DEFENDING TIME-SYMMETRIZED QUANTUM THEORY

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Abstract

Recently, several authors have criticized time-symmetrized quantum theory originated by the work of Aharonov et al. (1964). The core of this criticism was the proof, which appeared in various forms, showing that counterfactual interpretation of time-symmetrized quantum theory cannot be reconciled with the standard quantum theory. I argue here that the apparent contradiction appears due to inappropriate usage of traditional time asymmetric approach to counterfactuals, and that the contradiction disappears when the problem is analyzed in terms of time-symmetric counterfactuals. I analyze various aspects of time-symmetry of quantum theory and defend the time-symmetrized formalism.

1. Introduction. I shall discuss a pre- and post-selected quantum system, i.e. measurements performed at the time between two other measurements. The time-symmetric formalism for the description of such systems was proposed by Aharonov, Bergmann, and Lebowitz (ABL) (1964) and was developed in recent years. A partial list of references includes Aharonov et al. (1985), Aharonov and Vaidman (1990, 1991). Several authors criticized the time-symmetric approach to quantum theory in general and some of its particular applications. The most representative example is the work of Sharp and Shanks (1993). They presented a proof, which was later repeated and used by others, that the counterfactual interpretation of the ABL probability rule (Eq. 2 below), cannot be reconciled with the standard quantum theory. I shall claim here that the proof contains an error since it presuppose time asymmetry in order to reach a contradiction with time symmetry. The asymmetry was implicitly assumed through the conventional approach to counterfactual statements. I argue that for analyzing experiments on pre- and post-selected quantum system one should use time-symmetric counterfactuals and then no contradiction arises.

The plan of this work is as follows. In Section 2 I shall present a brief review of the time-symmetrized formalism and in Section 3 a brief review of the concept of counterfactual. In Section 4 I analyze possible counterfactual interpretations of the ABL rule. Section 5 is devoted to the analysis of the inconsistency proof of Sharp and Shanks and its variations. In section 6 I discuss related time asymmetry preconceptions in quantum theory. In section 7 the time symmetry (and asymmetry) of the process of quantum measurement is analyzed in order to give a rigorous context to the previous discussion about the time symmetry of the ABL rule. An application of the time-symmetrized approach which allows the definition of new concepts (which I call "elements of reality") is considered in section 8. Section 9 concludes the paper with a brief summary and some discussion of the time-symmetrized quantum theory in the framework of the many-worlds interpretation.

2. Time-Symmetrized Formalism. In standard quantum theory a complete description of a system at a given time is given by a quantum state $|\Psi\rangle$. It yields the probabilities for all outcomes a_i of a measurement of any variable A according to the equation

$$\text{Prob}(a_i) = |\langle\Psi|\mathbf{P}_{A=a_i}|\Psi\rangle|^2 \tag{1}$$

where $\mathbf{P}_{A=a_i}$ is the projection operator on the subspace defined by $A = a_i$. Eq. 1 is intrinsically asymmetric in time: the state $|\Psi\rangle$ is determined by some measurements in the past and it evolves toward the future. The time evolution between the measurements, however, is considered time symmetric since it is governed by the Schrödinger equation for which each forward evolving solution has its counterpart (its complex conjugate with some other well understood simple changes) evolving backward in time. The asymmetry in time of the standard quantum formalism is manifested in the absence of the quantum state evolving backward in time from future measurements (relative to the time in question).

Time-symmetrized quantum theory describes a system at a given time by a two-state vector $\langle\Psi_2||\Psi_1\rangle$. It yields the (conditional) probabilities for all outcomes a_i of a measurement of any variable A according to the generalization of the ABL formula (Aharonov and Vaidman, 1991):

$$\text{Prob}(a_i) = \frac{|\langle\Psi_2|\mathbf{P}_{A=a_i}|\Psi_1\rangle|^2}{\sum_j |\langle\Psi_2|\mathbf{P}_{A=a_j}|\Psi_1\rangle|^2}. \tag{2}$$

The time symmetry does not mean that a system described by the two-state vector $\langle\Psi_2||\Psi_1\rangle$ is identical, in regard to its physical properties, to a system described by the two-state vector $\langle\Psi_1||\Psi_2\rangle$.[1] The time symmetry means that $\langle\Psi_2|$ and $|\Psi_1\rangle$ enter the equations, and thus govern the observable results, on equal footings. For example, (almost) standard measurement procedure with weakened coupling (which we call *weak measurement*, Aharonov and Vaidman 1990) yields *weak values* defined as

$$A_w \equiv \frac{\langle\Psi_2|A|\Psi_1\rangle}{\langle\Psi_2|\Psi_1\rangle}. \tag{3}$$

When we interchange $\langle\Psi_2|$ and $|\Psi_1\rangle$ the weak value changes to its complex conjugate and this can be observed in certain experiments. In Eq. 2 and in Eq. 3 the two states enter on the same footing and the legitimacy of the application of these equations (especially of Eq. 2) is what I defend in this paper.

