Instantaneous Nonlocal Measurements

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ABSTRACT

It is shown, under the assumption of possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously. The measurement is based on teleportation method. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

Keywords: Quantum measurements, nonlocality

1. INTRODUCTION

Seventy years ago Landau and Peierls[1] claimed that the instantaneous measurability of nonlocal variables (i.e., variables which related to more than one small region of space) contradicts relativistic causality. Twenty years ago, Aharonov and Albert[2] showed that some nonlocal variables (e.g., the Bell operator, see below) can be measured instantaneously and that this does not contradict causality. They also showed explicitly how the possibility of performing instantaneous von Neumann measurements of some other nonlocal variables does contradict causality. The question: "What are the *observables* of relativistic quantum theory?" remains topical even today.[3]

A variable can obtain the status of an observable if it can be measured. However, the standard (von Neumann) definition of quantum measurement is too restrictive for defining a physical observable: the von Neumann definition requires that a quantum system, which was before the measurement in an eigenstate of the measured variable, does not change its state due to the measurement process. The existence of a *verification* measurement which yields the eigenvalue of a variable with certainty, if prior to the measurement the quantum system was in the corresponding eigenstate, is enough for giving the status of an observable for such a variable even if the state of the system is distorted due to the measurement procedure. (If, initially, the system is in a superposition or mixture of the eigenstates of the observable, then the verification measurement yields one of the corresponding eigenvalues according to the quantum probability law.)

The meaning of "instantaneous measurement" is that in a particular Lorentz frame, at time t, we perform local actions for a duration of time which can be as short as we wish. At the end of the procedure (arbitrary small period of time after t) there are local records which together yield the outcome of the measurement of the nonlocal variable. Although at this time there is no observer who knows the outcome of the measurement, since bringing all local records to one place requires a finite time, we consider the measurement as being completed. At the end of the measurement interactions, the information about the outcome of the measurement is classically recorded in the outcomes of local (irreversible) measurements. The fact that the records are not yet brought together does not mean that the measurement is not completed.

Note the difference with the case of *exchange measurements*[4] which can also be performed for all nonlocal variables. In an exchange measurement local operations of swapping lead to swapping between the quantum state of the composite system and the quantum state of the local separated parts of the measuring device. Although in the exchange measurement the information about the quantum state of the system is instantaneously transmitted to the measuring device, one should not say the the measurement is completed. Indeed, the outcome of the measurement is not written in the form of classical information and, in fact, the outcome of the quantum

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measurement does not exist yet: the exchange measurement can be reversed and the system can be brought back to its original (in general unknown) state.

In this paper, I will show that apart from variables related to the spread-out fermionic wave function, *all* nonlocal variables have the status of observables in the framework of relativistic quantum mechanics, i.e., all variables related to two or more separate sites are measurable instantaneously using verification measurements. This includes variables with entangled eigenstates and nonlocal variables with product eigenstates.[5]

Verification measurements have been considered before. It has been shown[6] that verification measurements of some nonlocal variables erase local information and, therefore, cannot be ideal von Neumann measurements. Recently, Groisman and Reznik[7] showed that there are instantaneous verification measurements for all spin variables of a system of two separated spin-$\frac{1}{2}$ particles. I will first explain how to perform verification measurement of such a variable using another method and then generalize it to an arbitrary nonlocal variable.

2. AN EXAMPLE OF TWO SPIN-$\frac{1}{2}$ PARTICLES

The method I present here uses teleportation technique.[8] I will start explaining it by describing the measurement of a nonlocal variable of two spin-$\frac{1}{2}$ particles located in separate locations A and B, whose eigenstates are the following product states:

$$\begin{aligned} |\Psi_1\rangle &= |\uparrow_z\rangle_A \quad |\uparrow_z\rangle_B, \\ |\Psi_2\rangle &= |\uparrow_z\rangle_A \quad |\downarrow_z\rangle_B, \\ |\Psi_3\rangle &= |\downarrow_z\rangle_A \quad |\uparrow_\theta\rangle_B, \\ |\Psi_4\rangle &= |\downarrow_z\rangle_A \quad |\downarrow_\theta\rangle_B, \end{aligned} \tag{1}$$

where $|\uparrow_\theta\rangle$ is an eigenstate of a spin pointing in a direction $\hat{\theta}$ making angle θ with the z axis.

