

The Reality in Bohmian Quantum Mechanics or Can You Kill with an Empty Wave Bullet?

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Several situations, in which an empty wave causes an observable effect, are reviewed. They include an experiment showing “surrealistic trajectories” proposed by Englert et al. and protective measurement of the density of the quantum state. Conditions for observable effects due to empty waves are derived. The possibility (in spite of the existence of these examples) of minimalistic interpretation of Bohmian quantum mechanics in which only Bohmian positions supervene on our experience is discussed.

KEY WORDS: Bohmian quantum mechanics; empty waves; surrealistic trajectories; protective measurements; many-world interpretation.

1. INTRODUCTION

I have my most vivid memories of Jim Cushing from “Bohmian conference” in 1995 which took place in Bielefeld, Germany and had the title “Quantum Theory without Observers”. The majority of participants were devoted Bohmians and most of the talks and discussions were about meaning and achievements of Bohmian quantum mechanics.^(1,2) At that time I already was an enthusiastic proponent of the Many-Worlds interpretation (MWI),⁽³⁾ and Bohmian interpretation, which close to the MWI, was very intriguing for me. Illuminating discussions with Jim Cushing led me to think more about Bohmian Interpretation.

I am still a strong proponent of the MWI.⁽⁴⁾ The main reason for this is not the philosophical advantage of the plurality of worlds, but a desire to view physics as a theory of everything. The main obstacle for this is the collapse of the quantum wave. Collapse introduces randomness into

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physics, it puts limits on predictive power of physics. There is no attractive proposal for a physical theory of collapse, and, moreover, it seems impossible to define when collapse occur. I get used to the idea of plurality of worlds, but a theory without collapse and with a single world is clearly a better theory of everything. In some sense, Bohmian quantum mechanics is such theory. (Note, however, that Bohm himself never viewed his theory in that way. I had elaborate discussion with him in South Carolina in 1989 in which he explained that his theory is another step in the evolution of physics and there will never be the final theory of everything). The main reason (apart from nonlocality of Bohmian mechanics) why I still prefer the MWI, is that it does not really eliminate the plurality of worlds. The formalism still has many, many “empty wave” worlds in which I walk, eat, slip and, in particular, write papers. Nevertheless, Bohmian mechanics achieves something that no other theory was able to do: to single out in a pretty natural way a single world out of the plurality of worlds in the MWI. (Note that in some special cases the Bohmian world might slightly differ from the world of the MWI⁽⁵⁾).

In the MWI, the Wave Function of the Universe is decomposed into superposition of branches in which the shape of the wave function yields a sensible picture, and the time evolution of the wave function of a branch yields a sensible story (with possible further branching). It is postulated that our experience corresponds to all branches with sensible stories.

In the Bohmian mechanics, or, at least, in my approach to Bohmian mechanics, it is postulated that our experience corresponds to Bohmian positions. Bohmian positions correspond to a sensible picture in three dimensions which evolves in time and yields a sensible *single* story. The same arguments which start from the locality of known interactions (which are frequently named as decoherence theory) yield plurality of particular sensible stories in the MWI and a single story (usually identical to one of the MWI stories) in the Bohmian mechanics.

In my approach our experience is related solely to Bohmian positions and not to the quantum state (the wave function). It is not a new approach: it seems to me that it is the *pilot wave* approach as Bell⁽⁶⁾ understood it, and maybe De Broglie imagined it. Bedard⁽⁷⁾ attributes this view also to Holland⁽⁸⁾, Maudlin⁽⁹⁾ and Albert.⁽¹⁰⁾

In this paper I am going to analyze the significance of empty waves in the light of recent results about position measurements which do not show Bohmian positions. I will conclude with discussion of the interpretation, advocating minimalistic approach according to which our experience supervene only on Bohmian positions (and not on the quantum

wave). I add a short discussion of Bedard's arguments against minimalistic interpretation in the appendix.