In order to explain how to obtain a quantum system described at a given time t by a two-state vector $\langle\Psi_2||\Psi_1\rangle$ we shall assume for simplicity that the free Hamiltonian of the system is zero. In this case it is enough to prepare the system at time t_1 prior to time t in the state $|\Psi_1\rangle$, to ensure no disturbance between t_1 and t as well as between t and t_2, and to find the system at t_2 in the state $|\Psi_2\rangle$. It is crucial that $t_1 < t < t_2$, but the relation between these times and "now" is not fixed. The times t_1, t, t_2 might all be in the past, or we can discuss future measurements and then they are all in the future; we just have to agree to discard all cases when the measurements at time t_2 does not yield the outcome corresponding to the state $|\Psi_2\rangle$.

Note the asymmetry between the measurement at t_1 and the measurement at t_2. Given an ensemble of quantum systems, it is always possible to prepare all of them in a particular state $|\Psi_1\rangle$, but we cannot ensure finding the system in a particular state $|\Psi_2\rangle$. Indeed, if the pre-selection measurement yielded a result different from projection on $|\Psi_1\rangle$ we can always change the state to $|\Psi_1\rangle$, but if the measurement at t_2 did not show $|\Psi_2\rangle$, our only choice is to discard such a system from the ensemble. Note also the asymmetry of the measurement procedures. The measurement device has to be prepared before the measurement interaction in the "ready" state and we cannot ensure finding the "ready" state after the interaction. We might use some intermediate system which interacts with the observed system such that this intermediate system has the essential

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[1]Note, however, that if we limit ourselves to "physical properties" which are results of standard ideal measurements (whose probabilities are governed by Eq. 2), this symmetry property holds too.

symmetry of the states before and after the interaction. But then the problem will move to the next level of the measurement procedure chain, and it always reaches the asymmetry, because, according to the definition of measurement, the observer does not know the result before the interaction but he does, after the measurement. These asymmetries, however, are not relevant to the problem we consider here. We study the symmetry relative to the measurements at time t for a given pre- and post-selected system, and we do not investigate the time-symmetry of obtaining such a system. The only important detail is that the measurement coupling at time t has to be time-symmetric, as is assumed in ideal quantum measurements. See more discussion below, in section 7.

3.Counterfactuals. There are many philosophical discussions on the concept of counterfactuals and especially on the time's arrow in counterfactuals. Probably, the theory of counterfactuals of Lewis (1973) receives the most attention. Based on his theory Lewis (1986) discusses the asymmetry of counterfactuals between past and future. He analyses mainly deterministic worlds and claims that indeterminism, in particular the indeterminism of the process of reduction of a quantum state in the process of quantum measurement, does not lead to an asymmetry:

> If there is a process of reduction of the wave packet in which a given superposition may be followed by any of many eigenstates, equally this is a process in which a given eigenstate may have been preceded by any of many superpositions. Again we have no asymmetry. (1986, 39)

I disagree with this argument. The superposition which precedes the measurement is uniquely defined by the classical records regarding measurements in the past. In any way, the number of possible superpositions is not comparable with the number of eigenstates, so no symmetry can be seen here.

Apart from the possible connection between indeterminism and an apparent time asymmetry of our world, the importance of the indeterminism of the standard quantum theory is that it opens room for counterfactual questions about results of measurements without involving "miracles", i.e. events in which physical laws breaks down. Although Lewis devotes a large part of his theory to considering these "miracles" which are irrelevant for our discussion, I do adopt the basic approach of his analyses of counterfactuals, i.e. the usage of the language of "possible worlds". It is not clear if this is the only way to go, but I find it fruitful and certainly legitimate.

Another important work on the subject was done by Bennett (1984). He reaches the conclusion (with which I tend to agree) that Lewis failed to *derive* the temporal asymmetry of counterfactuals from general principles. Bennett develops his "Unified Symmetric Theory" of counterfactuals. It is based on the concept of "T-closest P-world" which is the world "closest" (whatever it means) to the actual world at time T at which the proposition P is true.

4

There are many discussions of counterfactuals in quantum theory, mostly in the context of EPR-Bell type experiments. Some of the examples are Skyrms (1982), Peres (1993), Mermin (1989) (which, however, does not use the word counterfactual), and Bedford and Stapp (1995) who even present an analysis of a Bell-type argument in the formal language of the Lewis theory of counterfactuals. The common situation is that a composite system is described at a certain time by some entangled state and then an array of incompatible measurements on this system at a later time is considered. Various conclusions are derived from statements about the results of these measurements. Since these measurements are incompatible they cannot be all performed together, so it must be that at least some of them were not actually performed. This is why they are called counterfactual statements. Note that there is no requirement that none of them are performed, although it might be so. The actual world is specified by its state at the initial time so it fits the general framework of Bennett's theory of counterfactuals.

