

NONLOCAL MEASUREMENTS IN THE TIME-SYMMETRIC QUANTUM MECHANICS

LEV VAIDMAN

*School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Sciences,
Tel-Aviv University, Tel-Aviv 69978, Israel
vaidman@post.tau.ac.il*

IZHAR NEVO

*School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Sciences,
Tel-Aviv University, Tel-Aviv 69978, Israel*

Received 28 February 2006

Although for some nonlocal variables the standard quantum measurements which are reliable, instantaneous, and nondemolition, are impossible, demolition reliable instantaneous measurements of all variables are possible. It is shown that this is correct also in the framework of the time-symmetric quantum formalism, i.e. nonlocal variables of composite quantum systems with quantum states evolving both forward and backward in time are measurable in a demolition way. The result follows from the possibility to reverse with certainty the time direction of backward evolving quantum states.

Keywords: Quantum measurements; nonlocal measurements; time symmetry.

1. Introduction

There are causality constraints on nondemolition quantum measurements¹ which allow measurements of only certain classes of nonlocal measurements.² Recently it has been shown that in principle it is possible to perform an instantaneous measurement of an arbitrary nonlocal variable of two spin- $\frac{1}{2}$ particles³ and, in fact, of any composite system,⁴ provided we do not require that the measurement is non-demolition. Such demolition measurement distinguishes with certainty between eigenstates of the nonlocal variable, but, contrary to the standard definition of a quantum measurement, it might destroy the measured eigenstate. The possibility to perform such measurement provides physical meaning for nonlocal variables in the framework of relativistic quantum mechanics.

Recently, we are witness to a significant development of the time-symmetric formalism of quantum mechanics^{5,6} originated by Aharonov, Bergmann and Lebowitz.⁷ In this formalism, in addition to the standard quantum state evolving

forward in time from the time of a complete measurement on the system performed in the past, there are quantum states evolving backward in time from complete measurements performed in the future relative to the time in question. Moreover, in this framework one can define states evolving for one subsystem in one time direction and in the opposite time direction for another. Thus, one can define novel types of nonlocal variables, for which the eigenstates evolve backward in time or even in different time directions. In this work we show that such nonlocal variables are also measurable (in the sense of demolition measurements) and this result provides physical meaning for nonlocal variables and states of the time-symmetric quantum formalism.

2. The Concept of Nonlocal Measurements

Let us now state more precisely the problem we want to consider. The quantum system consists of two or more parts separated in space. We assume that we are equipped with measuring devices which allow to measure any local variable of the parts of the system. We also have unlimited resources of entanglement, the quantum channels which connect the sites in which separate parts of the system are located. Given an unlimited time, these resources would allow, through teleportation of quantum states of different parts to one location, a measurement of an arbitrary variable, but we require *instantaneous* measurement and thus there is no time to complete the teleportation of quantum states of different parts. The meaning of “instantaneous” is that shortly after the time of measurement, there are permanent records in the locations of the parts of the system, which together yield the value of the nonlocal observable.

The method should work for measurement of any variable of our composite quantum system. Examples of nonlocal variables are variables with eigenstates which are entangled (Bell operator), or variables with product state operators which cannot be measured via measurements of local variables (which coined the name *nonlocality without entanglement*).⁸ In fact, we can consider it as a definition of nonlocal variables: *variables which cannot be measured through measurements of local variables*.

3. Demolition Nonlocal Measurements

Some nonlocal variables are measurable in a sense of standard (nondemolition) quantum measurements (e.g. Bell operator.²) However, it is known that there are nonlocal variables which are immeasurable in this sense, and this is why we have to look for other definition of measurement: for demolition measurements all variables are measurable. In particular, projection operator on an arbitrary quantum state of the system is measurable and this provides physical meaning for quantum states of the composite system including nonlocal (entangled) states.

Let us recall the basic idea of nonlocal demolition measurements⁴ on the example of bipartite system where each part consists of several spin- $\frac{1}{2}$ particles. It consists of a sequence of “half teleportations” between the sites of the system, where, say, Alice

and Bob are located. “Half teleportations” are teleportations without corrections and thus they do not have a minimal time for implementation since they do not include transmission of information.

If a quantum state has a simple form

$$|\Psi\rangle = \Pi_i |s_i\rangle_i, \quad (1)$$

where $|s_i\rangle$ is either $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$, then it can be determined unambiguously through local measurement even if it was first “half teleported” from another place, provided we collect together the results of the spin measurements of the half-teleported state together with the results of Bell measurements of the half-teleportation procedure. Indeed, the only possible effects of the second half of teleportation procedure are changes of the total phase and spin flips. The phase is irrelevant and the effect of the spin flip can be corrected after the measurement, when the information about the results of the Bell measurements arrives.