Note that an instantaneous ideal von Neumann measurement of such a variable does contradict causality. Assume that at time t such an ideal measurement is performed. Then we can send superluminal signal from A to B in the following way. We prepare in advance the system in the state $|\Psi_1\rangle$ and agree that Bob at site B measures the spin z component of his particle shortly after time t. Now, in order to send a superluminal signal, Alice at site A can at a very short time before time t flip her spin. If she does so, then after the nonlocal measurement at time t, the system will end up either in state $|\Psi_3\rangle$ on in state $|\Psi_4\rangle$. In both cases Bob has a nonvanishing probability to find his spin "down", while this probability is zero if Alice decides not to flip her spin.

The first step of the verification measurement is the teleportation of the state of the spin from B (Bob's site) to A (Alice's site). Bob and Alice do not perform the full teleportation (which invariably requires a finite period of time), but only the Bell measurement at Bob's site which might last, in principle, as short a time as we wish. (I will continue to use the term "teleportation" just for this first step of the original proposal.[8]) In the teleportation procedure for a spin-$\frac{1}{2}$ particle we start with a prearranged EPR pair of spin-$\frac{1}{2}$ particles one of which is located at Bob's site and another at Alice's site, $|\Psi_-\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$. The procedure is based on the identity

$$\begin{aligned} |\Psi\rangle_1|\Psi_-\rangle_{2,3} = \frac{1}{2}(&|\Psi_-\rangle_{1,2}|\Psi\rangle_3 + |\Psi_+\rangle_{1,2}|\tilde{\Psi}^{(z)}\rangle_3 + \\ &|\Phi_-\rangle_{1,2}|\tilde{\Psi}^{(x)}\rangle_3 + |\Phi_+\rangle_{1,2}|\tilde{\Psi}^{(y)}\rangle_3), \end{aligned} \tag{2}$$

where $|\Psi_\mp\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle \mp |\downarrow\rangle|\uparrow\rangle)$, $|\Phi_\mp\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\uparrow\rangle \mp |\downarrow\rangle|\downarrow\rangle)$ are eigenstates of the Bell operator and $|\tilde{\Psi}^{(z)}\rangle$ signifies the state $|\Psi\rangle$ rotated by π around $\hat{z}$ axis, etc. Thus, the Bell operator measurement performed on the two particles in Bob's site "collapses" (or effectively collapses) to one of the branches of the superposition, the RHS of (2), and, therefore, teleports the state $|\Psi\rangle$ of Bob's particle to Alice except for a possible rotation by π (known to Bob) around one of the axes of teleportation: $\hat{x}_1$, $\hat{x}_2$, or $\hat{x}_3$. Bob choses the axes of teleportation (which define the eigenstates of the Bell measurement) in the following way: $\hat{x}_3 = \hat{z}$ and $\hat{x}_1$ is such that $\hat{\theta}$ lies in the plane of $\hat{x}_3$ and $\hat{x}_1$, see Fig. 1.