In the situations discussed in this work the actual world is specified by its state at two times. Thus, Bennett's basic concept of T-closest P-world cannot be applied directly, but the following quotation of Bennett seems very relevant:

> Here is an easier example. At T_1 I bet that when the coin is tossed at T_2 it will come up heads; and in the upshot it does just that; but this is a purely chance event, with no causally sufficient prior conditions. Now consider the conditional "If I had bet on tails at T_1 I would have lost." Everyone I have polled is inclined to say that that conditional is true, despite the fact that at some of the T_1-closest "I bet on tails" worlds the coin comes up heads [tails?[2]] at T_2. (Why does it come up heads [tails] at some of those worlds? Because, since the fall of the coin had no causally sufficient prior conditions, every "tails" ["heads"] world is indistinguishable, in respect of its state at T_1, from some "heads" ["tails"] world.) If I am to respect these judgments I must modify my theory,... (1984, 76)

In his modification of the theory (which he only sketches in a few lines) there is no formal symmetry between the times T_1 and T_2. But this is not because Bennett introduces temporal asymmetry here, but because the two times have different status. The time T_1 has a special status as the one at which various possibilities (the type of bet to be taken) have to be chosen, and this is why Bennett relates the concept of T-closest worlds to T_1 and not to T_2. In order to analyze temporal symmetry we have to consider a symmetric setup. Let us add another coin tossing at time T_0 prior to the time of the bet. Unquestionably, "everyone Bennett has polled" who suggested we accept that in all relevant (counterfactual) worlds the outcome of the coin tossing at time T_2 must coincide with that in the actual world would also suggest that the outcome of the coin tossing at time T_0 must coincide with that in the actual world. Thus, we can see that even a

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[2]It seems to me that there is some mismatch between "tails" and "heads" which I, hopefully, corrected in the brackets.

modified theory of counterfactuals of Bennett, which is suitable for the analysis of possible measurements performed between two other measurements, is symmetrical with regard to the past and the future of time T_1.

The example of Bennett is not identical to the problem of measurement performed on quantum system at the time between two other measurements. In Bennett's case there was no physical mechanism according to which the decision of which bet to make could influence the result of the coin toss.[3] In contrast, most intermediate quantum measurements change the probabilities for the results of the later measurement. However, it seems to me that the theory of counterfactuals which respects the actual event at time T_2 in spite of the fact that it should be identical with a specific (actual) result only in 50% cases is much more close to the theory of counterfactuals accepting an actual event at time T_2 when the probability is different and even influenced by the intermediate action, rather than to the theory which disregards the actual result at time T_2, as is suggested in alternative interpretations.

4. Counterfactual Interpretations of the ABL Probability Rule. In this section I shall consider three ways to interpret the "counterfactual interpretation". The first interpretation I cannot comprehend, but I have to discuss it since it was proposed and used in the criticism on the time-symmetrized quantum theory. I believe that I understand the meaning of the second interpretation, but I shall argue that it is not appropriate for the problem which is discussed here. The last interpretation is the one I want to adopt and I shall present several arguments in its favor.

Interpretation (a) *Counterfactual probability as the probability of the result of a measurement which has not been performed.*

Let me quote Sharp and Shanks:

> ...for, conditionalizing upon specified results of measurements of M_I and M_F, there is no reason to assign the same values to the following probabilities: the probability that an intervening measurement of M had the result m^j given that such a measurement in fact took place, and the probability that intervening measurement *would have had* the result m^j given that no such intervening measurement of M in fact took place. In other worlds there is no reason to identify $\mathrm{Prob}(M = m^j | \mathbf{E}_M[\psi_I^i, \psi_F^k])$ and $\mathrm{Prob}(M = m^j | \mathbf{E}[\psi_I^i, \psi_F^k])$.(1993, 491)

I can not comprehend the meaning of the probability for the result $M = m^j$ given that the measurement M has not take place. As far as I can see $\mathrm{Prob}(M = m^j | \mathbf{E}[\psi_I^i, \psi_F^k])$ has no physical meaning. Sharp and Shanks continue:

> (For a classical illustration, consider a drug which, if injected to facilitate a medical test at t, has an effect, starting shortly after the test and persisting

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[3]Moreover, one can make an experiment which will show that the bet decision does not change the probability of the result of the coin toss.

past t_F, on the the value of the tested variable. Suppose that it is unknown whether a test was conducted at t, but that a value for the tested variable is obtained at t_F. Using the value at t_F, we would estimate differently the value prior to t depending on whether we assume that a test did or did not take place at t.)

This might explain what they have in mind, but the argument does not hold since in many situations there is no quantum mechanical counterpart to the classical case of "the value [of a tested variable] prior to t" . In standard quantum theory *unperformed experiments have no results*, see Peres (1978).

Cohen and Hiley partially acknowledge the problem admitting that at least in the framework of the orthodox interpretation this is meaningless concept:

> In other words we cannot necessarily assume that the ABL rule will yield the correct probabilities for what the results of the intermediate measurements *would* have been, *if* they had been carried out, in cases where these measurements have not *actually* been carried out. In fact, this sort of counterfactual retrodiction has no meaning in the orthodox (i.e., Bohrian) interpretation of quantum mechanics, although it can legitimately be discussed within the standard interpretation [von Neumann (1955)] and within some other interpretations of quantum mechanics (see, for example, Bohm and Hiley[1993]).(1996, 3)

I fail to understand the interpretation (a) in any framework. Maybe, if we restrict ourselves to the cases in which the system at the intermediate time is in an eigenstate of the variable which we intended to measure, (but we had not), we can associate the probability 1 with such unperformed measurements. This is close to the idea of Cohen (1995) to consider counterfactuals in the restricted cases corresponding to consistent histories introduced by Griffith (1984). But, as far as I can see, interesting situations do not correspond to consistent histories, and therefore no novel (relative to classical theory) features of quantum theory can be seen in this way. It is possible that what Cohen and Hiley (1996) have in mind is the interpretation (b) which I shall discuss next.