2. CAN AN EMPTY WAVE KILL?

Consider a gedanken experiment in which a bullet splits its quantum wave at a “beam splitter” into two equal weight wave packets, one moving toward a cat, while another misses the cat. For simplicity, we consider equal, uniform density, spherical wave packets. Consider a situation that the Bohmian position of the bullet is inside the wave packet that misses the cat, see Fig. 1. It was generally believed that in this situation we should not worry about cat's health. However, recently, there were several works showing that empty waves can have observable effects. Hardy⁽¹¹⁾ discussed empty waves in the interaction free measurements,⁽¹²⁾ Englert *et al.*⁽¹³⁾ discovered “Surrealistic Bohmian trajectories” in which an empty wave leaves a trace of its trajectory and Aharonov *et al.*^(14,15) showed that in protective measurements^(16,17) one can observe the shape of the quantum wave, while the Bohmian particle is essentially at rest and does not visit the regions where the value of the wave function is measured.

An empty wave can, in the future, reach the Bohmian position of the particle and cease to be “empty”. Clearly, at this stage it can lead to

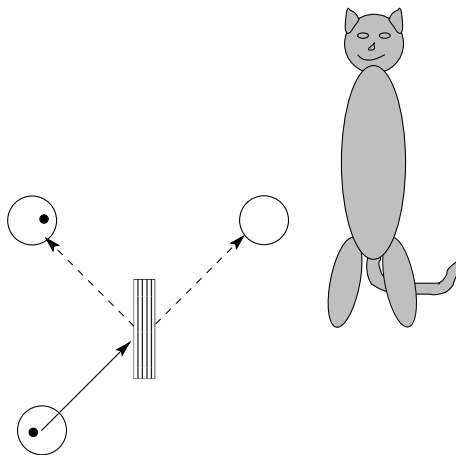


Fig. 1. A bullet is fired toward a cat, but after the beamsplitter only an empty wave of a bullet comes toward him. Bohmian particle position, signified by a black dot travels away with the other part of the quantum wave.

observable changes, i.e. to change Bohmian trajectory. Bell⁽⁶⁾ pointed out that usually it will be a dominant influence. Consider the two wave packets of the “bullet” which are forced to overlap again (as in a Mach-Zehnder interferometer without second beam-splitter), see Fig. 2. At the point of the overlap, the empty wave “grabs” the Bohmian particle. Indeed, this result can be immediately seen from Bell’s presentation of Bohm’s theory in the form of the pilot wave where the velocity of Bohmian position of a particle depends on the current density and the wave density at the location of the Bohmian particle:

$$\vec{v} = \frac{\vec{j}}{\rho}, \quad (1)$$

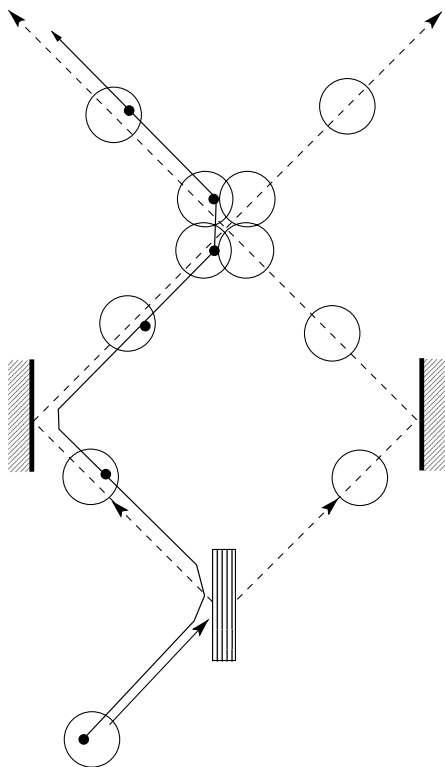


Fig. 2. Trajectory of a Bohmian particle in a Mach-Zehnder interferometer without second beamsplitter. At the meeting point of empty and nonempty wave packets, the Bohmian particle “changes hands” and continues to move with what was before an empty wave.

where $\rho(\vec{x}) = |\psi(\vec{x})|^2$, and $\vec{j} = \frac{\hbar}{2im} \{\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*\}$. The velocity in the region of the overlap of the two wave packets is given by

$$\vec{v} = \frac{\vec{j}_1 + \vec{j}_2}{\rho_1 + \rho_2} = \frac{\vec{v}_1 + \vec{v}_2}{2}, \quad (2)$$

where indexes “1” and “2” correspond to the two wave packets. The horizontal component of the velocity of the Bohmian particle vanishes during the time it is inside both wave packets. So, from the moment of the overlap starts the competition between the two wave packet: which one will keep the point inside it longer? When one wave packet leaves, the point continue to move with the velocity of the second wave packet and it remains inside it. Since at the beginning of the overlap, the Bohmian position is at the boundary of the empty wave packet and it is inside the other one, the empty wave has longer way to go and it always “wins” the competition: the empty wave “grabs” the Bohmian particle.