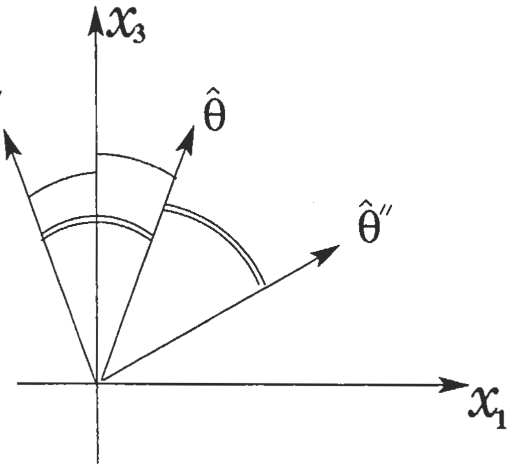

Figure 1. The axes of teleportations. Bob starts with teleportation choosing $\hat{x}_3$ and $\hat{x}_1$ such that $\hat{\theta}$ lies in the plane defined by the axes. Alice teleports back with $\hat{x}_3' = \hat{\theta}$. Bob continues with $\hat{x}_3'' = \hat{\theta}'$, etc.

The second step is taken by Alice. She can perform it at time t without waiting for Bob. She measures the spin of her particle in the z direction.

If the result is "up", she measures the spin of the particle teleported from Bob in the z direction and this completes the measurement. Indeed, the eigenstates of the spin in the z direction are teleported without leaving the z line. Thus, Bob's knowledge about possible flip together with Alice's results distinguish unambiguously between Ψ_1 and Ψ_2.

If the result is "down", Alice cannot perform a measurement on Bob's teleported particle because it has spin either along the line of $\hat{\theta}$ (corresponding to teleportation without rotation or rotation around $\hat{x}_2$) or along the line of $\hat{\theta}'$ obtained from the line of $\hat{\theta}$ by π rotation around $\hat{x}_1$ (or $\hat{x}_3$). In this case, Alice teleports Bob's teleported state back to Bob using a new teleportation axes defined by $\hat{x}_3' = \hat{\theta}$ and $\hat{x}_2' = \hat{x}_2$.

In the third step, Bob performs an action similar to that of Alice in step 2. He knows whether the spin state in θ direction was teleported to Alice along the $\hat{\theta}$ line or along the $\hat{\theta}'$ line. In the former case, the state teleported to him is still along the $\hat{\theta}$ line, so he completes the procedure by spin measurement in this direction. In the latter case, he receives the spin either along the $\hat{\theta}'$ line or along the $\hat{\theta}''$ line obtained by π rotation around the $\hat{\theta}$ axis. In this case, he teleports the particle back with the teleportation axes $\hat{x}_3'' = \hat{\theta}'$ and $\hat{x}_2'' = \hat{x}_2$.

Alice and Bob continue this procedure. If $\theta = \frac{k}{2^n}\pi$, the line $\hat{\theta}^{(n)}$ coincides with the line $\hat{\theta}^{(n-1)}$ because the angle between the lines is: $\theta^{(n)} - \theta^{(n-1)} = (2^n\theta)\text{mod}\pi$. In this particular case the process is guaranteed to stop after n teleportation steps. If the lines do not coincide, the probability that after n teleportations the result of the measurement is not known is 2^{-n}, so the probability of success can be made as large as we wish. There is no minimal time for performing all the steps of this procedure. Bob and Alice need not wait for each other: they only have to specify before the measurement the teleportation channels they will use. Note, that usually Alice and Bob will use only a small number of teleportation channels: they stop when both Alice and Bob make teleportations which do not change the line of the spin. Thus, this method requires less resources than the alternative approach.[7]

3. GENERAL CASE OF A VARIABLE OF A TWO-PARTS SYSTEM

The method I presented above can be modified for measurement of other nonlocal variables of two spin-$\frac{1}{2}$ particles. However, I will turn now to another, universal, method which is applicable to any nonlocal variable $O(q_A, q_B, ...)$, where q_A belongs to region A, etc. I will not try to optimize the method or consider any realistic proposal: my task is to show that, given unlimited resources of entanglement and arbitrary local interactions, any nonlocal variable is measurable.