Interpretation (b)

Counterfactual probability as the probability of the result of a measurement would it have been performed based on the information about the world in which the measurement has not been performed.

At time t_1 we preselect the state $|\Psi_1\rangle$. We do not perform any measurement at time t. We perform a measurement at time t_2 and find the state $|\Psi_2\rangle$. We ask, what would be the probability for the results of a measurement performed at time t in a world which is identical to the actual world at time t_1.

This is a meaningful concept, but I believe that it is not adequate for discussing pre- and post-selected quantum systems because it is explicitly asymmetric in time. The

7

counterfactual world is identical to the actual world at time t_1 and might not be identical at time t_2.

This interpretation of the ABL rule is clearly inconsistent with predictions of quantum theory. According to the orthodox or standard interpretation the information about the actual world at time t_2 is irrelevant since the state $|\Psi_1\rangle$ together with the requirement of no disturbance between t_1 and t define completely the probabilities of all possible measurements at time t. Therefore, the ABL formula for probabilities which includes explicitly dependence on the result of the measurement at time t_2 cannot be consistent with quantum theory.

One may speculate about possible modifications of quantum theory which reconstruct statistical predictions of standard quantum theory but include, in addition, some hidden variables which specify individual outcomes of seemingly random results of quantum measurements. Then the information about the results of the measurements at time t_2 in a run of the experiment without an intermediate measurement might add to the description of the actual world at time t_1. See, for example, the discussion about such a situation in the framework of the Bohm (1952) theory by Aharonov and Albert (1987). In the framework of a hidden variable theory, for estimating the probabilities of the result of measurements at time t we have to consider the sub-ensemble of the pre-selected systems at time t_1 which have hidden variables corresponding to the appropriate result of the measurement at time t_2 on the condition that no disturbance (and, in particular no measurement at time t) took place between t_1 and t_2. The question of consistency between the ABL rule and the predictions of quantum theory in such framework seems to be a nontrivial problem. One may just recall the difficulty in the framework of the Bohm (1952) hidden variable theory according to which the outcome of a spin measurement might depend not only on the hidden variable of the system, but also on the state of the measuring device, (see Albert 1992, 153-154). However, I shall present now a simple example which allows us to show the inconsistency between quantum theory and this interpretation of the ABL rule irrespectively of the details of the hidden-variable theory.

Consider a spin-1/2 particle pre-selected at time t_1 in the state $|\uparrow_z\rangle$ and post-selected at the time t_2 in the same state $|\uparrow_z\rangle$. We ask what is the probability for finding spin "up" in the direction $\hat{\xi}$ which makes an angle θ with the direction $\hat{z}$, at the intermediate time t. In this case, the hidden variables, even if they exist, cannot change that probability because any particle pre-selected in the state $|\uparrow_z\rangle$, irrespectively of its hidden variable, yields the outcome "up" in the post-selection measurement at time t_2. Therefore, the statistical predictions about the intermediate measurement at time t must be the same as for the pre-selected only ensemble (these are *identical* ensembles in this case), i.e.

$$\text{Prob}(\uparrow_\xi) = |\langle \uparrow_\xi \mid \uparrow_z \rangle|^2 = \cos^2(\theta/2). \tag{4}$$

The ABL formula, however, yields:

$$\text{Prob}(\uparrow_\xi) = \frac{|\langle \uparrow_z \mid \mathbf{P}_{\uparrow_\xi} \mid \uparrow_z \rangle|^2}{|\langle \uparrow_z \mid \mathbf{P}_{\uparrow_\xi} \mid \uparrow_z \rangle|^2 + |\langle \uparrow_z \mid \mathbf{P}_{\downarrow_\xi} \mid \uparrow_z \rangle|^2} = \frac{\cos^4(\theta/2)}{\cos^4(\theta/2) + \sin^2(\theta/2)} \tag{5}$$

We have obtain two different results. This shows that this interpretation of the ABL rule is incorrect.

Interpretation (c)   *Counterfactual probability as the probability for the results of a measurement if it has been performed in the world "closest" to the actual world.*

This is identical in form and spirit to the theory of counterfactuals of Bennett (1984), although the context of the pre- and post-selected quantum measurements is somewhat beyond what he considered. This interpretation is explicitly time-symmetric. The title, however, does not specify it completely and I shall explain what do I mean (in particular by the word "closest") now.

I have to specify the concept of "world". There are many parts of the world which do not interact with the quantum system in question, so their states are irrelevant to the result of the measurement. In our discussion we might include all these irrelevant parts, or might not, without changing any of the conclusions. There are other aspects of the world which are certainly relevant to the measurement at time t, but we postulate that they should be disregarded. Everything which is connected to our decision to perform the measurement at time t and all the records of the result of that measurement are not considered. Clearly, the counterfactual world in which a certain measurement has been performed is different from an actual world in which, let us assume, no measurement has been performed at time t. The profound differences are both in the future where certain records exist or do not exist and in the past which must be different since one history leads to performing the measurement at time t and another history leads to no measurement.[4] However, our decision to make the measurement is not connected to the quantum theory which makes predictions about the result of that measurement. We want to limit ourselves to the discussion of the time-symmetry of the quantum theory. We do not consider here the question of the time-symmetry of the entire world. Therefore, we exclude the external parts from our consideration.