The situation is different if the empty wave bullet on its way “kills” an “empty wave” cat, see Fig. 3. Even if the physics is such that the bullet goes through the cat without significant delay and the empty wave packet of the bullet comes in time to overlap with the non-empty wave packet, the empty wave does not grab the Bohmian particle in this case. Indeed, the velocity of the Bohmian particle in the region of the overlap is

$$\vec{v} = \frac{\vec{j}_1}{\rho_1} = \vec{v}_1, \quad (3)$$

where \vec{v}_1 is the velocity of the non-empty wave packet. The reason is that at the time of the overlap of the wave packets of the bullet, the wave packets of some parts of the cat’s body do not overlap. The wave packets of these parts, entangled with an empty wave bullet, move relative to the case of the undisturbed cat. The Bohmian positions of the particles in the cat’s body are that of an uninjured cat, and therefore, the wave packets of some particles of the body entangled with the empty wave packet of the bullet do not contain the Bohmian particle inside it. In the configuration space of all involved particles (of the bullet and of the cat) there are two wave packets but the Bohmian position belongs only to one of them.

3. SURREALISTIC TRAJECTORIES

The examples presented above are not too surprising (although we are not used in Newtonian mechanics to change in velocity without

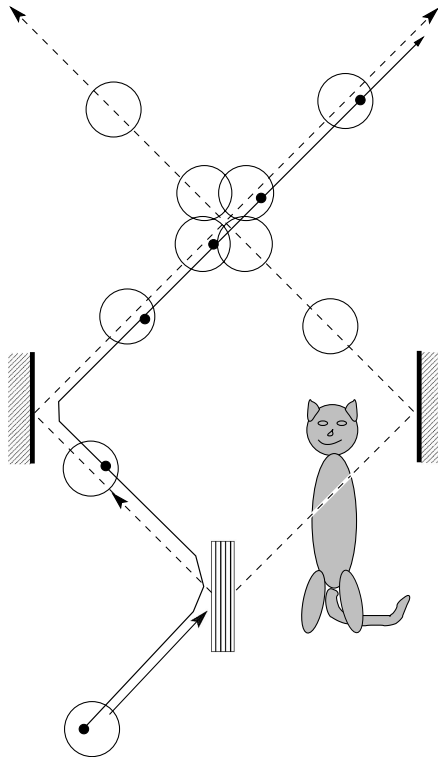


Fig. 3. Trajectory of Bohmian particle in a modified experiment. When the empty wave “kills” on its way, it does not influence the Bohmian trajectory in the future even if it overlaps with the non-empty wave as it was in Fig. 2.

interaction as in Fig. 2): the empty wave influences other objects only when it ceases to be an empty wave. A really surprising result was discovered by Englert *et al.*⁽¹³⁾ They realized that if (instead of killing a cat) the bullet will flip spins on its way, then the Bohmian trajectory will be as if the wave packets move in a free space, see Fig. 4. Nevertheless, the flipped spins show a different trajectory. There were many discussions regarding the meaning and significance of this example.^(18–22) When I first heard about this result, I did not believe it until I checked it myself.

My modification of this idea is to consider a very fast particle moving in a special bubble chamber in which the bubbles are developed slowly.