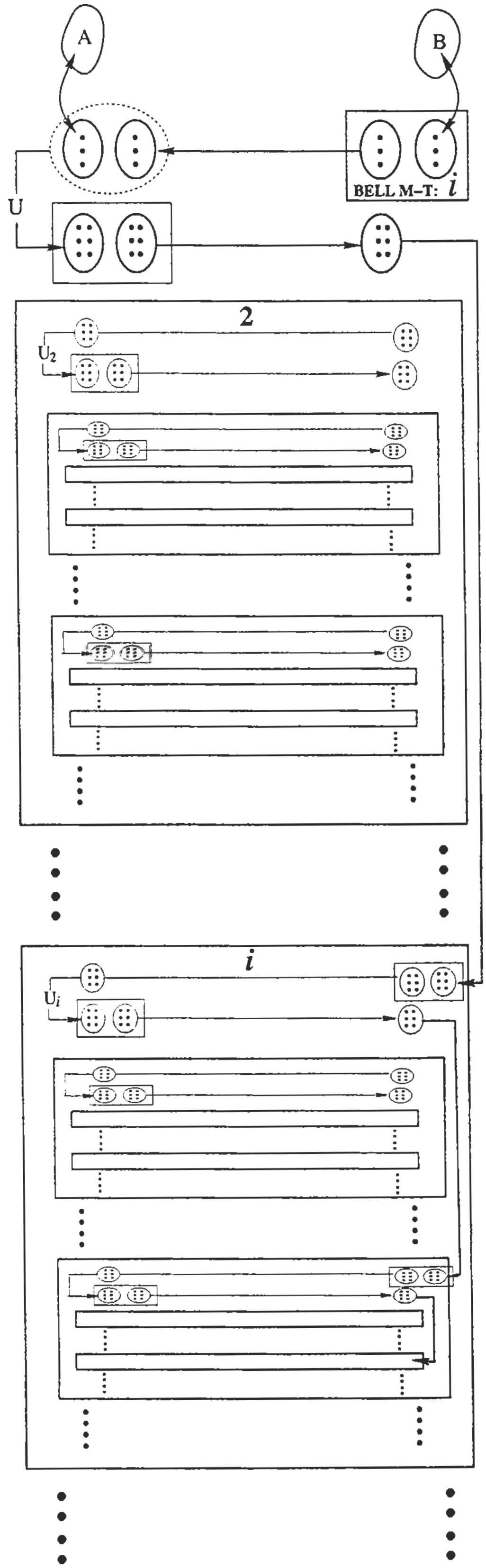

Figure 2. The scheme of the measurement of a nonlocal variable of a two-part system.

I will start with the case of a general variable of a composite systems with two parts. First, (for simplicity), Alice and Bob perform unitary operations which swap the states of their systems with the states of sets of spin-$\frac{1}{2}$ particles. In this way Alice and Bob will need the teleportation procedure for spin-$\frac{1}{2}$ particles only. Teleporting the states of all individual spins teleports the state of the set, be it entangled or not.

The general protocol is illustrated in Fig. 2. Bob teleports his system to Alice. Again, he does not send to Alice the results of his Bell measurements, but keeps them for his further actions; we signify the possible outcomes of these measurements by $i = 1, ...N$. The outcome $i = 1$ corresponds to finding singlets in all Bell measurements and in this case the state of Bob's system is teleported without distortion.

Alice performs a unitary operation on the composite system of her spins and the teleported spins which, under the assumption of non-distorted teleportation, transforms the eigenstates of the nonlocal variable (which now actually are fully located in Alice's site) to product states in which each spin is either "up" or "down" along the z direction. Then she teleports the *complete* composite system consisting of her spins and Bob's teleported spins to Bob. From now on this is the system which will be teleported back and forth between Bob and Alice. In all these teleportations the usual z, x, y basis is used. Hence, if the state is in the one of the product states in spin z basis, then it will remain in this basis.

If, indeed, Bob happened to teleport his spins without any distortion, i.e., $i = 1$ (the probability for which is $\frac{1}{N}$), Bob gets the composite system in one of the spin z product states and his measurements in the spin z basis that he now performs, complete the measurement of the nonlocal variable. If $i \neq 1$, Alice's operation does not bring the eigenstates of the nonlocal variable to the spin z basis, so Bob cannot perform the measurement and he teleports the system back to Alice following a protocol that we explain below.