What constitutes a description of a quantum system itself is also a very controversial subject. The reality of the Schrödinger wave, the existence or inexistence of hidden variables etc. are subjects of hot discussions. However, everybody agrees that the collection of all results of measurements is a consistent (although maybe not complete) description of the quantum system. Thus, I propose the following definition:

> *A world "closest" to the actual world is the world in which all measurements (except the measurement at the time t if performed) have the same outcomes as in the actual world.*

This definition overcomes the common objection according to which one should not consider together statements about pre- and post-selected systems regarding different measurements at time t because these systems belong to different ensembles. The difference

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[4]If a random process chooses between the two possibilities, then the past before this process might be identical.

is in their quantum state at the time period between t and t_2.[5] Formally, the problem is solved by considering only results of measurements and not the quantum state. The justification of this step follows from the rules of the game: it is postulated that the quantum system is not disturbed during the periods of time (t_1, t) and (t, t_2). Therefore, it is postulated that no measurement on the system is performed during these periods of time. Since unperformed measurements have no results, the difference between the ensembles has no physical meaning in the discussed problem.

From the alternatives I presented here, only interpretation (c) is time-symmetric. This is the reason why I believe that it is the only reasonable candidate for analyzing the (time-symmetric) problem of measurements performed between two other measurements.

A very serious study of time's arrow and counterfactuals, in particular, in the framework of quantum theory, was performed recently by Price (1996). Let me quote from his section "Counterfactuals: What should we fix?":

> Hold fixed the past, and the same difficulties arise all over again. Hold fixed merely what is accessible, on the other hand, and it will be difficult to see why this course was not chosen from the beginning. (1996, 179)

This quotation looks very much like my proposal. And indeed, I find many arguments in his book pointing in the same direction. However, this quotation represents a time asymmetry: "merely what is accessible" is, in fact, "an accessible past". But this is not the time asymmetry of the physical theory; Price writes: "no physical asymmetry is required to explain it." Although the books includes an extensive analysis of a photon passing through two polarizers – the classic setup for the ABL case, I found no explicit discussion of a possible measurement in between, the problem we discuss here.[6]

5. Inconsistency proofs. The key point of the criticism of the time-symmetrized quantum theory is the conflict between counterfactual interpretation of the ABL rule and predictions of quantum theory. I shall argue here that the proofs of the inconsistency are unfounded and therefore the criticism essentially falls apart.

The inconsistency proofs (Sharp and Shanks 1993; Cohen 1995; Miller 1996) have the same structure. Three consecutive measurements are considered. The first is the preparation of the state $|\Psi_1\rangle$ at time t_1. The probabilities for the results a_i of the second

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[5]If one is adopting our backward evolving quantum state, he can add that the systems are also different due to the backward evolving state between t and t_1.

[6]Price briefly and critically mentions the ABL paper. He writes (1996, 208): "What they [ABL] fail to note, however, is that their argument does nothing to address the problem for those who disagree with Einstein – those who think that the state function is a complete description, so that the change that takes place on measurements is a real change in the world, rather than merely change in our knowledge of the world." This seems to me an unfair criticism: the ABL clearly state that in the situations they consider "the complete description" is given by *two* wave functions, see more in Aharonov and Vaidman (1991). Moreover, it seems to me that the development of this time-symmetric quantum formalism is not too far from the spirit of the "advanced action" – the Price vision of the solution of the time's arrow problem.

measurement at time t are considered. And the final measurement at time t_2 is introduced in order to allow the analysis which uses the ABL formula. Sharp and Shanks consider three consecutive spin component measurements of a spin-1/2 particle in different directions. Cohen analyses a particular single-particle interference experiment. It is a variation on the theme of Mach-Zehnder interferometer with two detectors for the final measurement and the possibility of placement of a third detector for the intermediate measurement. Finally, Miller repeated the argument for a system of tandem Mach-Zehnder interferometers. In all cases the "pre-selection only" situation is considered.

It is unnatural to apply the time symmetrized formalism for such cases. However, it must be possible. Thus, I should not show that the time-symmetrized formalism has an advantage over the standard formalism for describing these situations; I should only show consistency. In the standard approach to quantum theory the probability for the result of the measurement of A at time t is given by Eq. 1. The claim of all the proofs is that the counterfactual interpretation of the ABL rule yields a different result. In all cases the final measurement at time t_2 has two possible outcomes which we signify as "1_f" and "2_f"; they are spin "up" or "down" and the click of the detector D_1 or D_2 respectively. The suggested application of the ABL rule is as follows. The probability for the result a_i is:

$$\text{Prob}(A = a_i) = \text{Prob}(1_f)\text{Prob}(A = a_i|1_f) + \text{Prob}(2_f)\text{Prob}(A = a_i|2_f), \qquad (6)$$

where $\text{Prob}(A = a_i|1_f)$ and $\text{Prob}(A = a_i|2_f)$ are the conditional probabilities given by the ABL formula, Eq. 2, and $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ are the probabilities for the results of the final measurement. There is no ambiguity about the probability of the intermediate measurement given the result of the final measurement, it is uniquely defined by the ABL formula. The error in the proofs is in the calculation of the probabilities $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ of the final measurement. In all three cases it was calculated on the assumption that no measurement took place at time t. Clearly, one cannot make this assumption here since then the discussion about the probability of the result of the measurement at time t is meaningless. Unperformed measurements have no results. Thus, there is no surprise that the value for the probability $\text{Prob}(A = a_i)$ obtained in this way comes out different from the value predicted by the quantum theory.