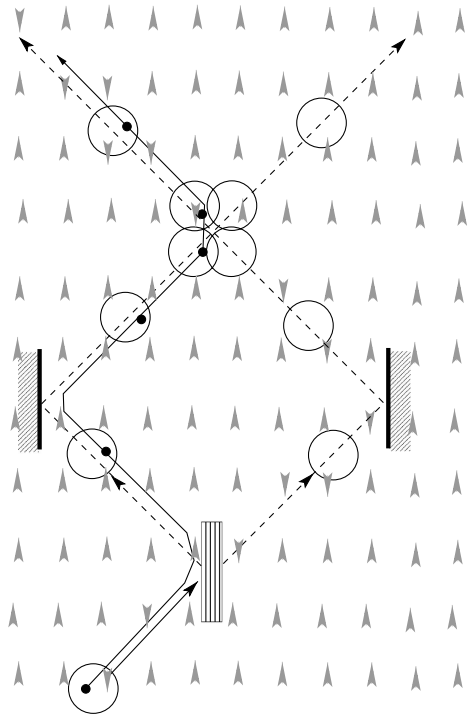


Fig. 4. Surrealistic trajectories. The trace of flipped spins and the actual trajectory of Bohmian particle are different. Which one is “surrealistic” is a matter of interpretation. (Note that flipped quantum waves of spins are only in one branch of the universal wave function, the branch corresponding to particular detection of the particle in the detector on the left.

During the time the particle moves inside the interferometer, the quantum states of electrons of excited atoms which later create the bubbles have no enough time to move out of the Bohmian positions of the electrons that are at rest at this time. The electron Bohmian positions are at rest because the excited states do not contribute to the Bohmian velocity when the Bohmian position of the wave packet of the particle moves in another place. The result of the experiment (which can be seen only much later) is a trace of bubbles corresponding to one trajectory while the trajectory of the Bohmian position is the other one, Fig. 5. The bubbles show the trajectory of the empty wave!

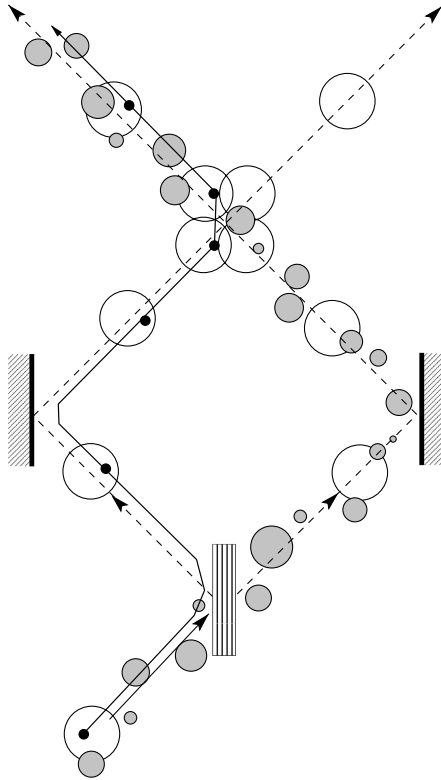


Fig. 5. The trace of (slow developing) bubbles shows trajectory which is different from the trajectory of the Bohmian particle. (Again, these are the bubbles of the branch of the wave function corresponding to a particular world). This is an example in which the “world” of the MWI is different from the “world” of Bohmian Mechanics: in the Bohmian world the particle moved in the left arm of the interferometer, while in the (postselected) MWI world it moved in the right arm.

4. PROTECTIVE MEASUREMENTS

Another situation in which position measurements do not show Bohmian positions are *weak measurements*^(5,23) and in particular, weak adiabatic measurements of position of a particle in nondegenerate energy eigenstates. Such measurements are called *protective measurements*.^(16,17,14,15) In protective

measurements we find, at the end, the wave function of the particle. In many energy eigenstates the Bohmian particle does not move, so it seems that the local values of the wave function obtained in the experiment arise without the Bohmian particle being at the vicinity of this location. However, it is not obvious that the measuring interaction in the process of the protective measurement does not move the Bohmian positions in such a way that the results of protective measurements could be explained as the time average of the presence of the Bohmian particle in a particular place. There have been an extensive analysis of this question and it has been shown that it is not the case, i.e. that the spacial profile of the wave function is obtained without the Bohmian particle being present in most of the regions of the non-vanishing wave function.

Consider a particle in a potential well, whose initial wave function is the ground state, Fig. 6. We assume that Bohmian position is in point A and we want to measure the density of the quantum wave at point B . If we introduce an adiabatic and weak perturbation of the potential which eventually goes to zero, we know that the wave function coincides at any moment with the ground state of the instantaneous Hamiltonian (we assume that the ground state is always nondegenerate). Our assumptions about the perturbation which is required for performing protective measurement of the density of the wave function at the vicinity of B ensure that the change in the wave function is small at all times and eventually vanishes. The lemma proved by Aharonov *et al.*⁽¹⁵⁾ tells us that the change in particle position is likewise small at all times. Thus, the perturbation of the potential at the vicinity of B due to the measurement will not change the Bohmian position significantly (which was originally at A) and will not bring it to B . So, for a Bohmian particle in a given position, we can probe the wave function in most other positions without the particle ever being present there.