Alice and Bob have numerous teleportation channels arranged in $N-1$ clusters numbered from 2 to N. Each cluster consists of two teleportation channels capable to teleport the complete system and $M - 1$ clusters of a similar type, where M is the number of possible outcomes of the Bell measurement for teleportation of the complete system. In turn, each of the $M - 1$ clusters consists of two teleportation channels and $M - 1$ further nested clusters, etc.

If in his first teleportation the result of Bob's Bell measurements is i, he teleports now the composite system back to Alice in the teleportation channel of cluster i. Alice does not know in which channel she gets the system back (if she gets it back at all). So she must work on all of them. She knows that if she does get the system in channel i, the result of the Bell measurement in Bob's first teleportation was i. Thus, she knows all the transformations performed on this system except for the last teleportation. Alice performs a unitary operation that transforms eigenstates of the nonlocal variable to product states under the assumption that the last teleportation was without distortion and teleports the system back to Bob.

Let us denote the result of the Bell measurement in Bob's last teleportation by i', $i' = 1, ..., M$. Again, for $i' = 1$ which corresponds to finding singlets for all Bell measurements, Bob performs the spin z measurement on the system which he receives in the teleportation channel of the cluster i. This then completes the nonlocal measurement. Otherwise, he teleports the system back in the channel of the sub-cluster i'. Alice and Bob continue this procedure. The nonlocal measurement is completed when, for the first time, Bob performs a teleportation without distortion. Since, conceptually, there is no limitation for the number of steps, and each step (starting form the second) has the same probability for success, $\frac{1}{M}$, the measurement of the nonlocal variable can be performed with probability arbitrarily close to 1.

4. CONCLUSIONS

The generalization to a system with more than two parts is more or less straightforward. Let us sketch it for three-part system. First, Bob and Carol teleport their parts to Alice. Alice performs a unitary transformation which, under the assumption of undisturbed teleportation of both Bob and Carol, transforms the eigenstates of the nonlocal variable to product states in the spin z basis. Then she teleports the complete system to Bob. Bob teleports it to Carol in a particular channel i_B depending on the results of the Bell measurement of his first teleportation. Carol teleports all the systems from the teleportation channels from Bob back to Alice in the channels (i_B, i_C) depending on her Bell measurement result i_C. The system corresponding to $(i_B, i_C) = (1, 1)$ is not teleported, but measured by Carol in spin z basis. Alice knows the transformation performed on the system which arrives in her channels (i_B, i_C) except for corrections due to the last teleportations of Bob and Carol. So she again assumes that there were no distortion in those, and teleports the system back to Bob after the unitary operation which transforms the eigenstates of the variable to product states in the spin z basis. Alice, Bob and Carol continue the procedure until the desired probability of successful measurement is achieved.

The required resources, such as the number of teleportation channels and required number of operations are very large, but this does not concern us here. We have shown that there are no relativistic constraints preventing instantaneous measurement of any variable of a quantum system with spatially separated parts, answering the above long standing question. In the thriving field of quantum communication this question is relevant for quantum cryptography and quantum computation performed with distributed systems.

Can this result be generalized to a quantum system which itself is in a superposition of being in different places? The key to this question is the generality of the assumption of the possibility to perform any local operation. If a quantum state of a particle which is in a nonlocal superposition can be locally transformed to (an entangled) state of local quantum systems, then any variable of the particle is measurable through the measurement of the corresponding composite system. However, while for bosons it is clear that there are such local operations (transformation of photon state to entangled state of atoms has been achieved in the laboratory[9]), for fermion states the situation is different.[10] If the transformation of a superposition of a fermion state to local variables is possible, then these local separated in space variables should fulfill anti-commutation relations. This is the reason to expect super-selection rules which prevent such transformations.

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