Straightforward calculations show that if one uses the formula (6) with the probabilities $\text{Prob}(1_f)$ and $\text{Prob}(2_f)$ calculated on the condition that the intermediate measurement has been performed, then the outcome is the same as predicted by the standard formalism of quantum theory. Consider, for example, the experiment suggested by Sharp and Shanks, the consecutive spin measurements with the three directions in the same plane and the relative angles θ_{ab} and θ_{bc}. The probability for the final result "up" is

$$\text{Prob}(1_f) = \cos^2(\theta_{ab}/2)\cos^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2)\sin^2(\theta_{bc}/2), \qquad (7)$$

and the probability for the final result "down" is

$$\text{Prob}(2_f) = \cos^2(\theta_{ab}/2)\sin^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2)\cos^2(\theta_{bc}/2). \qquad (8)$$

The ABL formula yields

$$\mathrm{Prob}(up|1_f) = \frac{\cos^2(\theta_{ab}/2)\cos^2(\theta_{bc}/2)}{\cos^2(\theta_{ab}/2)\cos^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2)\sin^2(\theta_{bc}/2)} \tag{9}$$

and

$$\mathrm{Prob}(up|2_f) = \frac{\cos^2(\theta_{ab}/2)\sin^2(\theta_{bc}/2)}{\cos^2(\theta_{ab}/2)\sin^2(\theta_{bc}/2) + \sin^2(\theta_{ab}/2)\cos^2(\theta_{bc}/2)}. \tag{10}$$

Substituting all these equations into Eq. 6 we obtain

$$\mathrm{Prob}(up) = \cos^2(\theta_{ab}/2). \tag{11}$$

This result coincide with the prediction of the standard quantum theory. It is a straight-forward exercise to show in the same way that no inconsistency arises also in the examples of Cohen and Miller.

Apparently, the motivation of the authors of the above inconsistency proofs for taking the expressions for the probabilities $\mathrm{Prob}(1_f)$ and $\mathrm{Prob}(2_f)$ based on the assumption that no measurement has been performed at time t follows from their interpretation of "counterfactual interpretation of the ABL rule" which was named (a) above. It seems that they consider that a necessary condition for a counterfactual is that it *has* to be contrary to what is in the actual world. In their view the only alternative to the postulate of "no measurement" is the postulate that a measurement has been actually performed.[7] I believe, however, that one can interpret counterfactuals without postulating that they are necessarily contrary to the actual world, see interpretation (c) above. Moreover, as I explained above, I find their interpretation (a) physically meaningless.

6. Time asymmetry prejudice.   In my approach the pre- and post-selected states are given. Only intermediate measurements are to be discussed. So the frequently posed question about the probability of the result of the post-selection measurement is irrelevant. It seems to me that the critics of the time-symmetrized quantum theory use in their arguments the preconception of an asymmetry. It is not surprising then that they reach various contradictions. Probably the first to go according to this line were Bub and Brown:

> Put simply, systems initially in the state ψ_I which are subject to an N measurement, and subsequently yield the state ψ_F after an M_F measurement, would not necessarily yield this final state if subjected to a measurement of M instead of N. (1986, 2338)

Their argument is valid (see Albert et al. 1986) in the context of the possible hidden variable theories which allow us to *predict* the results of measurements, but it should not

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[7]In this case the authors of the inconsistency proofs say that no contradiction arises, but also no interesting question can be asked.

be brought against proposals of a time-symmetrized formalism as it was done frequently later. Let me quote a few examples: Cohen writes:

> We have no reason to expect that, for example, the $N/4$ systems preselected by $|\psi_1(t_1)\rangle$ and post-selected by $|\psi_2(t_2)\rangle$ after an intermediate measurement of σ_{1y} would still have yielded the state $|\psi_2\rangle$ after an intermediate measurement of σ_{2x} or of $\sigma_{1y}\sigma_{2x}$ instead of σ_{1y}.(1995, 4375)

A consistent time-symmetric approach should question the pre-selection on the same footing as the post-selection; or rather not question any of them, as I propose.

Another asymmetry pre-conception lead to the "retrodiction paradox" of Peres:

> The asymmetry between prediction and retrodiction is related to the fact that predictions can be verified (or falsified) by actual experiments, retrodictions cannot. Retrodictions are counterfactual statements about *unperformed experiments*; in quantum mechanics, unperformed experiments have no results (Peres 1987). (1994, 23)

While I certainly agree that unperformed experiments have no results, I challenge the interpretation according to which counterfactual statements are necessarily about events which do not happen. The asymmetry considered by Peres (1994) is, in fact, between *prediction* based on the results in the past of time t in question and *inference* based on the results both in the past and in the future of time t. This inference he erroneously considered as retrodiction (see Aharonov and Vaidman 1995).