The procedure goes as follows. At the beginning, the state of Bob’s part of the system is “half-teleported” to Alice. Alice assumes that “half-teleportation” is, in fact, the full teleportation (there is a finite probability for results of Bell measurements corresponding to teleportation without correction) and transforms the eigenstates of the nonlocal variable to the orthogonal set of states in the form (1). Then, she half-teleports the transformed state to Bob. Bob knows the results of his teleportation Bell measurements and if they, indeed, were successful, he performs the spin measurements in the z basis. Together with the results of Alice teleportation Bell measurements these results yield the eigenstate of the measured variable. If the first Bob’s teleportation was not successful, he teleports the received state back to Alice in one of the numerous quantum channels according to the results of Bell measurements of the first teleportation attempt. Alice assumes that the second Bob’s teleportation was successful and performs unitary transformations on the outputs of all Bob’s channels to the form (1) and teleports each channel to Bob. Since every round adds finite probability for success (which equals one over the number of possible different outcomes of Bell measurements), the probability of the success after many rounds converges to 1. The method can be generalized to any number of sites of a composite quantum system.⁴

4. Demolition Nonlocal Measurements for Backward Evolving Quantum States

The method cannot be applied for measurements of backward evolving states by simple “time-reversal” of our operations. Indeed, in our procedure, Bob, starting from the second round, teleports the quantum state in a particular channel depending on the results of his previous Bell measurement. In the time reversed procedure, this corresponds to choosing the channel according to the result of a measurement which has not been performed yet. Note, that standard von Neumann measurement *is* applicable for measurement of backward evolving quantum state. The same procedure measures quantum states evolving forward and backward in time. It is true

also for nondemolition nonlocal quantum measurements,² but such measurements can be performed only for a very limited class of nonlocal variables. We remain with the question: Is it possible to measure (in a demolition way) an arbitrary variable for a backward evolving quantum state?

The answer is positive, and, moreover, we can measure in a demolition way any variable also for quantum states evolving in different time directions at different locations. The solution is very simple: the quantum state evolving backward in time can be transformed to a state evolving forward in time and since for quantum states evolving forward in time there is a measurement procedure which leads to a successful measurement with any desired probability, we have, at least theoretically, a solution for measurement of a nonlocal variable for quantum state with parts evolving in arbitrary time directions.

Any quantum system with a finite number of states can be mapped to a system of N spin- $\frac{1}{2}$ particles. So all what we have to do is to change the time direction of a spin- $\frac{1}{2}$ particle. To this end, we perform a Bell measurement on the particle and an ancilla and then (if needed) local operations acting on the ancilla to change the state of the two particles to a singlet $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$, making the two particle an EPR pair. In fact, we can prepare the singlet in any other way too. We also have to ensure that there is no disturbance between the time of Bell measurement and the time we receive the particle with the backward evolving state. Then, the ancilla obtains the time reversed state of the particle:

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \rightarrow -\beta^*|\uparrow\rangle + \alpha^*|\downarrow\rangle. \quad (2)$$

When we perform such time reversal for all parts of the system evolving backward in time, the eigenstates of the nonlocal variable with parts of the system evolving backward in time are transformed to a well defined mutually orthogonal states evolving forward in time. Thus, we obtain one-to one correspondence between the eigenstates of the variable with parts evolving in different time directions and eigenstates of a variable evolving forward in time. The latter we know how to measure in a demolition way, thus, we got a theoretical method for demolition measurement of an arbitrary nonlocal variable.

5. Quantum State with One Part Evolving Forward in Time and Another Part Evolving Backward in Time

Let us describe an example. In order to “prepare” an eigenstate of an operator corresponding to a quantum state evolving in part A forward in time and in part B backward in time, consider two “crossed” nonlocal (in space and time) measurements, Fig.1:

$$\begin{aligned} O_1 &\equiv \sigma_z^A(t_1) - \sigma_z^B(t_2), \\ O_2 &\equiv (\sigma_x^A(t_1 - \epsilon) - \sigma_x^B(t_2 + \epsilon)) \bmod 4. \end{aligned} \quad (3)$$

For time t , $t_1 < t < t_2$, part A of each eigenstate evolves forward in time and part B evolves backward in time:

$$\begin{aligned}
 O_1 = 2 : & \quad |\uparrow\rangle_A \langle\downarrow|_B, \\
 O_1 = -2 : & \quad |\downarrow\rangle_A \langle\uparrow|_B, \\
 O_1 = 0, \ O_2 = 0 : & \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle_A \langle\uparrow|_B + |\downarrow\rangle_A \langle\downarrow|_B), \\
 O_1 = 0, \ O_2 = 2 : & \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle_A \langle\uparrow|_B - |\downarrow\rangle_A \langle\downarrow|_B).
 \end{aligned} \tag{4}$$

