reservation utility of man  $i$  and  $u_j$  be the reservation utility of woman  $j$ . State the conditions for stability in terms of the dual problem. Show on a graph the set of imputations  $v_1, v_2$  that support the stable assignment.

3. Replace the data in problem 1 by

men	$m$
1	2
2	4

women	$f$
1	1
2	3
3	5

Which couples will marry and under what conditions? Show that there are two stable assignments such that two couples marry. Calculate the respective imputations.

4. Consider now the case with a continuum of men and women, where the mass of men is 1 and the mass of women is  $r > 1$ . The distribution of male attributes is uniform on  $[0, M]$  and the distribution of female attributes is uniform on  $[0, F]$  where  $F < M$ . Find the stable assignment. Which men and which women remain single? Explain.

5. Derive the integral formulas for the shares of married men and women in marital output. Explain the boundary conditions. Show that the difference between the husband's and wife's shares in the marital surplus declines along the stable assignment profile (use change of variables to write the husband's share as a function of  $f$ ). Show that at the top of the stable assignment profile the wife receives a higher share of the marital surplus. Explain.

6. Compare your results to the example in B.C.W. chapter 9 and explain the differences. Which example is more realistic?