If we are not considering a pre- and post-selected system then there is an asymmetry between prediction and retrodiction. For example (Aharonov and Vaidman 1990, 11-12), assume that the x component of the spin of a spin-1/2 particle was measured at time t, and was found to be $\sigma_x = 1$. While there is a symmetry regarding prediction and retrodiction for the result of measuring σ_x after or before time t (in both cases we are certain that $\sigma_x = 1$), there is an asymmetry regarding the results of measuring σ_y. We can predict equal probabilities for each outcome, $\sigma_y = \pm 1$, of a measurement performed after time t, but we cannot claim the same for the result of a measurement of σ_y performed before time t. The difference arises from the usual assumption that there is no "boundary condition" in the future, but there is a boundary condition in the past: the state in which the particle was prepared before time t. Maybe in a somewhat artificial way we can reconstruct the symmetry even here, out of the context of pre- and post-selected systems. We can "erase" the results of the measurements of the spin measurements in the past (Vaidman 1987, 61). In order to do this we perform at time t_0, before time t, a measurement of a Bell-type operator on our particle and another auxiliary particle, an ancilla. We ensure that no measurement is performed on the particle between t_0 and t (except the possible measurement whose result we want to consider) and we prevent any measurement on the ancilla from time t_0 and on. The Bell-type measurement correlates the quantum state of our particle evolving from the past with the results of the future

measurement performed on the ancilla. Since the latter is unknown, we obtain, effectively, an unknown past for our particle. Now, for such a system, if we know that the result of the measurement at time is $\sigma_x = 1$, we can also *retrodict* that there are equal probabilities for both outcomes of the measurement of σ_y performed before time t (but after time t_0). The time symmetry is restored.[8]

7. Time symmetry of the process of measurement. Obviously, in order to discuss a measurement at time t between two other measurements in a time-symmetric way, the process of measurement at time t must be time-symmetric. Usually, a measurement of a quantum variable A is modeled by the von Neumann (1955) Hamiltonian

$$H = g(t)pA, \tag{12}$$

where p is the momentum conjugate to the pointer variable q, and the normalized coupling function $g(t)$ specifies the time of the measurement interaction. The function $g(t)$ can be made symmetric in time (not that it matters) and the form of the coupling then is time-symmetric. The result of the measurement is the difference between the value of q before and after the measurement interaction. So it seems that everything is time-symmetric.

However, usually there is an asymmetry in that that the initial position of the pointer is customized to be zero (and therefore the final position correspond to the measured value of A). This seemingly minor aspect points to a genuine asymmetry. Of course, the initial zero position of the pointer is not a necessary condition; we can choose any other initial position as well. But, we cannot chose the final position. We *know* the initial position and we find out, at the end of the measurement the final position. We can introduce another step with symmetrical coupling, but we will not be able to remove the basic asymmetry: we do not know the result of the measurement before the measurement but we do know it after the measurement.

This asymmetry in time is an intrinsic property of the concept of measurement and it has no connection to the quantum theory. It is related to the arrow of time based on the increasing memory. See illuminating discussion of Bitbol (1988) of the process of measurement in the framework of the many-worlds interpretation (Everett, 57).

The symmetry aspect of the process of measurement which is important for our discussion is that a measurement at time t leads to identical forward and backward evolving states out of time t. Operationally, it means that under the assumption of zero Hamiltonian and and identical pre- and post-selected states (or mixtures), the probabilities for the result of any measurement performed before t (but after the pre-selection) is equal to that performed after time t (but before the post-selection). The measurement described by the Hamiltonian (12) has this time symmetry. Moreover, any "ideal" von Neumann measurement, which projects on the property to be measured and does not

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[8]The time symmetry is restored not just for the σ_y measurement, but for any spin measurement.

change the quantum state if it has the measured property, is symmetric in this sense. Recently Shimony (1995) proposed considering more general quantum measurements which do not change the measured property (so they are repeatable, the main property which is required from a "good" measurement) but which change the state (even if it has the measured property). Such measurements are intrinsically asymmetric in our sense, since given identical boundary conditions in the past and in the future they lead to different probabilities for the results of some measurements performed before or after the time of the generalized measurement. Clearly, the time symmetrized formalism is not applicable for such measurements as Shimony has showed.

8. Elements of Reality.

8. Elements of Reality. Until now we have discussed a situation in which we know the results of the measurements at t_1 and t_2, we know that there is no measurement or disturbance of the system between t_1 and t_2 except, may be, a measurement at time t. We also discussed the outcomes of that possible measurement. For the same system we can discuss several, in general incompatible, possible measurements at time t and this is why we consider it as counterfactual reasoning. Interesting novel (relative to a pre-selection only situation) structures emerge in this situation. In particular, there are situations in which several incompatible measurement (if performed) have certain results with probability 1. I have proposed to call them *elements of reality* (Vaidman 1993a, 1996). These elements of reality, contrary to the pre-selected situations might contradict the *product rule* (Vaidman 1993b), i.e., if $A = a$ and $B = b$ are elements of reality, $AB = ab$ might not be an element of reality.