Now, our time reversal operation (2) at time $t - \epsilon$ at part B transforms these eigenstates into:

$$\begin{aligned}
 & |\uparrow\rangle_A |\uparrow\rangle_B, \\
 & |\downarrow\rangle_A |\downarrow\rangle_B, \\
 & \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B), \\
 & \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B).
 \end{aligned} \tag{5}$$

This is a nonlocal variable evolving forward in time which cannot be measured in a nondemolition way,¹ but it can be measured in a demolition way.

Although the method explained above allows to perform measurements of any nonlocal variable, it requires unnecessary large resources. In order to make reliable measurement of forward evolving quantum state with parts located in several places we need huge amount of entanglement.⁴ In fact, if in particular location the quantum state evolves backward in time, the measurement procedure can be much simpler: we can save a lot of entanglement resources. Indeed, our task is to bring

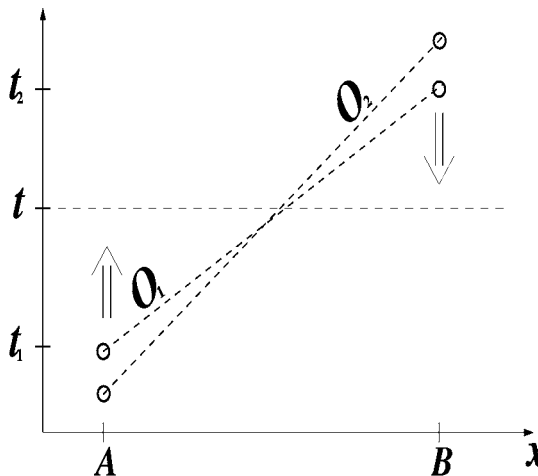


Fig. 1. The “preparation” of quantum state evolving forward in time in A and backward in time in B .

the states of all parts to one location. The quantum state evolving backward in time can be moved to another place in one step which also includes the change of the direction of the time evolution of the state. All what we have to do is to prepare a singlet with our particle and an ancilla located in the place to which we want to move the state of the particle. When there is a large distance between the two places it might require a significant time, but we have this time since we can prepare it in advance. If we cannot arrange in advance that one particle from the EPR pair is the part of our post-selected system, we can always perform local swap operation between the member of the EPR pair and the post-selected particle. Thus, the backward evolving quantum state or, the state with only one part evolving forward in time, can be measured in a demolition way using very moderate resources: $N - 1$ quantum channels for a N part system.

6. The Two-State Vector

Until now we discussed quantum states evolving at each space-time point in one time direction. In the framework of the time-symmetric formalism of quantum mechanics,⁵ the complete description of a quantum system is given by a two-state vector $\langle \Phi | | \Psi \rangle$ such that there are both forward and backward evolving quantum states at each part of the system. The most general description of a composite quantum system of N parts is the *generalized two-state vector*:⁹

$$\sum_i \alpha_{(i_1, \dots, i_N, j_1, \dots, j_N)} \prod_{n=1}^N \langle \Phi_{i_n} |_n | \Psi_{j_n} \rangle_n. \quad (6)$$

A generalized two-state vector might not be reducible to a two-state vector $\langle \Phi | | \Psi \rangle$. In this case, in order to “prepare” such state at time t it is necessary to have ancilla particles guarded from the time of collective measurement of our system and the ancilla before t and until collective post-selection measurement after time t . (Considering large enough system we can always arrive to a description by a two-state vector, like in standard quantum mechanics, considering large enough system we can always arrive to the description by a pure quantum state.)

The N part system described by a generalized two-state vector is equivalent to $2N$ part system with quantum state in half of the parts evolving forward in time and half of the parts evolving backward in time. Thus, preparing singlets with ancilla particles in the location of one of the parts of the system and their pairs at the locations of the other parts, we can bring all backward evolving parts of the quantum state to our chosen location. From this point we should proceed with the procedure of measuring nonlocal forward evolving state of N -part system.⁴

The conceptual possibility of measuring an arbitrary variable with eigenstates which are nonlocal generalized two state-vectors provides justification for ascribing physical meaning for nonlocal two-state vectors. Indeed, since all nonlocal variables can be measured, in particular, projection operators on all two-state vectors are measurable.