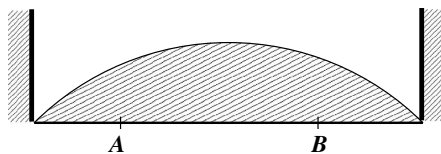


Fig. 6. Ground state of a particle in a one-dimensional box. We can measure the density of particle's quantum wave at the vicinity of B while the Bohmian position remains at the vicinity of A .

5. CONDITIONS FOR OBSERVABLE EFFECTS OF EMPTY WAVES

To summarize above examples let us state clearly the conditions at which empty waves cause an observable effect. There are three conditions:

- (i) Counterfactually, the wave should cause an observable effect if at the particular time the Bohmian particle was inside it (i.e. the wave packet was a non-empty wave). The meaning of “observable effect” is that some other system changes significantly its quantum state.
- (ii) At the time of the observation of the effect, the Bohmian particle should be inside the wave. (At this later time the wave is not in the interaction region, so the direct effect is absent, and we still can consider it as an effect caused by an empty wave).
- (iii) The change of the quantum waves of other objects (the “observable effect” of (i)) should be such that the spacial densities of their quantum waves are not changed significantly: they should not leave the locations of the Bohmian positions of the undisturbed objects.

Trivially, (i) takes place in all our examples: in Surrealistic trajectories spins are flipped, in the bubble chamber experiment the bubbles leave an observable trace and in the protective measurement, the pointer of the measuring device changes its state.

Figures 4 and 5 show that condition (ii) is satisfied both in the spin and in the bubble chamber experiments. In both cases, at the end of the experiment, the Bohmian particles are inside the wave packet which was an empty wave packet before.

In the experiment with spins, the spacial wave function of the spin particles remains without any change, i.e. (iii) is fulfilled. In the bubble chamber experiment there is some change in the spacial wave function of the particle, but it is insignificant. Indeed, (iii) is fulfilled due to the condition of the fast moving particle and slow developing bubbles.

In the analysis of protective measurement, there is a difficulty with defining “empty” and “non-empty” wave packets. We have to divide the quantum wave of the particle into two parts: one includes point *A*, the location of the Bohmian particle, and another includes point *B* where the measurement is performed. The problem is that, while the total wave function is essentially constant, the wave packet which is the part of the complete wave evolves in a nontrivial way. In particular, whatever part including *B* we take, it will very soon evolve and reach *A*, i.e. it will cease to be an empty wave. This explains how (ii) is fulfilled in protective

measurements. The basic property of weak measurements is that the position of the pointer of the measuring device has large quantum uncertainty (it is necessary for having small value of the conjugate momentum which appears in the interaction Hamiltonian). Thus, protective measurements fulfill property (iii).

So, can an empty wave of a bullet kill? The answer is that only a very special bullet can do this. First, it should later reach the location of its Bohmian position. It sounds as a difficult, but not impossible task. Second, it should not cause immediate change of Bohmian positions of particles in the cat's body, i.e. until the bullet reaches its Bohmian position. This tells us that the bullet cannot be a usual bullet, which makes holes immediately after it passes through the body. One might imagine that the bullet is just a single very fast particle. But then property (i) can hardly be satisfied. A single particle passing through a body does not kill.