One important aspect of these elements of reality is that they are Lorentz invariant. Their introduction solves an apparent contradiction which follows from the existence of Lorentz invariant elements of reality. Let me quote a recent paper by Cohen and Hiley:

> A further criticism by Vaidman (1993) leads to the conclusion that the gedanken experiment [of Hardy, 1992] does not lead to any contradiction, because it involves a pre- and post-selected quantum system, for which, it is claimed, the "product rule" does not apply. Unfortunately, as we show in a separate paper [apparently Cohen and Hiley 1996], Vaidman's analysis is not valid because it makes incorrect use of the formula of Aharonov, Bergman and Lebowitz (1964; Aharonov and Vaidman 1991).(1995, 76)

I believe that I have succeeded here in showing the legitimacy of applying the ABL formula for pre- and post-selected quantum systems with a choice of possible intermediate measurements which is exactly the situation considered in Hardy's gedanken experiment and thus defending my results (1993).

One might argue about the significance of these concepts beyond the philosophical construction of (jointly unmeasurable) "elements of reality", since it is impossible to perform incompatible measurements on a single system. I find the most important aspect of

these concepts in their relation to *weak measurements* (Aharonov and Vaidman 1990).[9] Weak measurements are almost standard measurement procedures with weakened coupling. Weak measurements essentially do not change the quantum states (evolving forward and backwards in time) of the system. Several weak measurements can be performed on a single system and they are compatible even though their counterparts, the ideal measurements are not compatible.

An example of an interesting connections between weak and strong (ideal) measurements is the theorem (Aharonov and Vaidman 1991) which says that if the probability for a certain value to be the result of a strong measurement is 1, then the corresponding weak measurement must yield the same value.[10] However, in general, the outcome of weak measurements might not be one of the possible outcomes of a strong measurement. The outcome of the weak measurement of a variable A is the weak value (Eq. 3) which might lie far away from the range of the eigenvalues of A. The weak value is not just a theoretical concept related to a gedanken experiment. Recently, weak values have been measured in a real laboratory (Ritchie et al., 1991).

9. Conclusions. In this paper I have defended the time-symmetrized quantum formalism originated by Aharonov et al. (1964) against recent criticism. The criticism followed from the pre-conception of time asymmetry which is the feature of the standard formalism of quantum theory and of the standard approach to counterfactual reasoning. I have argued, that in the context of the experiments on pre- and post-selected quantum systems, the time-symmetric counterfactual theory suggested by Bennett (1984) is the most appropriate. I introduced the time-symmetric counterfactual interpretation of the ABL rule and showed that it does not lead to any contradiction with the predictions of quantum theory.

I disagree in an essential way with a large number of recent works. In particular, contrary to the conclusions of Sharp and Shanks (1993), I believe that the time-symmetrized quantum theory "yields fresh insights about the fundamental interpretive issues in quantum mechanics". I base my belief on the research in which I took part and which allowed us to see numerous surprising quantum effects which are hidden in the framework of the standard approach in very complicated mathematics of some peculiar interference effects (e.g. Aharonov et al. 1987, 1990, 1993; Vaidman 1991). I see a novel rich structure in time-symmetrized quantum theory which suggests and solves surprising quantum problems (Vaidman et al. 1987; Vaidman 1996). It is also plausible that the time-symmetrized

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[9]The scope of this paper allows only to touch this broad issue, but one can find more on this subject in Vaidman (1996) where, in particular, I defined a new type of reality based on the concept of weak measurements.

[10]In such situation, when we discuss the outcomes of weak measurements, the statements about strong measurement have even a stronger counterfactual sense. The counterfactual reasoning about *unperformed* strong measurements (which correspond to elements of reality) helps us to find out the results of the *performed* weak measurements.

approach might help investigated the current problems of quantum gravity (see, for example, Unruh 1995).

The time-symmetrized quantum theory fits well into the many-worlds interpretation (MWI), my preferred interpretation of quantum theory (Vaidman 1993c, 1994). The counterfactual worlds corresponding to different outcomes of quantum measurements have in the MWI an especially clear meaning: these are subjectively actual different worlds. In each world the observers of the quantum measurement call their world as actual, but, if they believe in the MWI they have no paradoxes about ontology of the other worlds. Consider an illuminating example (this time of pre-selected only situation) by Mermin (1989) in which counterfactual reasoning lead him to wonder: can he help his favorite baseball team to win by watching their game on television? Or, can an action in one region change something in a space-time separated region? The counterfactual reasoning in the framework of the standard (single-world) interpretations lead him to the paradoxical answer "yes". The MWI answers that his action in one place causes different separation into worlds which include correlations between the two regions and therefore we have, this time a not surprising answer "yes". If we consider the problem from an external position, i.e. we consider the whole physical universe which incorporates all the worlds, then we obtain the expected answer "no".[11] Some things at the remote location become correlated to different things at the first location, but it does not change any measurable property in the remote location.

The MWI interpretation yields also a convincing answer to paradoxical situations considered by Penrose:

> What is particularly curious about quantum theory is that there can be actual physical effects arising from what philosophers refer to as *counterfactuals* – that is, things that might have happened, although they did not happened. (1994, 240)

According to the MWI, in the situations considered by Penrose, "things" did not happened in a particular world, but did happened in some other world (see Vaidman 1994). Therefore, they did took place in the physical universe and thus their effect on some other facts in the physical universe is not so surprising.

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[11]The answer "no" is expected because of the locality of physical interactions.

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