7. Meaning of the Backward Evolving Quantum State

Let us now add some discussion of the concepts described above. What do we exactly mean by “backward evolving state”? We need not to assume that at present there is actually a backward evolving state from the future measurement, i.e. that the future is fixed. The formalism is helpful even in a pragmatic approach when we consider a scenario with post-selection: At a particular time we perform measurements on each member of an ensemble of quantum systems. Later, somebody performs another measurement and *post-selects* a subensemble of systems with particular result of his measurement. There are no constraints on the post-selection measurement: any local or nonlocal variable can be measured, since there is no requirement that it will be instantaneous. All parts of the system can be brought to one place and then by assumption any measurement can be performed. Then, we discard all the results for systems which were not post-selected and consider only systems which were successfully post-selected.

Basic requirement of a measurement, and in particular of our demolition non-local measurement is that it reliably distinguishes the eigenstates of an observed variable, i.e. our measurement and the post-selection should yield the same results. Then, it seems that measurement of a variable for a backward evolving quantum state can be performed in a much simpler way: just *prepare* an eigenstate of this variable. Then, the post-selected ensemble, will consist only of cases in which the post-selection measurement yields the same result. Preparation of nonlocal state requires much less resources than its verification, since we can prepare the state of all parts of the system locally in one place and then bring them to separate parts at the right time. We assume throughout the paper that the free Hamiltonian is zero. In practice we can always compensate the free evolution.

However, preparation of a particular eigenstate is not good enough verification measurement of a backward evolving state. What we ask from a quantum measurement is not only that it yields the correct value of the measured variable if the state is the eigenstate of this variable, but also that if the state is a superposition of different eigenstates then the measurement yields one of them with the appropriate probability. If the backward evolving state is a superposition of several eigenstates of the measured variable, preparation of one particular state yields wrong statistics.

Backward evolving state defines probability of a measurement at present only if there is no quantum state evolving forward in time from the measurements in the past. This is usually not the case (while it is a usual situation for the time-reversed setup in which forward evolving quantum state is considered, since future is considered to be unknown). Thus, the past of the quantum system has to be erased, it should be unknown. This can be done using collective measurements on the system and ancilla particles, but then, the procedure which replaces the preparation is not simpler than the method described above.

8. Reversing the Direction of Time-Evolution of a Quantum State

What allowed us to solve the problem of measurability of nonlocal variables with arbitrary directions of time evolution for various parts of the system is a simple, but apparently new result: *we can change the time direction with certainty from backward evolving quantum state to forward evolving quantum state.*

Note that this is not so if we try to transform forward to backward evolving state. In order to reverse the forward evolving state we need to obtain a singlet as a result of Bell measurement performed on our particle and an ancilla. The probability for this is $\frac{1}{4}$. If the result is different, we cannot correct it to a singlet, since this operation depends on the result of the Bell measurement, but has to be performed *before* the Bell measurement.

Although we do not consider here continuous systems (every real system can be approximated well as a system with a finite number of states), it is interesting to note that we can, conceptually, to perform a time reversal of a backward evolving quantum state of a continuous system $\Psi(q)$ by preparing an original EPR state (the state of the Einstein, Podolsky, Rosen paper¹⁰) for the particle and an ancilla:

$$|q - \tilde{q} = 0, \quad p + \tilde{p} = 0\rangle. \quad (7)$$

Then, the backward evolving quantum state of the particle will transform into a complex conjugate state of the ancilla:

$$\Psi(q) \rightarrow \Psi^*(\tilde{q}). \quad (8)$$

If the particle and the ancilla are located in different locations, then such operation is a combination of time reversal and teleportation of a backward evolving quantum state of a continuous variable.¹¹

Acknowledgments

This research was supported in part by QAP European Integrated Project.

References

1. S. Popescu and L. Vaidman, *Phys. Rev. A* **49**, 4331 (1994).
2. Y. Aharonov, D.Z. Albert and L. Vaidman, *Phys. Rev. D* **34**, 1805 (1986).
3. B. Groisman and B. Reznik, *Phys. Rev. A* **66**, 022110 (2002).
4. L. Vaidman, *Phys. Rev. Lett.* **90**, 010402 (2003).
5. Y. Aharonov and L. Vaidman, *Phys. Rev. A* **41**, 11 (1990).
6. Y. Aharonov and L. Vaidman, in *Time in Quantum Mechanics* J.G. Muga et al. eds., pp. 369-412 (Springer 2002).
7. Y. Aharonov, P. G. Bergmann and J. L. Lebowitz, *Phys. Rev. B* **134**, 1410 (1964).
8. C. H. Bennett *et al.*, *Phys. Rev. A* **59**, 1070 (1999).
9. Y. Aharonov and L. Vaidman, *J. Phys. A* **24**, 2315 (1991).
10. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
11. L. Vaidman, *Phys. Rev. A* **49**, 1473 (1994).