6. INTERPRETATION

In Surrealistic Bohm trajectories⁽¹³⁾ as well as in the other examples described above, a seemingly correct experiment shows one trajectory, while calculations yield that Bohmian trajectory is different. Nevertheless, I do not see a direct contradiction with the minimalistic approach to Bohmian theory in which our experience supervene solely on Bohmian position. I believe that a Bohmian proponent has a good defense in the following argument: conceptually, in the framework of the Bohmian theory these experiments are *not* good verification measurements. Prediction of Bohmian theory for the motion of the particle is a vector function of time $\vec{r}(t)$. To test it we have to test the location of the particle at different times. Since "reality" corresponds only to Bohmian positions, we have to read the locations using Bohmian positions of the measuring device at that time. In all our surprising examples Bohmian particles of measuring devices moved only much later, not at the time in which the particle position was observed. When the Bohmian position of the measuring device was measured at the same time (as in the example presented in Fig. 2), no surprising behavior was observed. So, the Bohmian picture in which our experience supervene on Bohmian positions is consistent. There are no experiments in which a "good" measurement of position (a measurement that records the position of a particle at a particular time using Bohmian positions of the measuring devices), shows results which are inconsistent with calculated Bohmian positions.

Still, these surprising examples make Bohmian approach less attractive: We see that there are important causal structures which cannot be explained

using Bohmian positions alone, without explicit description of the quantum wave. For me it adds to the objections of the proponent of the MWI to the Bohmian approach. It leaves in the formalism the structure of all parallel worlds, but claims that they are not related to our experience. But in these empty worlds the wave in the shape of Lev Vaidman might also write a paper in the empty wave copy of the *Foundations of Physics Journal*, so how You, the reader, know that this is not such an empty wave world?

7. APPENDIX: BEDARD'S ARGUMENTS

The abstract of Bedard's paper is:

According to the traditional presentation of Bohm's interpretation we have immediate epistemic access to particle properties but not wave function properties, and mental states, pointer states, and ink patterns supervene on particle properties alone. I argue that these claims do not make physical sense, and I offer an alternative account that does.

What I accept or postulate (in the framework of my understanding of Bohmian mechanics) is that mental states supervene on particle properties alone. My motivation is not to get "classicality" as Bedard suggests: the experiments show that Nature does not follow the laws of classical physics, so there is no reason to put physics into a classical picture. My reason to turn to Bohm is to find a way of seeing a single world corresponding to the formalism of the physical theory of the universe, since I see that many physicists and philosophers have considerable difficulty with accepting numerous parallel worlds which we do not observe directly.

Bedard's arguments have already been criticized by Dickson.⁽²⁴⁾ As far as I can understand his philosophical jargon my refutation of Bedard is similar, but I believe it will be helpful to write here my arguments too.

Bedard's objection is that Bohmian positions at a single moment and without additional information of the properties of the particles are not enough to describe the reality. This is a correct statement, but Bohmians do not claim the opposite. The task of Bohmian (as well as any other) interpretation is to find correspondence between the mathematical formalism of the physical theory and our experience. Since conscious experience requires some period of time, we *have* to consider trajectories at some period of time and not just an instantaneous configuration for describing (defining) objects. Thus, an object made out of electrons only, in a configuration of a (real) cat made of electrons, protons, neutrons, etc., will cease to have the configuration of a cat long time before it can be perceived as a cat. The configuration of Bohmian particles have the shape of a cat for a considerable time if, and only if, they related to the right

kind of particles and they have appropriate quantum wave. It is possible to imagine Universe with different physical interactions in which my last statement is not true. But for physical interaction we have in our Universe it is true. Philosophical arguments of Dickson tell us that the situation in our Universe is relevant. Except for some very specific situations which are difficult to arrange and which probably were never arranged in real laboratories, everything we see or perceive in some other way is described correctly by trajectories of Bohmian particles.

Bedard claims that there are problems also with color and television screen pictures. I do not think that it is so: I expect no conceptual problem with defining Bohmian positions for photons. However, I can also avoid this discussion using the research of perception by our brains made by Aicardi *et al.*⁽²⁵⁾ in order to answer the criticism of Albert and myself⁽²⁶⁾ of a Ghirardi, Rimini and Weber collapse proposal.⁽²⁷⁾ We pointed out that in a Stern-Gerlach experiment in which the particle with the spin hits a fluorescent screen, the GRW collapse might not take place until the light from the screen comes to our eyes in spite of the fact that macroscopic number of atoms become excited in this process. Aicardi *et al.*⁽²⁵⁾ answered that inside the brain, in the process of perception, numerous cells move macroscopic distance depending on what we see, so at least inside the brain one can find the shape of Bohmian particles corresponding to what we have seen. Thus, it is feasible that our mental states supervene on particle positions alone.

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